

MATHEMATICAL FOUNDATIONS OF TIME SERIES ANALYSIS

THEORY AND CASES

Carlos Polanco

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Mathematical Foundations of Time Series Analysis

Theory and Cases

Authored by

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MATHEMATICAL FOUNDATIONS OF TIME SERIES ANALYSIS

Theory and Cases

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Without Big Data, you are blind and
deaf and in the middle of a freeway.

– Geoffrey Moore

1946 –

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FOREWORD

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Dr. Carlos Polanco's book provides a concise examination of the fundamental topics addressed in "MATHEMATICAL FOUNDATIONS OF TIME SERIES ANALYSIS: THEORY AND CASES." The author effectively clarifies the fundamental principles of time series analysis and illustrates their pragmatic implementation in diverse scientific fields via perceptive case studies. This piece of work serves as evidence of Dr. Polanco's ability and capacity to simplify the complex realm of time series for a wide readership, effectively connecting theoretical concepts with practical implementations. This book is certain to prove to be an indispensable resource for readers as they delve into the realm of time series analysis.

Vladimir N. Uversky

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FOREWORD

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Dr. Carlos Polanco's book offers a clear and accessible overview of the mathematical concepts presented in MATHEMATICAL FOUNDATIONS OF TIME SERIES ANALYSIS: THEORY AND CASES. Through straightforward explanations and relevant case studies, the author guides readers in understanding the essential principles of time series analysis and its practical applications across various scientific fields.

Whether you are new to the subject or already familiar with time series analysis, this book serves as a valuable resource. Dr. Polanco's engagement with the topic creates an educational environment that is both informative and practical.

This book provides a reliable reference to help you navigate the complexities of time series analysis and apply it effectively in your studies or professional work. It offers the tools and insights needed for a practical understanding of the core principles that drive this field.

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PREFACE

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This book is intended to provide a rigorous introduction to the mathematical foundations of the Time series analysis. It is designed for readers who are either new to the field or possess preliminary knowledge and seek to enhance their understanding of essential concepts and analytical methodologies within the discipline.

The material is structured systematically to offer a comprehensive overview of fundamental topics, including classical mechanics, thermodynamics, electrodynamics, and quantum mechanics. Each theoretical framework is presented with precise mathematical foundations of the time series analysis and is accompanied by an explanation of the physical laws that govern each domain.

The text emphasizes mathematical accuracy and conceptual clarity. Each chapter contains a selection of carefully designed exercises and problems aimed at facilitating the application of theoretical concepts and fostering the development of analytical skills essential for a thorough engagement with the subject matter. Readers are encouraged to approach these exercises with diligence, as they provide important practice in mastering the theoretical constructs presented.

The chapters are organized to build sequentially upon prior knowledge, allowing for a gradual progression in both complexity and depth of the content. Summaries at the end of each chapter encapsulate the central concepts and serve as a concise reference for reviewing key ideas.

This work aspires to function not only as an academic resource but also as a source of intellectual engagement for those pursuing further study in physics. Readers are encouraged to critically examine the material, to consider its implications, and to cultivate an inquisitive approach toward the fundamental structure of the universe.

The author would like to acknowledge the Faculty of Sciences at Universidad Nacional Autónoma de México and Instituto Nacional de Cardiología for providing cases and examples.

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List of Symbols

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Symbol	Description	Page
$f(x) : \mathbb{R} \rightarrow \mathbb{R}$	Real-Valued Function	3
$f(t) : \mathbb{R} \rightarrow \mathbb{R}$	Time Series Function	3
Domain (f)	Domain of the Function	5
Graph (f)	Graph of the Function	5
Range (f)	Range of the Function	5
IQR	Interquartile Range	8
$Q_1 \cdots Q_4$	Quartiles	8
$T(t)$	Trend Component	12
T_{mean}	Mean Estimation	13
$\sum_{i=1}^n (T(t) - T_{mean})^2$	Unexplained Variation	13
$S(t)$	Seasonal Component	14
$C(t)$	Cyclical Component	16
$I(t)$	Irregular Component	17
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$N(t)$	Noise Component	35
$\frac{T_L(t) \times (C(t))}{I(t)}$	Prediction Form	38
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$f : \mathbb{R}^n \rightarrow \mathbb{R}$	Multivariate Real-Valued Function	49
Graph(f) _{$x(t)$}	Graph of Restricted Function	50
r	Correlation Coefficient	52

I

BASIC CONCEPTS

The first part of this book introduces the reader to the concepts, properties, and main components of a time series $Y(t)$. It defines each of these components in detail, analytically and geometrically, isolating them from the time series. We introduce the Least squares method to approximate the analytical function and utilize visualization techniques to enhance our understanding of the studied phenomenon. This section includes the study of analytical functions in the plane and in space using the multidimensional panel data technique. It has been ensured that the section is self-contained so that the unfamiliar reader can follow the topic.

CHAPTER 1

Time Series

Abstract: This chapter reviews in detail the definition and components of a real-valued function $f(x)$. A Time Series Function $f(t)$ is defined later with its components and limitations. We also review the quality of the data points in a Time Series Graph. Finally, we introduce the Trend components $T(t)$, Cyclical components $C(t)$, Seasonal components $S(t)$, and Irregular components $I(t)$, as elements of a Time Series Graph.

Keywords: Cyclical component, irregular component, least squares method, seasonal component, trend component

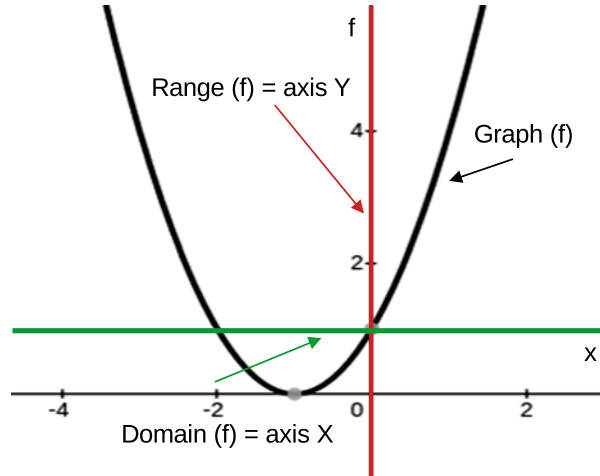
1.1. Introduction

In this chapter, we will review the concept of Time Series function $f(t)$, which is a real-valued functions $f(x)$ whose independent variable is time t . We will identify the *domain*(f), *image*(f), and *graph* (f) and we will discriminate the Discrete and Continuous Time Series Graphs. We will study different types of graphs with some examples to define the four components: Trend component $T(t)$, Cyclical component $C(t)$, Seasonal component $S(t)$, and Irregular component $I(t)$ as the elements of a Time Series function $f(t)$. Finally, we will study several techniques to ensure that the data points are reliable and reflect the phenomena studied.

1.2. Real-Valued Functions

Definition 1.1. A function f , is a **well-defined rule** that relates two sets $f : A \rightarrow B$, where each element in set A corresponds to a **unique** element in set B .

A **real-valued function** $f(x)$ is particularly a numerical function i.e., $f : \mathbb{R} \rightarrow \mathbb{R}$, where its independent variable $x \in \text{domain}(f) \subset \mathbb{R}$ and its dependent variable $f(x) \in \text{image}(f) \subset \mathbb{R}$. The graph of the real-valued function f is $\text{graph}(f) = \{(x, f(x)) \in \mathbb{R}^2 \mid x \in \text{domain}(f) \text{ and } f(x) \in \text{image}(f)\}$ (Fig. 1.1).

Figure 1.1: Graph(f) and its components.

1.3. Time Series Functions

Definition 1.2. A **Time Series function** is a real-valued function $f(t)$ (Def. 1.1) whose independent variable $t \in$ the domain (f) and represents the time. $Graph(f)$ (Def. 1.1) is named **Time Series graph** i.e., $graph(f)$ shows the behavior of the phenomenon over a long period i.e., $domain(f)$.

Note: 1.1. In the Time Series Analysis, we will use the nomenclature above mentioned for the succession of the data points in $image(f)$ generated by the Time Series function f , the geometrical representation of $graph(f)$, the data points evaluated by the Time Series function f , or the Time Series function f itself (Def. 1.1), to avoid confusion.

The procedure to know the behavior of a Time Series function f is to break it into sums or products of the real-valued functions $f_i(t)$, such that function f is interpreted as sums (Eq. 1.1) or products (Eq. 1.2).

$$f(t) = f_1(t) + \cdots + f_n(t) \quad (1.1)$$

$$f(t) = f_1(t) \times \cdots \times f_n(t) \quad (1.2)$$

The transformation of a Time Series function $f(t)$ into components of a sum or product $f_i(t)$ enables the elimination of some terms $f_i(t)$, so the behavior of the remaining functions can be easily identified.

Let the Time Series function $f(t) = f_1(t) + \cdots + f_n(t)$ (Eq. 1.1); if you subtract from both sides of Eq. 1.1 the term $f_1(t)$, you will get Eq. 1.3. Note that if you graph the function (Eq. 1.3) in the Y axis, you will get the term $f(t) - f_1(t)$ and the

graph will correspond to the Time Series function $f_2(t) + \dots + f_n(t)$, which **is not** $f(t)$.

$$f(t) - f_1(t) = f_2(t) + \dots + f_n(t) \quad (1.3)$$

Now consider the Time Series function $f(t) = f_1(t) \times \dots \times f_n(t)$ (Eq. 1.2), if you divide both sides of Eq. 1.2 by the term $f_1(t)$, you will get Eq. 1.4. Note that if you graph the function (Eq. 1.4) in the Y axis, you will get the term $\frac{f(t)}{f_1(t)}$ and the graph will correspond to the Time Series function $f_2(t) + \dots + f_n(t)$, which **is not** $f(t)$.

$$\frac{f(t)}{f_1(t)} = f_2(t) + \dots + f_n(t) \quad (1.4)$$

We will use these two procedures in the following sections.

1.3.1. Limitations

The following assumptions are associated with the analysis of a Time Series, it is advisable to carefully review them before using this method.

1. Patterns or regularities observed in the past are supposed to continue in the future.

Note Special care should be taken in the size of the chosen $domain(f)$, so the repetition of the regularities in the Time Series $graph(f)$ can be observed.

2. If $domain(f)$ is divided into intervals i.e., discretize the time interval, these intervals have to be evenly distributed throughout the $domain(f)$ of the Time Series function f .

Note This can be achieved if the $domain(f)$ of the Time Series function f is divided into equal intervals.

3. There should not be “lost data” sets for the Time Series function f .

Note If this condition is not met, all the results of the Time Series function are compromised, so before using this procedure it must be verified that the set of data is uniform and without “lost data”.

4. Time Series analysis **should not** be used to predict events of random nature e.g., terrorist attacks, Tsunami disasters, or any other disaster, among others.

Note These types of events do not have regularity; therefore, they should not be studied with a Time Series Analysis.

1.4. Time Series Graphs

A $graph(f)$ of a Time Series function f (Fig. 1.2) is mainly used to study the behavior of a phenomenon. In this graph, you can see a Time Series Graph (Fig. 1.2) that can be considered **discrete**, since its $domain(f)$ is discrete.

CHAPTER 2

Time Series Analysis

Abstract: In this chapter, we review the decomposition of a Time Series function $f(t)$ from the exclusion of the Trend component $T(t)$, Cyclical component $C(t)$, Seasonal component $S(t)$, and Irregular component $I(t)$ with the quotient and subtraction methods. We also deepen the Time Series Analysis from the exclusion in a different order, according to the nature of the phenomenon studied.

Keywords: Cyclical component, cyclical decomposition, irregular decomposition, least squares method, linear squares method, nonlinear squares method, seasonal decomposition, time series analysis, time series graph, trend decomposition

2.1. Introduction

In this chapter, we will interpret a Time Series function $f(t)$ as the sum or product of the Trend component $T(t)$, Seasonal component $S(t)$, Cyclical component $C(t)$, and Irregular component $I(t)$ introduced in Chapt. 1.

Thus, the Time Series function $f(t)$ will be treated as the sum of the functions (Eq. 2.1) or the product of the functions (Eq. 2.2).

$$f(t) = T(t) + C(t) + S(t) + I(t) \quad (2.1)$$

$$f(t) = T(t) \times C(t) \times S(t) \times I(t) \quad (2.2)$$

A function $f(t)$ product type (Eq. 2.2) can be transformed into a sum function (Eq. 2.1) with the logarithmic property $g(t)h(t) \rightarrow \log g(t) + \log h(t)$ in both sides of Eq. 2.2 as Eq. 2.3.

$$\begin{aligned} f(t) &= T(t) \times C(t) \times S(t) \times I(t) \\ \log f(t) &= \log T(t) + \log C(t) + \log S(t) + \log I(t) \end{aligned} \quad (2.3)$$

The decomposition process shown in this chapter will enable the exclusion of the components of the Time Series function $f(t)$, so it will be possible to study each component separately or the interaction of two or more components in a *graph* (f).

Carlos Polanco

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2.2. Multiplicative and Additive Decompositions

We have to consider that in the additive hypothesis, the four components are independent. For instance, the trend component does not condition the effect of the Seasonal or Cyclical components; whilst in the multiplicative hypothesis, the elements are interrelated with each other. Thus, the Seasonal or Cyclical components are added as a percentage of the Trend component and not as an independent value. In summary, to determine whether the Time Series has an additive or multiplicative scheme, we have to analyze the Cyclical component amplitude. If it increases with the trend component, it is a multiplicative model; if it remains constant, it is additive.

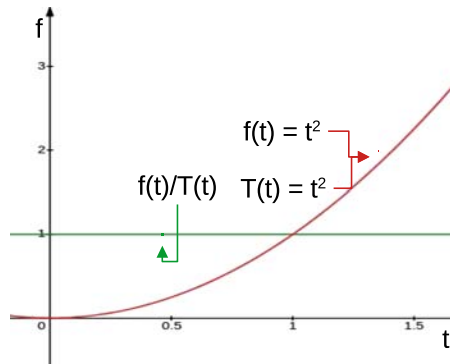
Each of these decompositions will be analytically and graphically illustrated with different examples that show the convenience of using either the additive or the multiplicative format of $f(t)$, as well as the decomposition order to be used.

2.2.1. Multiplicative Decomposition Form

We will start with a function that represents the experimental data $f(t) = t^2$ (Table 1), and as Trend component, we will have the function $T(t) = t^2$ (Ex. 1). Note that $f(t)$ and $T(t)$ are the same functions.

Now, we will divide $f(t)$ by function $T(t)$ (Eq. 1.1). Since $f(t)$ and $T(t)$ are identical, the quotient will be $1 \forall t \in \text{domain}(f)$ i.e., function $\frac{f(t)}{T(t)} = 1$; and its geometrical representation is given in Fig. 2.1.

Figure 2.1: Graphs of $f(t)$, $T(t)$, and $\frac{f(t)}{T(t)}$.

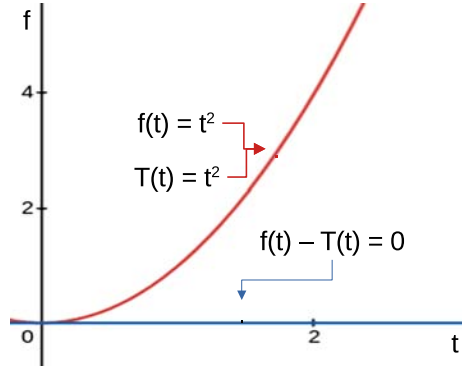


2.2.2. Additive Decomposition Form

Now, we are going to exclude from function $f(t) = t^2$ (Table 1) the Trend component represented with function $T(t) = t^2$ (Ex. 1). Note that both functions are the same.

To get this, we subtract from function $f(t)$ function $T(t)$ (Eq. 1.1), where $t \in \text{domain}(f)$. Since $f(t)$ and $T(t)$ are identical, the difference will be $0 \forall t \in \text{domain}(f)$ i.e., function $f(t) - T(t) = 0$. Its geometrical representation is Fig. 1.1, and Fig. 2.2.

Figure 2.2: Graphs of $f(t)$, $T(t)$, and $f(t) - T(t)$.



2.2.3. Basic Stages for Decomposition

The decomposition of the Time Series function $f(t)$ in four components, is done by subtracting or dividing each component in $f(t)$. This procedure is named **Time Series Analysis**.

The order for the decomposition of the components of the function $f(t)$ will be $T(t)$, $S(t)$, $I(t)$ and $C(t)$, so they define the terms of the function (Eq. 2.4).

$$f(t) = T(t) \times S(t) \times I(t) \times C(t) \quad (2.4)$$

The decomposition of the Trend component $T(t)$ will be the function (Eq. 2.5).

$$\frac{f(t)}{T(t)} = S(t) \times I(t) \times C(t) \quad (2.5)$$

Then, we exclude the Seasonal component $S(t)$ (Eq. 2.6).

$$\frac{f(t)}{T(t) \times S(t)} = I(t) \times C(t) \quad (2.6)$$

We will identify the Seasonal component with the regularities in a year.

CHAPTER 3**Interpolation and Extrapolation**

Abstract: This chapter will review in detail the main **real-valued functions** and their **maps**. Both are defined and geometrically interpreted. All concepts are discussed, exemplified, and illustrated with graphics. The concept of function permeates all chapters of this ebook since there is a function embedded in all mathematical operators. We strongly advise the reader to review all examples and do all the exercises included.

Keywords: Cyclical decomposition, irregular decomposition, linear squares method, multivariate components, noise component, nonlinear squares method, polynomial extrapolation, polynomial interpolation, time series analysis, trend decomposition

3.1. Introduction

In this chapter, we will review the forecasting technique named **Extrapolation** that helps to calculate previous or subsequent values in a data set, and an estimation named **Interpolation** that calculates the value between two already existent in a data set.

We will study both techniques from a polynomial function degree 5, built with the Least Squares Method (Sect. 1.7.1), computationally implemented with the Progs. A.2- A.5. Since we are using a polynomial approximation, both techniques will be called **Polynomial Interpolation** or **Polynomial Extrapolation**.

These techniques can be used in any data set, considering that a data set of a Time series is the most representative set of a phenomenon studied. However, as previously studied, a Time Series represents four phenomena combined: the Trend $T(t)$, Cyclical $C(t)$, Seasonal $S(t)$, and Irregular components $I(t)$.

To these four components we can add a fifth component called the **Noise component** $N(t)$ that will have a greater range of presence than an isolated event such as the Irregular component $I(t)$.

The Noise component is usually associated with the “startup” or “warming up” of a process in an initial phase.

An example of this component is a pandemic outbreak that occurs at any time and lasts at least 10 years affecting everything.

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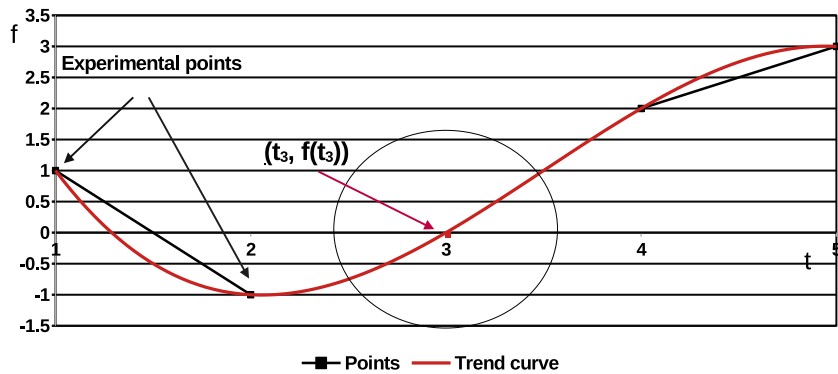
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In summary, a Time Series is a phenomenon that represents the interaction of multiple variables, not only one. Therefore, it is important to note that the forecast using these techniques is restricted to all of them.

3.2. Polynomial Interpolation

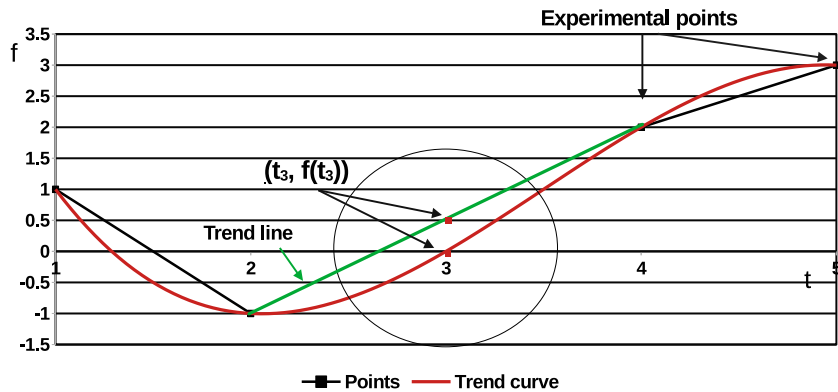
Interpolation estimates an intermediate outlier from the data set (Fig. 3.1) from a polynomial function f evaluating it from point t_3 i.e. the outlier will be $(t_3, f(t_3))$.

Figure 3.1: Missing point $(t_3, f(t_3))$ and trend curve (red).



In the graph of the function $f(t_2, f(t_2))$ and $(t_4, f(t_4))$, two points can be joined by an infinite number of curves (Fig. 3.2). Here, we used a line that is a polynomial function of degree 1, so the point $(t_3, f(t_3))$ changes position.

Figure 3.2: Missing point $(t_3, f(t_3))$, trend curve (red), and trend line (green).

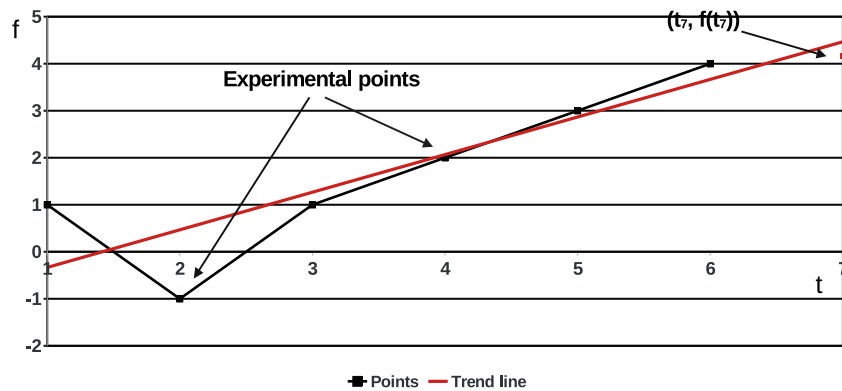


3.3. Polynomial Extrapolation

Extrapolation estimates an outlier at the far left or far right of the data set in a polynomial function f , evaluating it at the point t_i i.e., the outlier $(t_i, f(t_i))$ will be at either end of the function or both.

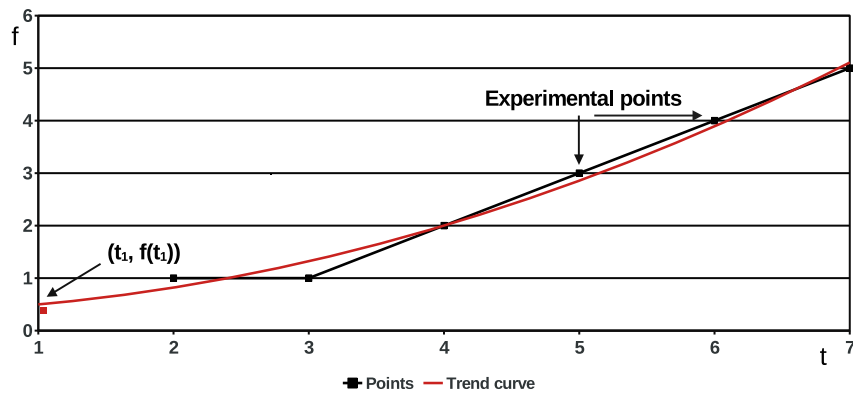
This is a value at the far right $(t_7, f(t_7))$ that represents a future value estimated with the polynomial function f (Fig. 3.3).

Figure 3.3: Missing point $(t_7, f(t_7))$ and trend curve (red).



This is a far left value $(t_1, f(t_1))$ and represents a past value estimated with the polynomial function f (Fig. 3.4).

Figure 3.4: Missing point $(t_1, f(t_1))$ and trend curve (red).



Extreme values help to know the past or future trend of a Time Series Graph.

CHAPTER 4

Multivariate Time Series

Abstract: This section examines the development of a multivariate time series function $f_{i,j}(t)$ that encapsulates the impact of one variable on two or more variables. We present the elements that constitute the multidimensional space and demonstrate how multidimensional panel data can facilitate the analysis. Each multivariate time series function $f_{i,j}(t)$ is derived using the multivariate least squares method to analyze the trend $T(t)$, cyclical $C(t)$, seasonal $S(t)$, and irregular components $I(t)$.

Keywords: Correlation coefficient, cyclical component, interquartile range, IQR, multidimensional panel data, multivariate time series function, outliers, seasonal component, time series forecasting, time series function

4.1. Introduction

This section will examine the elements of a time series function $f(t)$, where the independent variable t is specifically confined to the $\mathbb{R}^2$ plane or the $\mathbb{R}^3$ space. Subsequently, we will elucidate the analytical and geometrical interpretation to confine the function $f(t)$ to a sequence of points $x(t) \in \text{domain}(f)$. Utilizing the sequence of points from $\text{graph}(f)$, we will construct the time series function f through the least squares approach, and the components of the time series will be delineated. Ultimately, we shall illustrate its utility with a practical example.

4.2. $f : \mathbb{R}^n \rightarrow \mathbb{R}$

A **multivariate real-valued function** $f(x_1, \dots, x_n)$ is a numerical function e. $f : \mathbb{R}^n \rightarrow \mathbb{R}$ where the independent variable $(x_1, \dots, x_n) \in \text{domain}(f) \subset \mathbb{R}^n$ and the dependent variable $f(x_1, \dots, x_n) \in \text{image}(f) \subset \mathbb{R}$. The Graph of the real-valued function f is $\text{graph}(f) = \{(x_1, \dots, x_n, f(x_1, \dots, x_n)) \in \mathbb{R}^{n+1} \mid (x_1, \dots, x_n) \in \text{domain}(f) \text{ and } f(x_1, \dots, x_n) \in \text{image}(f)\}$.

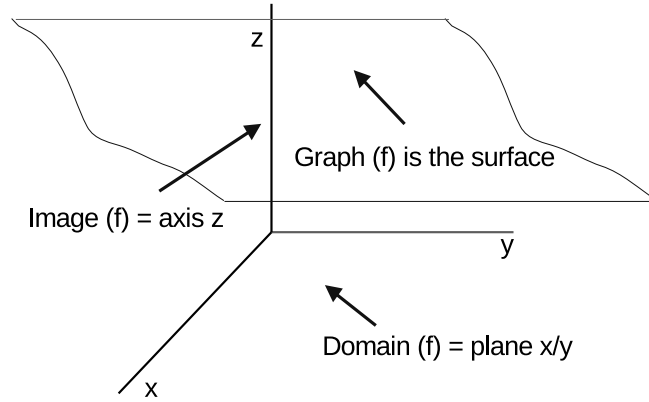
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4.2.1. $\text{Graph}(f)$

Particularly, if $n = 2$, graph f is a surface (Fig. 4.1).

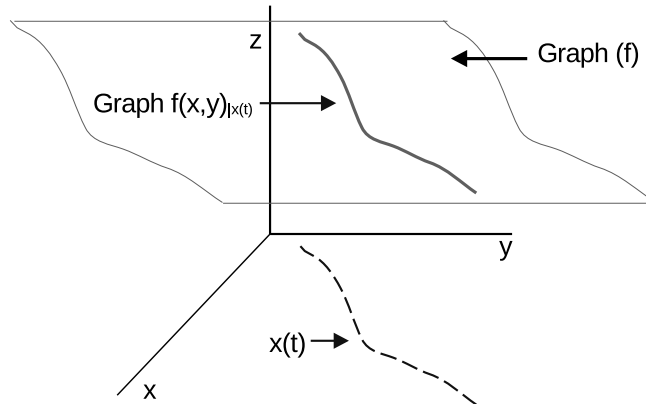
Figure 4.1: Graph of function $f(x,y)$.



4.2.2. $\text{Graph}(f)|_{x(t)}$

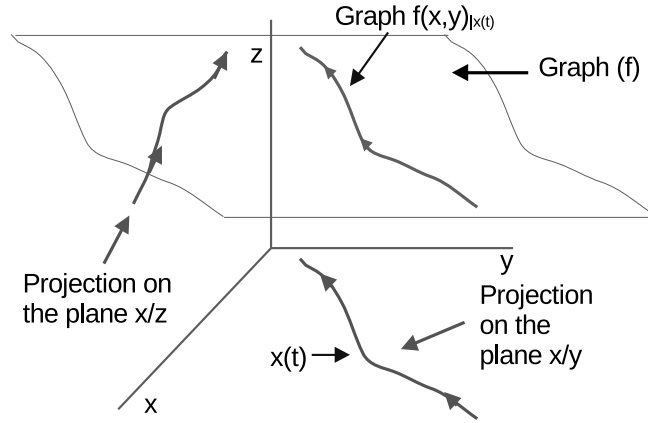
The graph of the multivariate real-valued function $f(x_1, \dots, x_n)$ is transformed from a plane in the space $\mathbb{R}^{n+1}$ to a curve in the same space when its evaluation is restricted to a sequence of points $x(t) \in \text{domain}(f)$. The $\text{graph}(f)|_{x(t)}$ (Fig. 4.2) illustrates the case of a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$.

Figure 4.2: Graph of function $f(x,y)$ restricted to $x(t)$.



The arrangement of points depicted by the curve $x(t)$ can be aligned to indicate either a progression or regression in time (Fig. 4.3). The $\text{graph}(f)|_{x(t)}$ exhibits a consistent direction. The projections of the curve in the planes of space $\mathbb{R}^3$ illustrate the relationships among the variables $(x, y, f(x, y))$ (Fig. 4.3).

Figure 4.3: Oriented curve of function $f(x, y)$ restricted to $x(t)$ and its projections over the planes x/y , x/z , and y/z .



Despite the abstraction required to geometrically understand the functions $f: \mathbb{R}^2 \rightarrow \mathbb{R}$, it is not practical to use this notation to represent a Multivariate Time Series function, since in higher dimensional spaces, it is not possible to obtain any geometrical representation. Therefore, in the next section, we will introduce a procedure named **Multidimensional Panel Data**, which is highly intuitive.

4.3. Multidimensional Panel Data

Using a Multidimensional Panel Data (Table 1), it is possible to observe the changes that occur in one factor over time in comparison to what happens in another factor. Simply examining the $\text{Graph}(f)|_{x(t)}$ is equivalent to the same thing that was done previously. A time series function $f(t)$ can be used to analyze the numerical progression shown in Table 1. This function can then be broken down into the components that were discussed earlier.

II CASES

The second part of the book introduces the reader to application examples that use time series to understand the phenomenon under study. The cases studied in detail correspond to the following areas: financial risk analysis, mathematical epidemiology, astronomical stellar classification, weather forecasting, commercial goods supply chain, and environmental pattern recognition.

CHAPTER 5**Financial Risk Analysis**

Abstract: This chapter will study in detail the behavior of oil and non-oil imports and exports in México from 1993 to 2022. The Time Series Graphs of these factors are decomposed into the Trend $T(t)$, Cyclical $C(t)$, Seasonal $S(t)$, and Irregular $I(t)$ components. We will carry out the conclusive analysis from the graphical results, not from the economic point of view. All concepts are discussed and illustrated with graphics. We strongly advise the reader to review these concepts in the previous chapters.

Keywords: Cyclical decomposition, irregular decomposition, seasonal decomposition, time series analysis, time series graph, trend component, trend decomposition

5.1. Introduction

Financial risk analysis is the review and evaluation of the probability that a drastic change happens in one or several factors and its effect on the financial risk study. Determining this change and its impact is not evident; usually when a factor is altered, it also impacts other factors.

Here, we will review the behavior of the oil and non-oil imports and exports in México from 1993 to 2022, with the Time Series method.

5.2. Oil and Non-oil Imports and Exports in México

Although the source is cited for each example presented here, the reader should keep in mind that other factors may influence the outcomes. However, the number of variables has been restricted to focus on the mathematical method.

The following graphs geometrically describe the oil (Fig. 5.1) and non-oil (Fig. 5.2) imports and exports in México from 1993 to 2022.

In a first analysis of Fig. 5.1, we can see that in July 2009, the curve of domestic oil exports reaches a maximum value -though there are other local maximum values-. On the other hand, and in May 2013, there is a sharp change in the curve of the domestic oil exports i.e., before this date, the curve of the domestic oil exports is

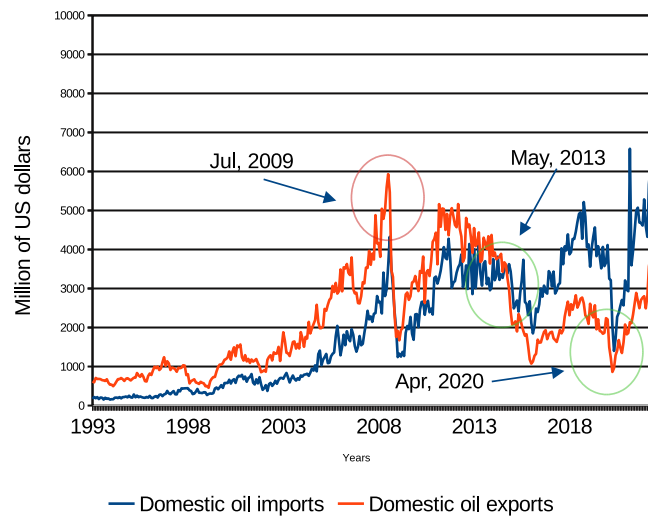
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above the curve of the domestic oil imports; after this date, the trend is reversed. Finally, in Apr 2020, the domestic oil exports and imports have a minimum value. We can also see that at the far right of the graph, the domestic oil imports have a maximum value, however, as it is at the end of the range, we will not study it as its regularity cannot be verified.

In both cases, the linear trend is positive.

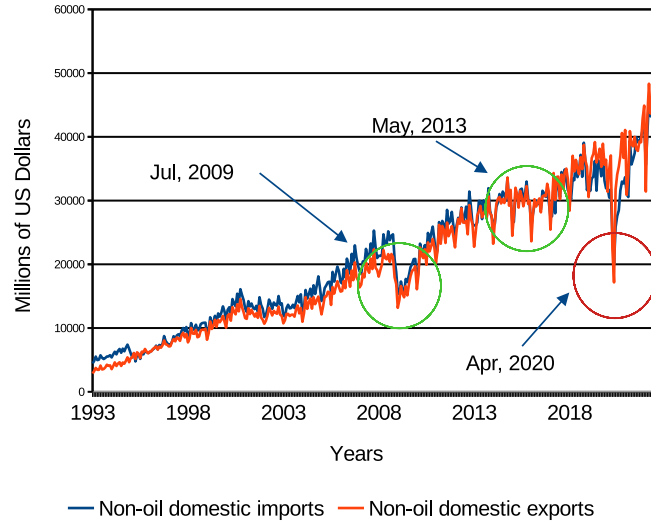
Figure 5.1: Imports and exports of oil in México represented by Time function $f(t)$ [5].



In the graph, the domestic non-oil exports and imports curve (Fig. 5.2) have a sharp fall in the trend of Jul 2009. Then, in May 2013, we can see that the curve of the domestic non-oil exports changes trend i.e., before that date, the curve representing the domestic non-oil exports is above, and after the trend date, of the curves is reversed. In April 2020, both curves, domestic non-oil exports, and imports, reached the minimum value. At the far right of the graph, we can see the maximum values for the non-oil exports and imports, however, they are not considered as there is not enough information to confirm the regularity.

In both cases, the linear trend is positive.

Figure 5.2: Imports and exports of non-oil in México represented by the Time function $f(t)$ [5].



5.3. Trend Component

Here, we plot the graph of the Trend Component $T(t)$ for the domestic oil and non-oil imports and exports and calculate the function $\frac{f(t)}{T(t)}$ for each one of them.

In the curves $\frac{f(t)}{T(t)}$ of the domestic oil exports and imports (Fig. 5.3), it is geometrically seen that in July 2009, the domestic oil exports and imports, reached a maximum value, and in May 2013, the trend of the curves is reversed. The curve representing the domestic oil exports is above the oil imports, after that date, the trend changes. In April 2020, we can see that the curves of the domestic oil exports and imports reached a minimum value.

CHAPTER 6

Mathematical Epidemiology

Abstract: This chapter will review in detail the overall mortality and births in México between 1994 and 2021, to explain the catastrophic behavior associated with Epidemiology. The Time Series graphs of the factors will be decomposed only into the Trend component $T(t)$, and Irregular component $I(t)$, and from the graphic results, we will carry out an analysis to explain the excess mortality. All concepts are discussed and illustrated with graphics. We strongly advise the reader to review the concepts in the previous chapters.

Keywords: Cyclical component, cyclical decomposition, deaths, graph(f), irregular decomposition, overall mortality, time series, time series analysis, time series graph, trend decomposition

6.1. Introduction

The analysis of mortality in México is based on the number of deaths [6] and births [7] reported by Instituto Nacional de Estadística y Geografía (INEGI) between 1994 and 2021. From both factors, we explain the extreme epidemiological event that occurred in 2020.

As a strategy, we will analyze the general birth and death figures with the Time Series methodology. We will express both factors as Time Series functions and we will decompose them into the Trend Component $T(t)$ and the Irregular Component $I(t)$ to find an explanation for the behavior of the excess mortality at the end of this period.

6.2. Overall mortality in México

Although the source is cited for each example presented here, the reader should keep in mind that other factors may influence the outcomes. However, the number of variables has been restricted to focus on the mathematical method.

The graphs geometrically describe the behavior of the deaths reported by INEGI (Fig. 6.1) in México between 1994 and 2021.

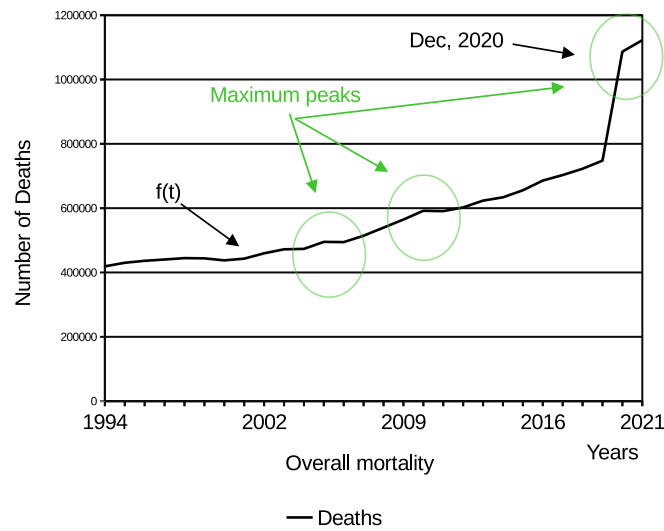
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In the first analysis (Fig. 6.1), we can see that in December 2020, the curve of the overall mortality has a maximum value, though we can also see other local maximum values. Between the years 1995 and 2017, the graph shows an upward trend with a slow growth and the curve shows an upward concavity, which means a slowdown.

The linear trend is all positive.

Figure 6.1: Overall mortality in México [6] represented by the Time function $f(t)$ and the maximum points (green) in the graph.



6.3. Overall births in México

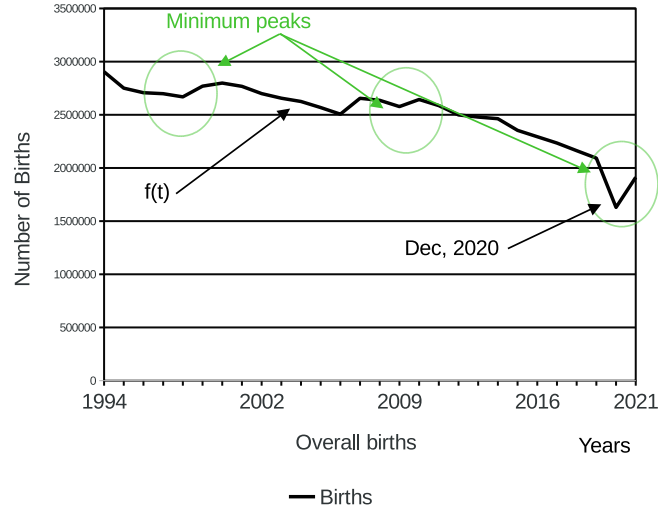
Although the source is cited for each example presented here, the reader should keep in mind that other factors may influence the outcomes. However, the number of variables has been restricted to focus on the mathematical method.

The graphs geometrically describe the behavior of the births reported by INEGI (Fig. 6.2), in México between 1994 and 2021.

In Fig. 6.2, we can see that in December 2020, the curve of the overall births has a minimum value, though other local minimum values are also shown. Between 1995 and 2017, there is a downward trend with a slow decrease and the curve shows a downward concavity, which means an acceleration.

The linear trend is all negative.

Figure 6.2: Overall births in México [7] represented by the Time function $f(t)$ and the minimum points (green) in the graph.



6.4. Trend Component

Then, we calculated the linear Trend Component $T(t)$ of the overall mortality in México (Fig. 6.3) and function $\frac{f(t)}{T(t)}$ for the overall mortality studied (Fig. 6.4).

In the curve $\frac{f(t)}{T(t)}$ of the overall mortality (Fig. 6.3), we can geometrically see that the maximum value in December 2020 does not correspond to the Trend Component. We can also see that in the first months of 1994, the line of the linear Trend Component $T(t)$ starts below the graph of the Time Series $f(t)$; this is usually due to very high maximum values and also to the upward concavity of the curve.

CHAPTER 7**Consumer and Producer Price Index**

Abstract: This chapter will review in detail the behavior of the Consumer Price Index, Producer Price Index, and Inflation in México between 2008 and 2022. The Time Series Graphs of these factors will be decomposed into the Trend component $T(t)$, Cyclical component $C(t)$, Seasonal component $S(t)$, and Irregular component $I(t)$. From the graphic results, we will carry out a conclusive analysis that will not be from the economic point of view. All concepts will be discussed and illustrated with graphs. We strongly advise the reader to review these concepts in previous chapters.

Keywords: Consumer price index, producer price index, cyclical component, cyclical decomposition, graph(f), irregular component, irregular decomposition, time series graph, trend decomposition

7.1. Introduction

This chapter reviews the impact that sharp changes have on the graphs of the Consumer Price Index (CPI) [8] (Fig. 7.1), the Producer Price Index (PPI) [9] (Fig. 7.2), and Inflation [9] with a Time Series. For this purpose, we have taken the figures from 2008 to 2022 in México.

The Consumer Price Index (CPI) is the price of a weighted average market basket of consumer goods and services purchased by households [10], and the Producer Price Index (PPI) measures the average changes in prices received by domestic producers for their output [11].

The PPI is often used to calculate real growth by adjusting inflated revenue sources, and the CPI is often applied to calculate changes on the cost of living by adjusting revenue and expense sources.

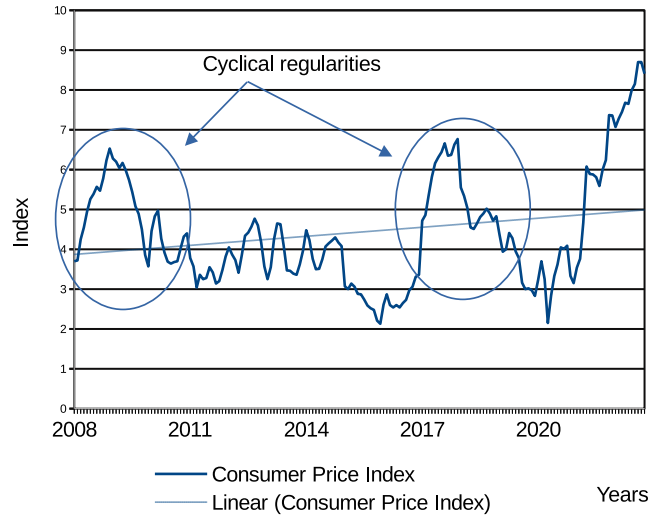
Inflation refers to a rise in the prices of goods and services in the economy over time, eroding purchasing power for both consumers and businesses.

The following analysis is based on the graphs for the CPI and PPI to later include the graph for inflation.

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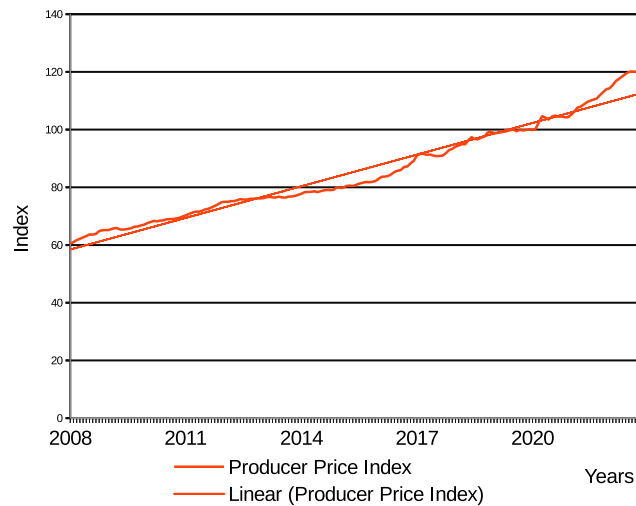
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Figure 7.1: Consumer Price Index (black) in México represented by the Time function $f(t)$ [8].



In the graph of the Consumer Price Index (Fig. 7.1), there is a slightly upward trend, whilst in the Producer Price Index (Fig. 7.1), the trend is upward with no cyclical movement.

Figure 7.2: Producer Price Index (black) in México represented by the Time function $f(t)$ [9].



In an ideal scenario, both graphs of the CPI and PPI should correlate so that only a translation between both can be observed. However, in this case, there is a correla-

tion between the Consumer Price Index with the factor of Inflation, as we will see later.

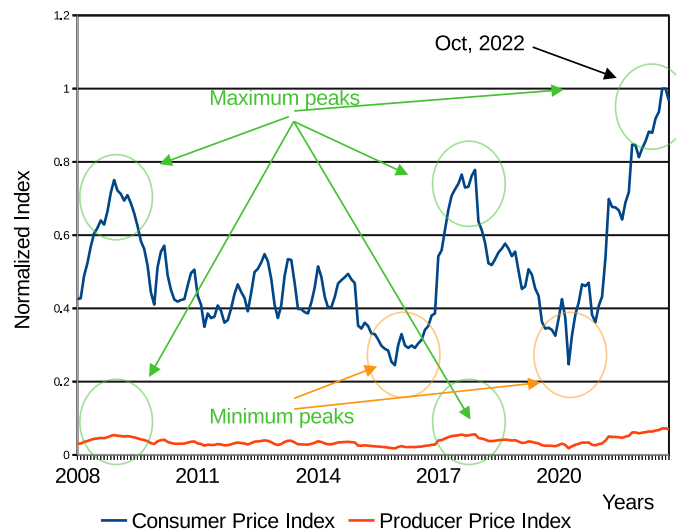
7.2. Consumer – Producer Price Index and Inflation in México

Although the source is cited for each example presented here, the reader should keep in mind that other factors may influence the outcomes. However, the number of variables has been restricted to focus on the mathematical method.

Initially, the graphs of the Consumer Price Index and the Producer Price Index (Sect. 7.1) in México between 2008 and 2022 are normalized with the maximum value of each set of data (Fig. 7.3).

In Fig. 7.3, it can be seen that the maximum points of the factors PPI and CPI correlate in three different dates, being the maximum in October 2022. We can also see that there are two local minimums in the Consumer Price Index and the Producer Price Index that correlate.

Figure 7.3: Normalized Consumer Price Index [8] (black), Producer Price Index [9] (red), maximum peaks (green), and minimum peaks (orange).



In Fig. 7.4, we included the graph of the normalized inflation to the normalized graphs of the Consumer Price Index and Producer Price Index, thus we can see a similarity of the trend in the Consumer Price Index and Inflation.

CHAPTER 8**Seismicity Pattern Recognition**

Abstract: This chapter will review the behavior of earthquakes in México with the Time Series method from 1901 to 2022. The Time Series Graph will be decomposed into the Trend $T(t)$, Cyclical $C(t)$, Seasonal $S(t)$, and Irregular component $I(t)$ functions. Based on the results from the graphs, we will carry out an analysis to find graphical regularities. All concepts are discussed, and illustrated with graphics. We strongly advise the reader to review them in the previous chapters.

Keywords: Cyclical component, cyclical decomposition, irregular component, irregular decomposition, seismicity pattern recognition, time series, time series analysis, time series graph, trend component, trend decomposition

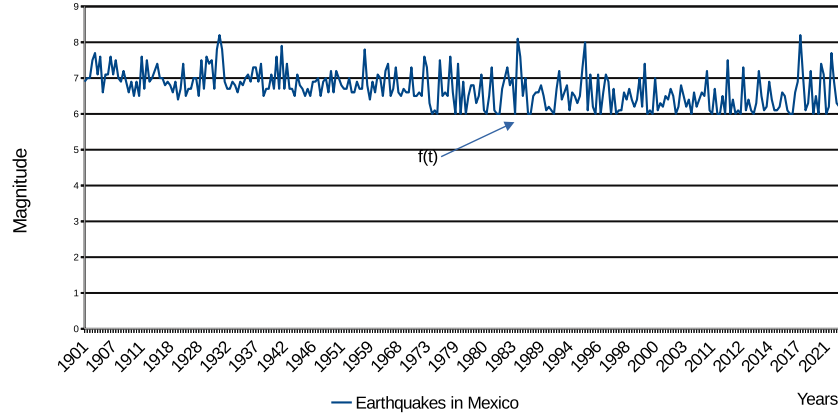
8.1. Introduction

In this chapter, we review the regularities identified in the record of earthquakes that represented a risk to the population from 1900 to 2020 in México. For this purpose, from Servicio Sismológico Nacional [12], we got all the information about all surface earthquakes greater than 5.5 degrees on the Richter scale that occurred in the Mexican territory that have been verified. We expressed this information in a graph as a Time Series function (Fig. 8.1).

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Figure 8.1: Earthquakes in México from 1901 to 2022 represented by Time function $f(t)$ [12].



It is important to note that although the information recorded seems graphically large (Fig. 8.1), it is due to the length of the range i.e., 122 years since most years only have a couple of events logged.

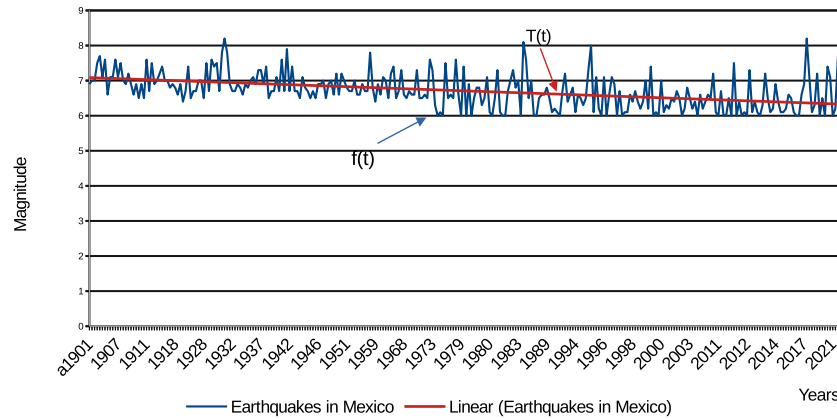
8.2. Seismicity in México

Although the source is cited for each example presented here, the reader should keep in mind that other factors may influence the outcomes. However, the number of variables has been restricted to focus on the mathematical method.

An earthquake is the abrupt movement of the Earth caused by the release of energy accumulated over a long period. Usually, the movements are slow and imperceptible, however, some displacements release a large amount of energy when one of the plates moves sharply against another, breaking it and causing the earthquake [13]. México is a country where earthquakes with these characteristics frequently occur.

The danger of an earthquake is related to the depth it has. The most destructive are those that occur at a few kilometres deep and their intensity is greater than 5.5 degrees on the Richter scale. We will review the earthquakes that occurred between 1900 and 2022 in México [12] to see if there is any correlation between frequency and intensity in the earthquakes of this region. For this purpose, we have graphed the Time Series function $f(t)$ and the Linear Trend Graph $T(t)$ (Fig. 8.2) that show a slight downward trend in the intensity.

Figure 8.2: Time Series function $f(t)$ and Trend Component $T(t)$ of earthquakes in México.

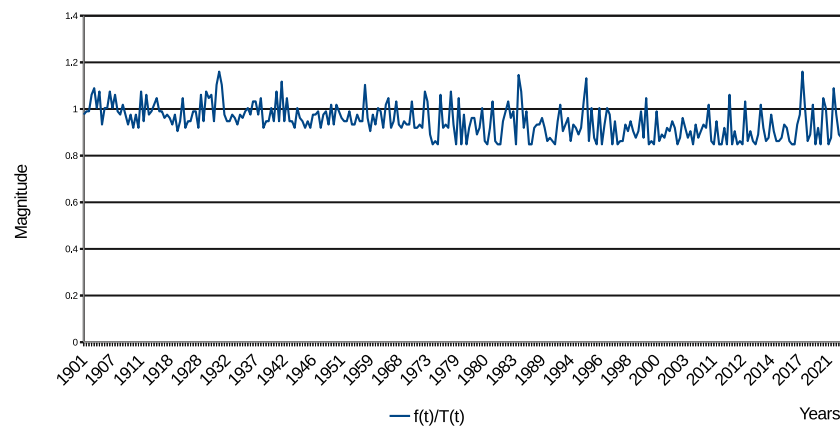


The graphs (Figs. 8.1, 8.2) reviewed show some cycles and a downward trend that we will decompose into the Time Series function and its components.

8.3. Trend Component

We get the Trend Component $T(t)$ of the number of incidences of earthquakes in México by separately calculating function $\frac{f(t)}{T(t)}$ (Fig. 8.3).

Figure 8.3: $\frac{f(t)}{T(t)}$ function of earthquakes in México.



CHAPTER 9**Population Density**

Abstract: This chapter will review in detail the behavior of the population density in México from 1990 to 2020. The Time Series Graph of this factor will be decomposed into the Trend $T(t)$, Seasonal $S(t)$, Cyclical $C(t)$, and Irregular $I(t)$ components. From the graphical results, we will analyze and obtain will get a method to calculate the population density in the future and the possible consequences. All concepts are discussed and illustrated with graphics. We strongly advise the reader to review the concepts in previous chapters.

Keywords: Cyclical decomposition, graph(f), irregular component, irregular decomposition, overall mortality, time series, time series analysis, time series graph, trend component, trend decomposition

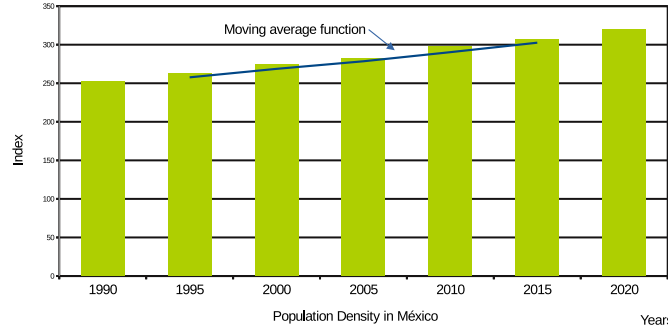
9.1. Introduction

In this chapter, we will review the regularities found in the population density graphs of each State in México between 1990 and 2020. For this purpose, we have taken all the records related to this factor from Instituto Nacional de Estadística Geografía e Informática [14] and we have classified them by year and State. These graphs have then been expressed as a Time Series function of the average value (Fig. 9.1).

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Figure 9.1: Population Density behavior in México [14].



9.2. Population Density

Although the source is cited for each example presented here, the reader should keep in mind that other factors may influence the outcomes. However, the number of variables has been restricted to focus on the mathematical method.

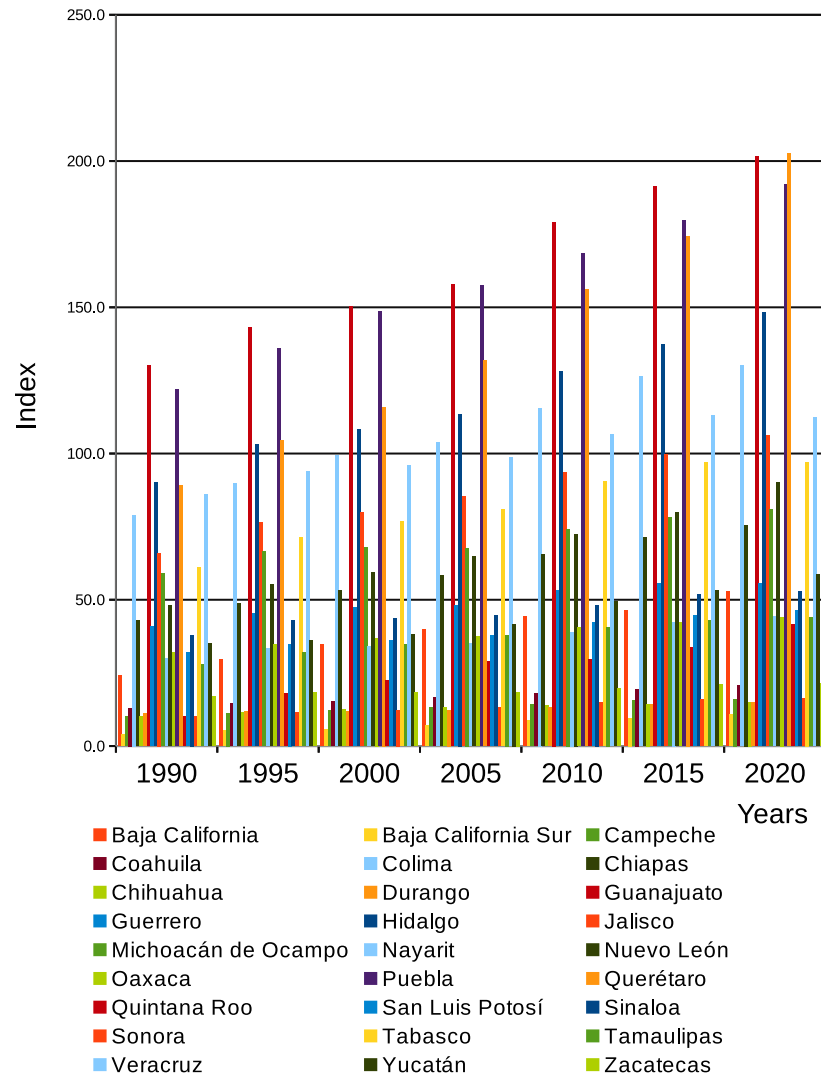
In a world of finite resources, population density should be a major issue; however, when it is eventually reported by the media, it is usually a secondary issue. Population density is related to the allocation of resources (normally by State) to the inhabitants. An irregular distribution of the population provokes a lack of resources in some places, severely affecting the food supply and the attention to vulnerable groups in those locations.

In this sense, population density is a pointer to the quality of life the inhabitants of a country, state, or territory have since the more densely populated a place (high density) the lower the quality of life, if there is no adequate urban and social planning. On the other hand, a scarcely populated region (low density) in a developing country implies the deficient or null distribution of the goods and services necessary for survival.

Given the population difference between the States in México, they were divided into three groups: those with low density (Fig. 9.2), those with high density (Fig. 9.3), and finally the area with the highest density, which is México City (Fig. 9.4) with a density 30 times higher than the average of the low-density States (≈ 200).

From Fig. 9.2, we can see that 27 of the 31 States in México have a very low population density. This means a limited or null supply of public services such as piped water, electricity, or telephone.

Figure 9.2: México States with low population density [14].



In Fig. 9.3, we can see that 3 of 31 México States have high population density. High values imply a high demand per square kilometre of the public services mentioned. To meet this demand, the State has to transport these services from other States.

CHAPTER 10

Future Uses

Abstract: This chapter reviews how the Time Series method, studied in this work, can be applied to different fields and it explores its use with several approaches. It also examines how the applicability of this method can affect diverse disciplines with different impacts. The implementation of the Time Series method can even have the option of self-learning programming. However, it is worth it to emphasize the fact that this method has limited results when it is used to forecast events.

Keywords: Cyclical component, cyclical decomposition, graph(f), irregular component, irregular decomposition, time series, time series analysis, time series graph, trend component, trend decomposition

10.1. Scope

A Time Series function $f(t)$ has four components i.e., $f(t) = T(t) + S(t) + c(t) + I(t)$. Each one of them represents a particular type of fluctuation in the graph $f(t)$, so by clearing the component to be identified, we can analyze the function.

Although the theory and application cases studied in this book show its effectiveness in knowing a particular phenomenon, the Time Series method is far from helping to forecast by extrapolation or interpolation of the graph $f(t)$.

In a strict sense, the graph of a function can show the regularities of a phenomenon, and its decomposition can effectively explain a phenomenon in a sector of the domain; however, to extend the graph of the function to determine the past or future trend of a known range is, to put it simply, a leap into the void.

Consider the following sequence: 1, 2, 3, 4, 5, 6. Would you bet that the next number is 7? There is no information to know what number is next, the sequence could be 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, or 3, 4, 5, 6. There is an infinite number of possibilities.

The last two options show that the forecast of a future or past event from a present event or in a determined range requires much more attention and the use of other operators, such as stochastic variable functions.

Stochastic functions enable the association of an element with a value of the probability of occurrence. For instance, consider a real-valued function $f(x)$ representing

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the sequence 1, 2, 3, 4, 5, 6 i.e., $f(x) = x$, whose $\text{domain}(f) = \{1, 2, 3, 4, 5, 6\}$. If we evaluate element 7, it will be $f(7) = 7$ and its accuracy will be that the result could only be true or false. If we use a stochastic function $f(x) = x \pm 30\%$ for value 7, we will get $f(7) = [4.9, 9.1]$.

This means that the function has a range of values, including 7 or 5. Note that these two values forecast the seventh element of the first and the last sequence mentioned.

Are stochastic functions the answer to forecast the present or future trend of a factor or variable? The answer is NO. Since the percentage (%) we can give when evaluating a function of a specific value x i.e., $x \pm y\%$, is related to the level of knowledge you have of the phenomenon studied and this knowledge is precisely what we want to find out from the function, so this is a contradiction.

What can help is to train the stochastic function with an initial value and that this value is adjusted as the results are accepted or rejected. This type of self-adjusting function will have a convergence radius that will lead to a representative function of the phenomenon studied and will help to progressively understand it. A family of Time Series functions can be used for this purpose.

In summary, the Time Series Method is an efficient procedure to understand what happens in a limited range of time, and the information it provides about the phenomenon can be used to feed a stochastic function that gradually adjusts the percentage value of the forecast.

10.2. Main Possibilities

This book is formed by two sections: the first introduces the Time Series Method defining a Time Series function $f(t)$; it then shows its components and how they can be used, and later it describes how this function can be extended to a greater number of variables or factors. The second section has a set of cases from different fields, completely solved and developed explicitly so the reader can understand the strategy to make the most of the Time Series Method.

The reader will not find a unique strategy; in these cases, the phenomenon studied will indicate the approach taken with the Time Series function. Below, we will recount the main strategy for each case.

Chap. 5, Financial Risk Analysis case: The Time Series method showed a cyclical regularity in oil imports and exports. This regularity is not evident in the original graphs. The reversed predominance of the domestic oil exports over the imports in May 2013 (Fig. 5.1) can be partially explained by its cyclical component (Fig. 5.5). A careful revision of each component and its adjustment will enable the construction of a Time Series function $f(t)$ (Sect. 5.6) that can give information over a short period. This period can be two months or so, bearing always in mind that an irregular event can affect the analysis.

Chap. 6, Mathematical Epidemiology case: The use of the Time Series method based on the graphs of the overall mortality (Fig. 6.1) and births (Fig. 6.2) in México, showed a drastic fall in the overall births in December 2020 and, simultaneously, high growth in the overall mortality in the same month. It did not show any cyclical or seasonal events, although when calculating the irregular component in both graphs (Figs. 6.7 and 6.8), it showed a match at the same time in December

2020. In this case, we can see that it is possible to combine the two Time Series functions, to form $f(t)$ that can model the net growth i.e., $f(t) = f_{mortality} + f_{births}$, or to build another where $f(t_{mortality}, t_{births}) : \mathbb{R}^2 \rightarrow \mathbb{R}$. This function will show if the population gets old or young.

Chap. 7, Consumer and Producer Price Index and Inflation case: The analysis of the behavior of the Consumer Price Index, Producer Price Index, and the Inflation in México between 2008 and 2022 with the decomposition of its Times Series graphs showed two important issues: the cycles in the Consumer Price Index and the correlation between Inflation and the Producer Price Index. The latter will reduce the number of factors to study issues: the Consumer and the Producer Price Index. When calculating the graph of these factors for the United States of America (Fig. 7.12), we can see that the graphs do not correlate in a way that there is a translation between them. This lack of translation negatively impacts the purchasing power of the consumer.

Chap. 8, Seismicity Pattern Recognition case: The study of the surface earthquakes with a magnitude greater than 5.5 degrees on the Richter scale that occurred in México between 1900 and 2022, showed in the Linear Trend component a decrease in their intensity in the time range studied (Fig. 8.3) and a quarterly cycle in its Seasonal component (Fig. 8.4). Both results are important since they help to take measures to protect the population and orient them on what they can do when an earthquake of the characteristics studied takes place.

Chap. 9, Population Density case: It describes the results found on the population density factor for México between 1990 and 2020, classifying its States in high, and low population density. Twenty seven out of the 31 States in México with low population density have continuous growth. We can see the same trend in the 3 States with high population density. México City shows a much larger rate of growth, 30 times larger than the rate of the low population density States. The Seasonal and Cyclical components did not provide more information about the phenomenon, and the Trend component shows an increasing linear behavior. When the population density in Spain was graphed, we could also see an increasing linear behavior, but with high and low density, it was almost equally distributed in all its regions. In the case of Finland, the growth is almost non-existent and there is a very low population density in its territory. These two countries are considered developed; therefore, their population density does not affect the sufficiency of goods and services, which is not the case in México.

In these examples, the Time Series Method made it possible to explore and have a better understanding of the phenomena studied. The Time Series function $f(t)$ can even be used, in a very restrictive way, to interpolate or extrapolate the trend of the function.

The theory introduced in this ebook, is not abundant; however, it focuses on the main components of the Time Series Method that we consider enough to address any project. Nevertheless, since it is not a definitive method, a refined analysis of the components will be necessary in each case studied.

There are many other applications of the Time Series method in different fields. This work aims to introduce the theory with practical cases, motivating its use with the simplicity of the Function Theory involved and its ability to address multiple variables simultaneously.

APPENDIX A

Computational Programs

Abstract: In this chapter, we present three computational programs written in Fortran-95, designed to address various aspects related to matrix analysis. The first program is a Cramer's rule matrix calculator, allowing for the solution of systems of linear equations using this technique. The second program focuses on the generation of matrix A and its parameters b_i , providing an efficient tool for creating customized matrices. Lastly, an introduced program tackles the unexplained variation component in the context of data analysis. These Fortran-95 programs offer effective and reliable solutions for matrix problems and data analysis, expanding the capabilities of researchers and professionals in the field.

Keywords: Cramer rule matrix calculator program, FORTRAN-95, Matrix A, parameters b_i program, Linux script, unexplained variation component program

A.1. Introduction

In the fields of computer science and computational sciences, the Fortran-95 programming language has played a crucial role in the development of highly efficient and trustworthy programs. In this chapter, we will investigate a collection of Fortran-95 programs that address various aspects of mathematical analysis. Through the power of computational calculation, these programs have been created to solve particular problems and provide accurate solutions.

We will begin by introducing the Cramer's Rule Matrix Calculator program, which is an indispensable instrument in linear algebra. This software is based on the well-known Cramer method for solving linear equation systems using matrices. Users will be able to input coefficient matrices and obtain the corresponding solutions for unknown variables using this program. We will investigate how Fortran-95 facilitates the efficient and accurate calculation of Cramer's rule.

Next, we will examine the program for generating the matrices A and b_i . This application is especially helpful for solving optimization problems and fitting curves. The program allows users to generate a matrix A with specific parameters (b_i) to meet the requirements of their specific problem. We will investigate how Fortran-95 facilitates the efficient construction and manipulation of matrices, paving the way for more sophisticated mathematical solutions.

This chapter will also include a Fortran-95-developed Linux script. This script automates specific duties by interacting with the Linux operating system using Fortran-95's capabilities. We will investigate how Fortran-95 can be used not only for sophisticated mathematical calculations, but also to leverage the potential of the Linux platform and improve data handling and processing efficiency.

We will conclude by discussing the Unexplained Variation Component program. This program focuses on variance analysis and provides a valuable instrument for evaluating the contribution of various factors to the variation in a dataset. We will demonstrate how Fortran-95 can be used to calculate and interpret the unexplained variation components, allowing for a more in-depth comprehension of the data and its variability.

By examining these Fortran-95 computer programs, readers will gain a better understanding of the mathematical analysis applications of this potent programming language. In addition, they will benefit from specific examples and applications that will strengthen their abilities to use Fortran-95 to solve problems in the scientific and engineering disciplines.

A.2. Linux script

```
1      ! /bin/sh
2      #Author Carlos Polanco
3      #Date July, 2022
4
5      # Step 1
6      #Generation of matrix A and parameters b_i
7      gfortran pgmccl.f -o primero
8      ./primero
9
10
11     # Step 2
12     #Solution by Cramer Rule Matrix Systems Calculator
13     and Laplace Theorem
14     clear
15
16     rm todo.py
17
18     cat partePY1.py > todo.py
19     resultado='head -1 parteprimera.dat \'
20     echo "      $resultado" >> todo.py
21     echo " " >> todo.py
22
23     resultado16='head -1 partesegunda16.dat \'
24     echo "      $resultado16" >> todo.py
25
26     resultado26='head -1 partesegunda26.dat \'
27     echo "      $resultado26" >> todo.py
28
29     resultado36='head -1 partesegunda36.dat \'
30     echo "      $resultado36" >> todo.py
31
32     resultado46='head -1 partesegunda46.dat \'
33     echo "      $resultado46" >> todo.py
34
```

```
35 resultado56=`head -1 partesegunda56.dat`  
36 echo "    $resultado56" >> todo.py  
37  
38 resultado66=`head -1 partesegunda66.dat`  
39 echo "    $resultado66" >> todo.py  
40  
41 cat partePY2.py >> todo.py  
42  
43 python todo.py | grep "A" > salida.dat  
44 cut -d"=" -f2 salida.dat > salida2.dat  
45  
46 head -1 salida2.dat > a0.dat  
47 head -2 salida2.dat | tail -1 > a1.dat  
48 head -3 salida2.dat | tail -1 > a2.dat  
49 head -4 salida2.dat | tail -1 > a3.dat  
50 head -5 salida2.dat | tail -1 > a4.dat  
51 head -6 salida2.dat | tail -1 > a5.dat  
52  
53 # Step 3  
54 #Calculation of the polynomial function y_est  
55 gfortran pgmccl2.f -o segundo  
56 ./segundo  
57  
58 Output  
59 -2.488 13.579 -18.054 13.854 41.409  
60 25.345 162.795 1095.012  
61 Tendencia no lineal: 0.999768794  
62
```

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