

# SMART KINEMATICS FOR MODERN ENGINEERING STUDENTS



**Yevsey Gutman**

**Bentham Books**

# **Smart Kinematics for Modern Engineering Students**

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## **Smart Kinematics for Modern Engineering Students**

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## FOREWORD I

Kinematics as a field of engineering science has been known since the Renaissance, the time of Leonardo da Vinci. Nevertheless, it has gained the most popularity with the development of commercial packages of CAD/CAM systems, and as emphasized in the book, kinematics became a very efficient tool in systems engineering, particularly for multi-degree-of-freedom mechanical systems.

So, if it is so important for designers, then how can it be used in everyday practice, and where it possible to learn about the critical parts of that discipline? The person who is trained on this subject is called a “kinematician” and very often plays the role of a system engineer. There are many traditional technical disciplines that are taught in colleges today, such as mechanical engineering, electrical engineering, software engineering, and so on, but there are no courses in universities where they teach a discipline called systems engineering. The main reason for this phenomenon is that to be qualified as a system engineer, the engineer must gain experience in many different areas. One of these areas is knowledge of kinematics, since kinematics provides the model (even with certain assumptions) of the mechanical system to be designed and developed. From that perspective, this Book on Kinematics teaches the students or engineers some specific practical “tricks” using a large variety of real examples with real applications.

So, the first note here, is the value of this book, which is an illustration of the main steps to be established and solved on the road to designing practically any mechanical system.

Now, what must a modern engineer possess (in terms of knowledge and experience) to fully and efficiently utilize all benefits of “Kinematics?” As it is rightly stated in the Book, there are two or three disciplines that would give the winning ticket on every project involved with multi-degree freedom systems. One of them is the absolute fluency in the manipulation of homogeneous 4 by 4 matrices of coordinate transformation. The second topic, based on the opinion of the author of this book, is the knowledge about different types and configurations for kinematic pairs or joints. Based on the number of applications presented in the book, everyone should be convinced that experience in numerical analysis, specifically in different methods of optimization is a must.

The book demonstrates these main steps in eleven different applications with a very detailed, almost step-by-step explanation of different strategies chosen for a given application. It covers applications from the automotive field, robotics, and a variety of moving platforms, including the basics for driving simulators. The most intriguing is Chapter 11, devoted to the comparison of kinematics between mechanical joints and human biomechanical ones, illustrated by the temporomandibular (TMJ) joint. This justifies the title of the book “Smart Kinematics....”

The material in this book will be very important to gain knowledge about modern methods in mechanical engineering for undergraduates with some prerequisites in the area of linear algebra and matrix methods. It would be very helpful to graduate students, as well as for engineers already graduated from the university environment.

**Alex Kapelevich**  
President, AKGears, LLC

## FOREWORD II

The question with which the Author opens the book is important and necessary.

The events that developed after the 60s of the last centuries with the advent of Robotics posed new challenges and drew attention to old problems that were since then thought of as merely theoretical speculation and not very significant in practice. As an example, to find all the solutions of the 7R mechanism, which later became a strategic issue for the efficient control of industrial robots. Issues that highlighted the importance of Kinematics, not as a subsection of Dynamics, but as an essential part of its foundations.

Indeed, Jack Phillips in his book *Freedom in Machinery* (1985) states “The study of mechanisms is important because the geometry of mechanical motion (*i.e.* Kinematics, n.d.r.) is often the crux of a real machine’s design”.

Kinematics under the impulse of new methodologies opened the field to crucial problems such as singularities of mechanisms and their mathematical representation, key problems to deeply understand how to control mechanical systems effectively.

From the first pioneering works of Fyodor Litvin and Joseph Duffy, important methods of investigation were developed since the 80s that strongly contributed to the understanding of many problems. Just to mention a few: the kinematic analysis of both the closed chain 7R mechanism (Hong-Yu Lee and Chong-Gao Liang) and the mechanism known as the Gough–Stewart mechanism or, more synthetically, as 6-6 platform (Manfred Husty), have been completely solved; new important closed chain mechanisms (mechanisms later called parallel mechanisms) have been synthesized (Clément Gosselin); the synthesis of spatial mechanisms has regained new vigour and interest (Michael McCarthy), and Kinematic analysis and synthesis are increasingly applied in robotic devices for motor rehabilitation.

The title of Gutman’s book may seem provocative and, reading the index, the content is anomalous and very ambitious. However, reading the chapters, gradually one realizes the value that the approach used by the Author can have on teaching the subject to the students, the next designers.



The book stands out of the ordinary, out of the traditional way of presenting the Kinematics to university students. Likewise unexpected things from what we are familiar with, the first reader's reaction can be of surprise, rejection, criticism, *etc.*, but looking with more attention and free from prejudices, he/she can glimpse the truest content and the potential the book can have in the high-level educational process. However, not everything is easily acceptable, but the whole work represents an approach to the subject absolutely worthy of serious consideration.

Putting in the foreground some applications to teach the basic concepts of Kinematics may seem inefficient, but here lies the point. It is well known the dualism deductive method *vs* inductive method. This book, indeed, combines the two methods in an original and certainly efficient way for teachers who want to grasp the most creative and positive aspects of both, just adapting them to their educational realities without prejudices.

Gutman's work deals with the study of Kinematics from a completely original point of view, very different from the traditional method consolidated in recent decades. It tries to stimulate the interest of students, to whom the book is mainly addressed, starting from real problems and stimulating the deepening of the theory to better understand its application.

In essence, compared to the classic approach from theory to application, the book moves in several cases in a reverse direction, that is, from application to theory (inductive rather than deductive method).

The book reflects the experience of the Author, firstly a university researcher and then a technician with a long experience gained in the world of advanced motion simulators (in an international leading company).

After the introductory part, where the theoretical bases to deal with kinematic analysis are exposed, the book presents the fundamental problems of a designer, namely the optimization and the analysis of complex industrial systems, to end with important and complex applications related to biomechanical problems.

The subject is largely presented by referring to notable industrial examples, then gradually introducing in Chapter 2 the necessary theoretical concepts to solve the problem at hand. Whilst Chapter 3 refers to the theory in a very general and abstract way. Then it treats the influence of manufacturing errors on the mobility of a mechanism, a very interesting issue, applied to the Uhing mechanism (RCCC) taken as an example.

In general, optimization is not covered in Kinematics textbooks. The treatment of Gutman's book in Chapter 4 does so with great clarity and usefulness for the student, who can easily understand that a real system is always the result of an optimization; either based on an empirical process, which is refined over the years, or, more effectively, on mathematical models easily implemented in computer codes that even low skilful designers can use.

Particular attention to stability, control and trajectory planning of robots is devoted in Chapter 5, while methods for solving the position analysis of robots with 6 degrees of freedom (dof) are exposed in Chapter 6, highlighting their most critical aspects. A gantry system is also presented as an example to show how to deal with the singular configurations of a mechanism. The interesting concept of Phantom dof is introduced in Chapter 7.

Chapters 8, 9 and 10 are devoted to the analysis of complex systems (testing machines) where the methods set out in the previous chapters are used.

Chapter 11 shows how very complex biomechanical systems (tibio femoral joint, knee, ankle, *etc.*) can be modelled by kinematic elements of an equivalent mechanism, which can be of the utmost importance to design and implement dental prostheses, and to plan surgical interventions.

The book subject is frequently treated colloquially as if one were lecturing students. This makes the exposition less formal, maybe less attractive for the researcher and the professional scholar, but perhaps more accessible and interesting for the students who deal for the first time with these concepts that are anything but immediate (*i.e.* the modelling of mechanical systems (mechanisms) and the solution of systems of nonlinear equations).

For the depth of the topics and the analytical tools used, the book should be understood for an advanced kinematics course for Master and PhD students.

In conclusion, the book is quite different from most books on Kinematics available on the market. It treats basic and advanced elements of the theory (as many other books do) plus very interesting industrial applications, which are rarely found in the academic literature.

The book represents a balanced synthesis of basic elements of three-dimensional kinematics and examples of application from the industrial world that are very interesting and certainly of keen interest to teachers and a stimulus for students. In particular, students can come across with complex concepts (of kinematics),

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whose usefulness is frequently not fully understood, applied to the resolution of real problems of strong industrial importance, and then get tangible evidence of their relevance.

This book provides a relevant contribution to education. It is a novelty in the panorama of the books on mechanism Kinematics.

In my opinion, I believe that Gutman's work represents a valid tool for teachers and provides a significant contribution to the high-level training of today's students, who will be the designers of tomorrow.

**Vincenzo Parenti Castelli**  
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## **PREFACE**

This book is a collection of real examples to illustrate the use of kinematics in the field of multi-degree of freedom systems. The author has more than 50 years of experience in this area.

The theoretical material for this book has been developed on the knowledge and experience the author has obtained by working as a graduate student first and then as a closest colleague with two scientists: Prof. F.L. Litvin and Prof. S.A. Rodionov. Their contribution to the mathematics, optimization, differential geometry, surface behavior, advanced kinematics and engineering in general is well known around the world. Prof. S.A. Rodionov demonstrated the ability to navigate through the world of multi-parametric non-linear systems to develop optimal solutions to the problem. Additionally, Prof. F.L. Litvin opened the possibility and opportunity to combine a very deep knowledge in mathematics with extremely practical applications in mechanical engineering, especially in robotics and gearings. Both scientists were very strong advocates in using advanced matrix methods whether for linear algebra in optimization or for coordinate transformation and displacement models for mechanisms.

The author has paid attention through examples demonstrating the importance kinematic knowledge as the first but very critical step in designing and developing any multi-DOF systems as a part of systems engineering effort. Three major topics are highlighted in the book: use of matrix transformations between different domains, methods of optimization and the ability to develop displacement models to analyze system behaviors.

The results presented in this book have been complemented with research work published by the author and his colleagues. The manuscript of this book has been used to teach courses for undergraduate and graduate students at the University of Minnesota where the author has taught as an adjunct professor and also at the University of Information, Mechanics and Optics in St. Petersburg.

The author would like to express his deep gratitude to scientists, engineers and managers at MTS Systems Corporation where the author had opportunities to engage in very interesting and challenging projects over his career. Among them are Gary Gronert, Al Clark, Dr. Richard Lund, William Langer, Hugh Sparks,

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Brad Thoen, Judy Carmen and Charles Anderson. Special thanks to Judy Taylor and Dr. Gary Gutman for reading the manuscript and helping with editing the book.

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## DEDICATION

*This book is dedicated to the memory and the legacy of Professor Faydor L. Litvin as a world-class scientist, an outstanding researcher, a dedicated teacher, and a friend for his contribution to the world of kinematics.*

## INTRODUCTION

When we analyze or model practically any mechanical system, we have to deal with a few physical phenomena, such as the type and number of joints, flexibility of the links that connect those joints, friction within joints, and effects of the system's inertial parameters, including the components' mass and acceleration. The entire picture of the system's behavior can be simple or very complicated based on, a) the complexity of the mechanism, and b) the depth of the details to be investigated. Practically any design, analysis or modeling project would start with learning something about the system under investigation, something that would help the researcher or engineer start thinking about the apparatus involved.

In many cases, the engineering analysis starts from a pencil sketch where the general terms and critical attributes for this mechanism are presented, sometimes in rudimentary form. Among the engineer's first thoughts would likely be a question about how the mechanism would move. At this point, it would be too early to ask very many other questions or to suggest answers, as we are at this mechanical system's very beginning.

The first step is to understand how the system would move, which links could be considered prime movers, and how many of them the system would require to function properly?

Modern kinematics is the part of mechanical engineering where the engineer can apply and combine CAD systems, computer science, and established numerical methods to answer these questions.

It is very natural for the engineer to start with kinematics, given that almost any other discipline involved in design and modeling of a mechanical system would require knowledge about its motion. This particular step in kinematics is called the "displacement model." As engineers, we cannot "jump" over this step, regardless of the system's complexity, size, or application.

In simple terms, kinematics is the collection of methods used to analyze, describe, and present the functionality of a mechanism under so-called "simple" service conditions: no load or friction or masses, and with all components infinitely stiff. Based on the Pareto principle, nearly 80% of all results (or the outcome of any design) would come from just 20% of effort, including errors or other misdiagnosed issues. Investing in kinematics during the design stage would bring

the biggest return—a successful system. Thus, kinematics should always be part of the engineering phase of system development.

Kinematics is so effective because it does not require any expensive hardware or time-consuming experiments that would be difficult to simulate. Its assumptions are very clear. These days, commercial CAD/CAM packages and computer capability make kinematics the winning choice in any engineer's or researcher's toolbox.

Several components of kinematics are presented here to help both learning and practicing designers, including graduate and undergraduate students. These components are:

- 3D depiction of the mechanism, regardless of the degree of sophistication (from very detailed CAD systems to very simple stick models), with the ability to establish the coordinate systems that the application requires,
- knowledge about joints and their “correct” use and applications within the mechanism,
- what are the skills required to be a fluent operator of matrix transformation between different domains, and
- knowledge and experience in numerical methods, such as the optimization methods required to find solutions for displacement models by solving nonlinear systems.

Those and other associated topics are discussed and demonstrated within this book and include typical examples based on more than 45 years of experience in the field.

## **CHAPTER 1**

The main emphasis in this chapter is to show how different vectors are presented in the 3-dimensional (3D) world, including the construction of coordinate systems and coordinate transformation between different domains. These principles are demonstrated with examples of successive coordinate transformations and demonstrations of the object's orientation in 3D space. An introduction to the main properties of 3D surfaces, parametrization of normal vectors, and a plane tangential to any point on the surface are also discussed here.



## **CHAPTER 2**

Descriptions of the major joints commonly used in engineering designs and their applications are discussed here. Methods to develop the displacement model using closed and open contours are also presented. This chapter also offers detailed discussion and examples of a very common mechanism—the four-bar linkage. The Povodkovi mechanism is used to demonstrate some kinematic issues, with methods for solving the displacement model included at the end of the chapter.

## **CHAPTER 3**

The Uhing mechanism, with its given topology, is widely used in different industries and is considered here. However, this mechanism has limited ability to function with existing errors in mounting conditions, which can lead to fatal cases during service. A few design changes, including the configuration of some joints, are presented here.

## **CHAPTER 4**

This chapter focuses on the problem of solving multivariable systems within highly nonlinear equations, a skill required of any researcher faced with solving a displacement model in practically any kinematics application. The most common optimization methods are discussed and presented here, as well as a comparison of those methods.

## **CHAPTER 5**

A full 6 DOF robotic arm with a number of design considerations is presented here. A range of motion and cross-coupling issues, including domain mapping, are topics for consideration. Trajectory control with instability issues is demonstrated in this chapter, and Jacobian methods for the 6 DOF robotic arm are introduced here.

## **CHAPTER 6**

This chapter presents the major steps to develop the general-purpose 6 DOF robotic arm, with the main goal to demonstrate the critical aspects of the arm as a mechanism. Direct and inverse tasks are also presented here with a few alternative solutions, and different methods are analyzed and compared.

## **CHAPTER 7**

A typical design for a generic Light Manufacturing Tool (LMT) is presented and discussed, and a number of major coordinate systems are established to enable the development of the displacement model. The discovery and description of singularity and the singularity cone are a critical part of the displacement function for LMT as a system. Two types of singularity are presented, with a discussion on potential problems. A strategy for controlling the system within the singularity cone and adapting an extra DOF when required is another topic here, along with the introduction of phantom DOF and an analysis of its limitations in real applications.

## **CHAPTER 8**

A 6 DOF testing platform is common in many applications, and this chapter discusses the 6 DOF platform as a closed kinematic chain mechanism versus a 6 DOF robotic arm, which is an open chain mechanism. In a table application, the specimen or the table itself is supported by a number of actuators attached to the stationary base (wall and floor). Actuators in a platform system are the prime movers that use some sort of parallel motion to provide overall system motion. In a robotic arm, most of the links in the mechanism are connected in series to provide the desired motion of the end effector. Jacobian methods are common for analyzing the 6 DOF table or platform, and both the direct and inverse tasks are presented here.

## **CHAPTER 9**

Synergistic systems, also known as a Stewart platform, are used in a number of applications such as flight simulators and driver training simulations. Sometimes the Stewart platform is also used as an alternative solution for traditional table-type platforms. Both Stewart platforms and table platforms are closed kinematic chain mechanisms with supporting actuators acting in parallel with all 6 DOF to position the end effector (upper platform) at the desired location with a given trajectory. The application under consideration in this chapter is based on a Stewart platform in conjunction with a 2 DOF x-y table with a dome mounted on a rotary table on top of the platform. As a result of this configuration, it becomes a 9 DOF system, where some of the extra DOFs are used for the so-called washout algorithm used during the simulation.

**CHAPTER 10**

Analysis of tire behavior under real road conditions represents a very important subject for car and truck performance. Tire quality influences fuel economy, performance, and, even more important, safety issues. There are many examples of test equipment that simulate road conditions and bring tire testing capability to the laboratory-controlled environment. One of the first tire certification characteristics is the rolling resistance. It determines the efficiency of the tire on the road. The other characteristic is tire durability or, practically speaking, material testing of the design under vertical load and at different highway speeds. This performance verification becomes even more critical and sophisticated when steering and camber angles are applied during the test. Most of those parameters can be obtained under a number of real-time road conditions on the tire tester. Although using a large road wheel is not ideal for all aspects of these tests, it does represent the most economical solution to the problem.

**CHAPTER 11**

This chapter provides an analysis of a biomechanical joint, in this case, the temporomandibular joint (TMJ), to establish its kinematics in the same way as for mechanical joints. A short introduction to TMJ topology is presented with an attempt to determine the number of degrees of freedom (DOF). This chapter also discusses the instrument used to uncover the TMJ's "inside" design and establish "patient-specific information," along with the process to set up the digital recorder used to record and preserve the trajectory. A system of three-sensor clusters, each with three digital sensors, is used to monitor and record the motion, and the displacement model for this recorder is also developed here. In addition, this chapter discusses a general type of Space Location System (SLS) used in many measurement systems with tracking modules and target arrays.

**CHAPTER 1****Coordinate Transformation**

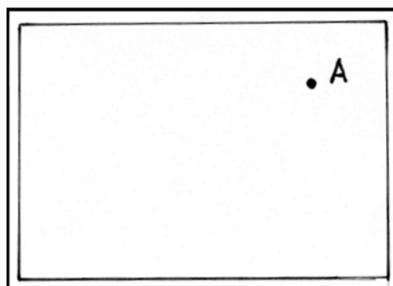
**Keywords:** Cartesian coordinate system, Coordinate system, Coordinate transformation, Curvilinear coordinates, Directional vector, Degrees of freedom, Matrix operations, Normal vector, Origin, Parametrization, Projections, Radius vector, Reference coordinate system, Relative coordinate system, Right-handed coordinate system, Skew coordinate system, Tangency, Tangent vector, Three-dimensional (3D) space, Translational coordinates, Vector operations.

**INTRODUCTION**

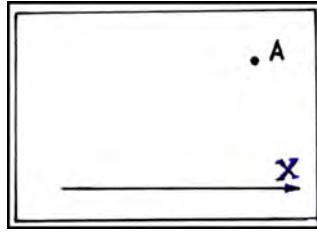
The subject of this chapter is how different vectors are presented in the 3-dimensional (3D) world including the construction of coordinate systems and coordinate transformation between different domains. These methods are supported with examples of successive coordinate transformations and demonstrations of the object's orientation in 3D space. An introduction to the main properties of 3D surfaces, parametrization of normal vectors, and a plane tangential to any point on the surface are also discussed here.

**POINTS IN SPACE**

The position of points in space plays a very important role in motion analysis. First, we must state some basic definitions. In Fig. (1), we find point "A" in the "window "W." This alone gives no information about point A's position in space.

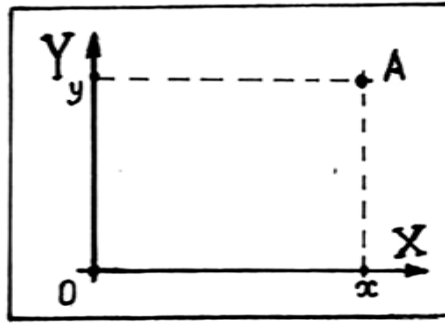


**Fig. (1).** A single point A in space.



**Fig. (2).** A first landmark in space.

This situation is like looking at a map too closely and finding no distinguishable features that we can use to judge the relative position. By zooming out, we find familiar landmarks. Similarly, in this case, our first "landmark" is a single distinguished line (Fig. 2), which we mark with an "X".



**Fig. (3).** Two coordinates in space.

Continuing to zoom out, we discover a second line "Y" (Fig. 3). Now we can establish the position of point "A" relative to the two lines "X" and "Y."

We usually use two numbers to represent the perpendicular distance from point "A" to each line. These distances are called coordinates (Fig. 4) and are designated by a lowercase "x" and "y", respectively. The x-coordinate is called the *abscissa*, and the y-coordinate is called the *ordinate*. The line "X" is called the *x-axis*, and the line "Y" is called the *y-axis*. This system of two axes is a *coordinate system*, and the intersection of these two defined axes is the *origin* of the coordinate system.

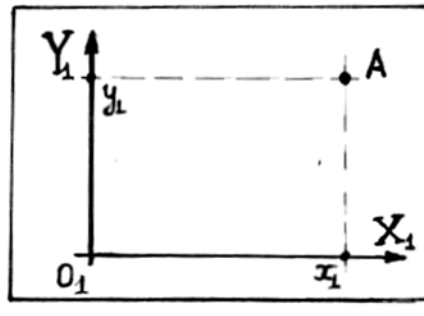


Fig. (4). Abscissa and ordinate for point A.

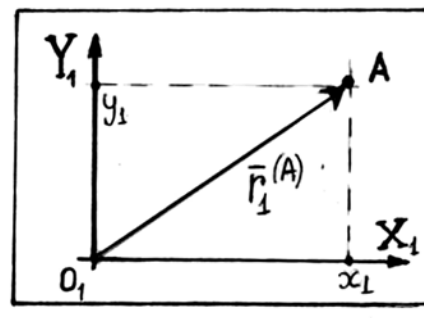


Fig. (5). Radius vector.

### Radius Vectors

As shown above, a set of two numbers (coordinates) represents the position of any point in a given coordinate system  $S_1$ . The position of the point "A" (Fig. 5) can be equivalently defined by directing a line segment that starts at the origin of  $S_1$  and terminates at point "A." This is the *radius vector*  $\bar{r}_1^{(A)}$ , which also has two projections onto the axes of  $S_1$ :

$$\bar{r}_1^{(A)} = \begin{Bmatrix} \mathbf{r}_x^{(A)} \\ \mathbf{r}_y^{(A)} \end{Bmatrix} \quad (1)$$

where:

$\mathbf{r}_x^{(A)}$  is the x-coordinate of point "A"

---

**CHAPTER 2****Pairs and Joints**

**Keywords:** Absolute movement, Closed contour method, Cylindrical pair, Displacement model, Five-by-five method, Four-bar linkage, Joints, Kinematic pairs, Links, Open vector contour method, Pairs, Parameters of motion, Phantom DOF, Povodkovyi mechanism, relative movement, Revolute pair, Sliding pair, Spherical pair, Three-by-three method, TLMWHIP.

**INTRODUCTION**

This chapter provides a discussion on major joints commonly used in engineering designs and their applications. Methods to develop the displacement model using closed and open contours are also presented. This chapter also offers detailed discussion and examples of a very common mechanism—the four-bar linkage. The Povodkoviy and direct-contact mechanism are used to demonstrate some kinematic issues, with methods for solving the displacement model included at the end of the chapter.

**KINEMATIC PAIRS OR JOINTS**

A *link* is a rigid part, which has no internal moving parts but with elements which can be used to joint this link with other links. A *joint* is a structural unit in which relative movement or a transfer of motion between links occurs. Thus, a joint (also called a “kinematic pair”) is a “coupling” that connects two links together, and allows two links to move relative to each other. A *Kinematic chain* is made up of collection of links with the joints between them [11, 12, 17, 18, 19, 21]. A *mechanism*, as formally defined, is a kinematic chain where one link is fixed.

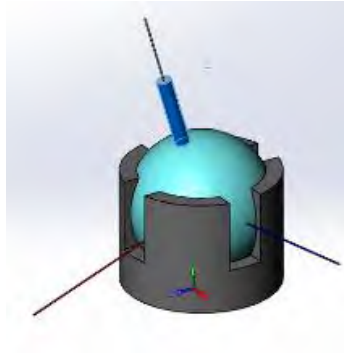
Kinematic pairs are divided into *higher* and *lower* types according to the region and nature of contact between the surfaces that make up the joint. This region of contact is necessarily the totality of the points of tangency between the surfaces of the joint.

Higher kinematic pairs have points (or lines) as their nature and regions of tangency, while lower pairs are in contact with each other using surfaces. The position of lines, which need not be straight, is vague in this classification system. We will group pairs that have lines as their regions of contact with the higher kinematic pairs.

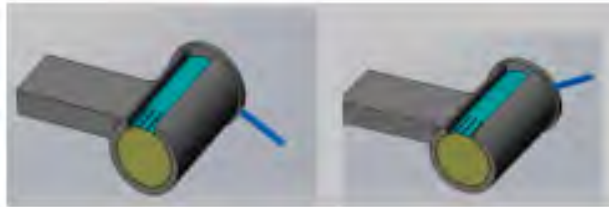
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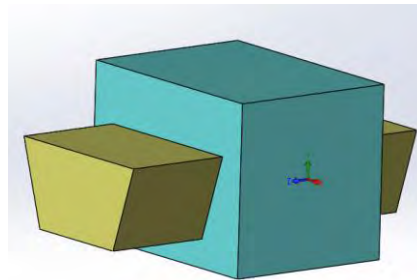
Examples of lower kinematic pairs are displayed in a few figures. Fig. (1) is a ball and socket joint with three degrees of freedom (DOF).



**Fig. (1).** Ball joint.



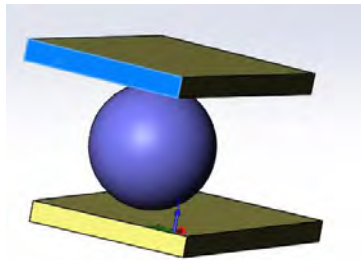
**Fig. (2).** Hinge.



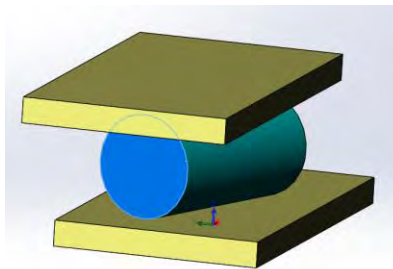
**Fig. (3).** Prismatic joint.

Meanwhile, Fig. (2) shows a hinge with a revolute pair providing one DOF, and Fig. (3) shows a dovetail as another example of lower kinematic pairs with one DOF.

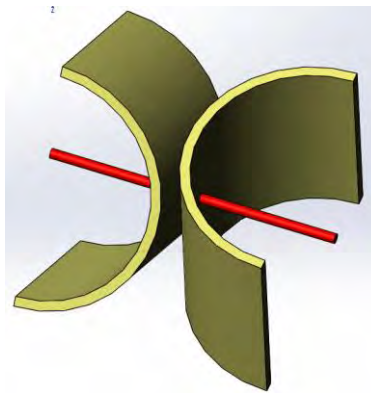




**Fig. (4a).** Point contact.



**Fig. (4b).** Line contact.



**Fig. (4c).** Contact between surfaces.

Higher kinematic pairs, though they lack such well-established names, are displayed in Fig. (4a) for a point contact between a ball and plane. Fig. (4b) shows a line contact between the cylinder and a plane and finally, Fig. (4c) depicts a point contact between two surfaces. We also have to consider the “design” of the kinematic pairs as closed and open joints.

**CHAPTER 3****Using Mechanisms-Displacement Model for Two Spatial Lever Mechanisms with Different Design Considerations**

**Keywords:** Articulation, Assembly conditions, Coordinate transformation, Degrees of freedom, Displacement model, Lever mechanism, Links, Pairs, Mating conditions, Mechanism, Open contour method, Position functions, Reference coordinate system, Shaft angle, Shaft distance, Spatial lever mechanism, Uhing mechanism, Vector projection.

**INTRODUCTION**

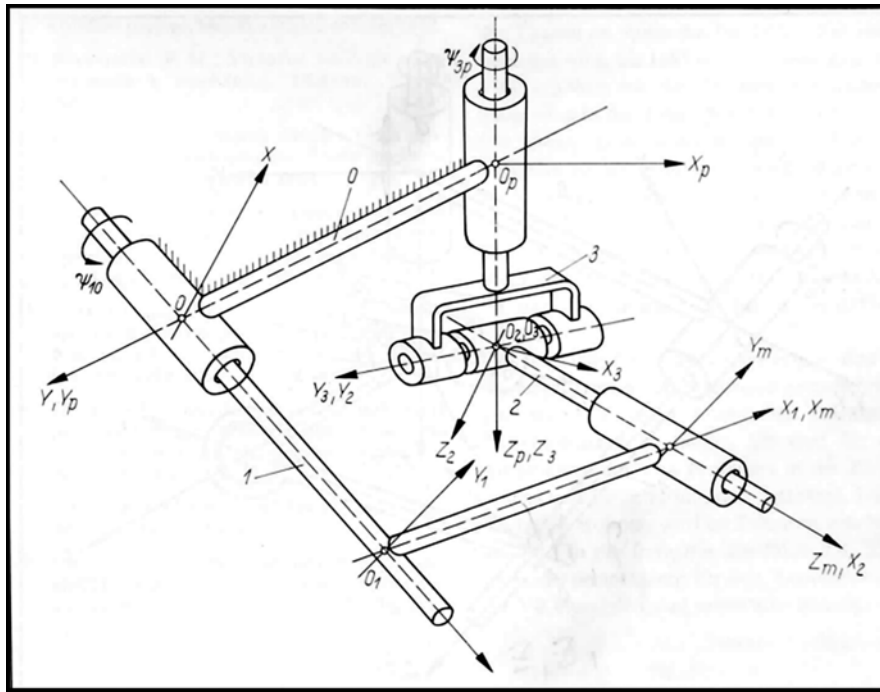
Determining the position of the links in three-dimensional lever mechanisms is one of the most complicated tasks in mechanism theory. Various methods have been developed to solve this task [1, 5]. A simplified method for determining the position functions of spatial lever mechanisms has recently been proposed [6], which is based on the conventional open contour method.

**Mechanism 1**

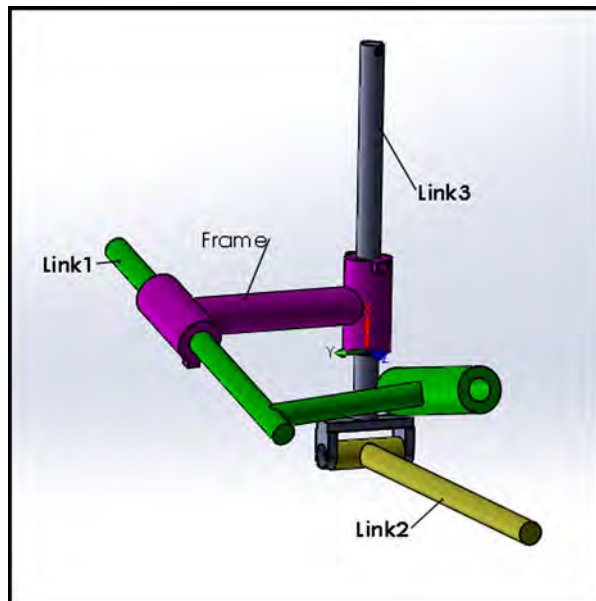
This method was used to determine the position functions of a four-link lever mechanism (Fig. **1a** and **1b**) first presented by Dr. Uhing [8]. The mechanism discussed here has been found in many applications within light industrial machine tools [7].

Depending on the relationships selected for some of the mechanism's design parameters, the drive member (Link1) rotates in either direction, which causes the driven member (Link3) to rotate. The goal of this analysis is, on the one hand, to examine the position function of the mechanism's elements (mechanism I) and to uncover the causes responsible for its high sensitivity to assembly errors.

To keep the shaft members from jamming, we can change the "structure" of Mechanism 1 to offer Mechanism 2 with similar functionality but without this disadvantage. This mechanism will be discussed later.



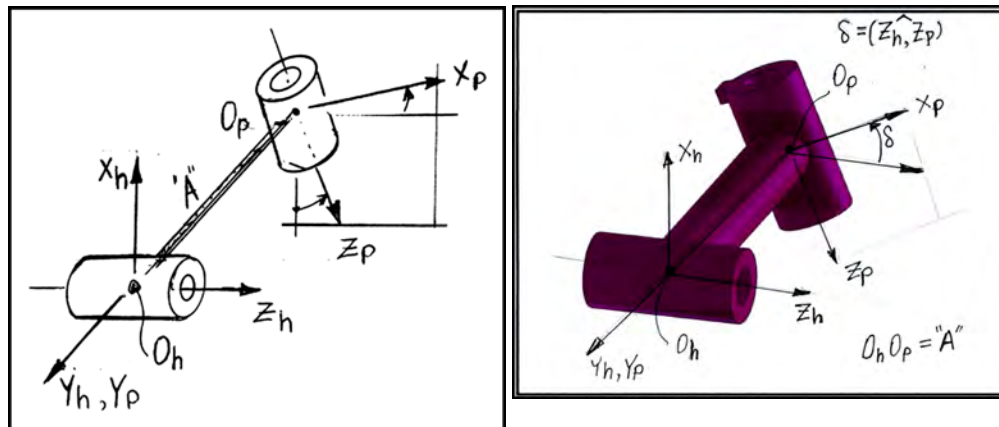
**Fig. (1a).** Main coordinates.



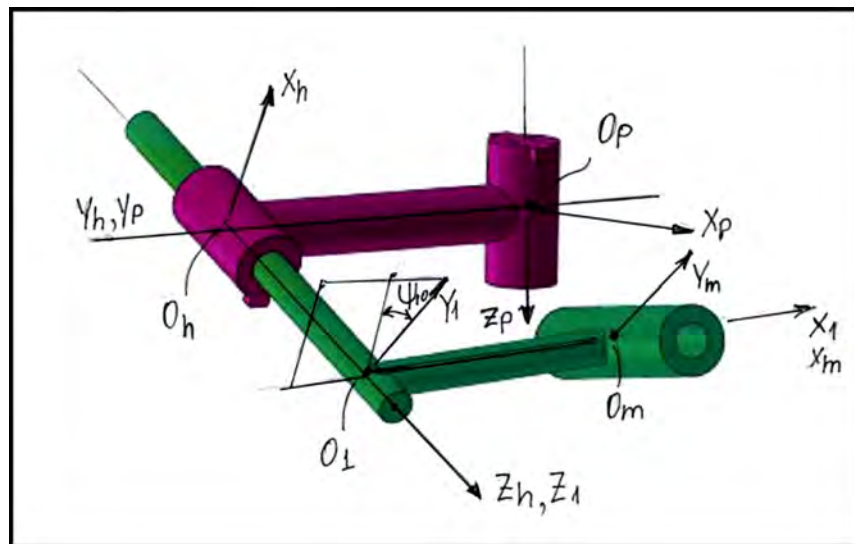
**Fig. (1b).** Original linkage.

**Coordinate Systems**

Several coordinate systems will be used here to determine the position functions of Mechanism 1. Two of these are stationary coordinate systems  $S_h$  and  $S_p$ , which are rigidly connected to the frame (Fig. 2). Four other coordinate systems are movable, with  $S_1$  and  $S_m$  fixed to Link1 (Fig. 3). Correspondingly, coordinate systems  $S_2$  and  $S_3$  are rigidly connected to Link2 and Link3.



**Fig. (2).** Main base/frame with geometry.



**Fig. (3).** Link1 within the frame.

## Optimization

**Keywords:** Advanced DLSM, DLSM, Finite difference method, Finite element method, Gradient-based method, Lagrange multiplier, Lagrangian, Linear approximation, Matrix of effect, Newton's method, Nonlinear equations, Optimization, Objective function, One-parameter optimization, Probe, Probe of derivatives, Search price, Singular matrix, Singular value decomposition, Optimization trajectory, Weighting functions.

### INTRODUCTION

We have seen that a number of mechanical engineering problems can be realized as problems that involve the optimization of certain properties. Until now, we have had to leave these examples unfinished, with only the promise that they could be completed with optimization tools—but no longer.

The solution of many engineering problems that involve the analysis and optimal synthesis of space mechanisms can be formulated as solutions of systems of nonlinear equations. Such problems include (a) finding the contact point in space gearings, (b) defining the displacement function for links in space mechanisms when it is impossible to find position by direct or final form; and (c) finding optimal design parameters for mechanisms to satisfy the desired synthesis criterion. Thus, the problem of finding solutions for systems of nonlinear equations is a central problem of analysis and synthesis, and it is one that can be dealt with separately.

In this chapter, we will develop a range of techniques that allow us to cope with the kinds of problems that have stumped us in the past. These results are mainly outgrowths of the minimum and maximum problems that are an integral part of an elementary calculus curriculum. Many of the fundamental theoretical results presented in this chapter are direct extensions of the above-mentioned methods, but not all of them. Among these issues are the fundamental points discussed [1-5]. It would be a good idea to now recount various places in the foregoing text where we noted that some sort of optimization process was necessary.

Newton–Raphson method, used in many places within this chapter is named after Isaac Newton (born c.1643) an English mathematician and physicist, and Joseph Raphson (born c. 1668) English mathematician of Irish descent.

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The first reference to this method could be found in Joseph Raphson's work "*Analysis Aequationum Universalis*", published in 1690. This work describes a method for approximating the roots of an equation, known today as the Newton–Raphson method.

Isaac Newton developed a very similar idea in his "*Method of Fluxions*", written in 1671, but not published until 1736, nearly 50 years after Raphson's Analysis.

There was a dispute about which implementation was better, however, the method used in modern mathematics carries the names of both mathematicians Newton–Raphson.

In numerical computing the Levenberg-Marquardt algorithm is used for solving of non-linear problems. It is also known as a Damped Least-Squares Method (DLSM). This algorithm was published first by Kenneth Levenberg [3] in 1944 and then, 40 years later in 1963, it was published by Donald Marquardt [15]. In this chapter, it is called DLSM by Levenberg.

We will consider comparative estimates of the best-known optimization methods and attempt to discover the diversity of solutions obtained by these methods and how to eliminate their problems. This part describes the main steps in the optimization algorithm that was used for the analysis and synthesis of some mechanisms [6-10, 13].

Following some introductory and preliminary material, the first part of the text was devoted to analyzing mechanisms. It was here that the need for optimization techniques first arose. The first analysis example, that of a four-bar linkage, led to the simplest of all trigonometric equations:

$$A \sin \phi_2(\phi_1) + B \cos \phi_2(\phi_1) = C \quad (1)$$

This was easily solved in a closed form, and we went on to more complicated mechanisms. There we encountered a variety of methods, all of which led to the formation of systems of nonlinear (*transcendental*) equations that we could not conceivably solve in closed form or, rather, that we would not want to even attempt to solve.

The other systems that we found were five-by-five and three-by-three. These are rather small systems, and their special nature, though not as simple as the four-bar linkage, made the job at hand even easier. The general case, which we also

analyzed, promised much greater complexity and showed us a true need to find a method for solving these systems.

## OBJECTIVE FUNCTION

The most natural way to solve such systems quickly emerged. The method we discovered, at least schematically, is very simple: guess the solution of the system and plug it in to see whether it is correct. This is an iteration scheme, where we minimize the difference between our provisional solution and the real solution with successively better guesses. This in turn can be seen as an optimization process, where we attempt to minimize the error of our guesses. We call this function (in more rarified terminology) the *objective function*.

Our next task was a synthesis problem, where we were faced with an even greater optimization task. First, we chose a set of values for the constraint equations, then we solved the constraint equations to find the components of the dimensional parameter vector. Next, we inserted these dimensional parameters into the constraint equations to see how well we had done.

After this, the iteration process took over, which produced successively better results until we had a satisfactory set of dimensional parameters. At that level of complexity, even the four-bar linkage could not be synthesized without making the methods of this chapter available, because the system of equations for analyzing even such a simple mechanism proved too formidable for a closed-form solution.

Having realized the need for optimization, we now begin investigating the process itself. Because all our practical problems eventually reduce to the problem of solving systems of nonlinear equations, that is the aspect of optimization we will now focus on.

We must start with a few preliminary definitions. This means clarifying our concept of what a system of nonlinear equations is and what the word "optimization" actually means. We begin with a system that has a vector in an  $n$ -dimensional variable space " $\mathbf{W}$ " and places it in a vector in  $m$ -dimensional function space " $\mathbf{Y}$ ". Symbolically, this can be represented by the system:

$$\begin{aligned} y_1(w_1, w_2, \dots, w_n) &= y_1 \\ \dots & \\ y_m(w_1, w_2, \dots, w_n) &= y_m \end{aligned} \tag{2}$$

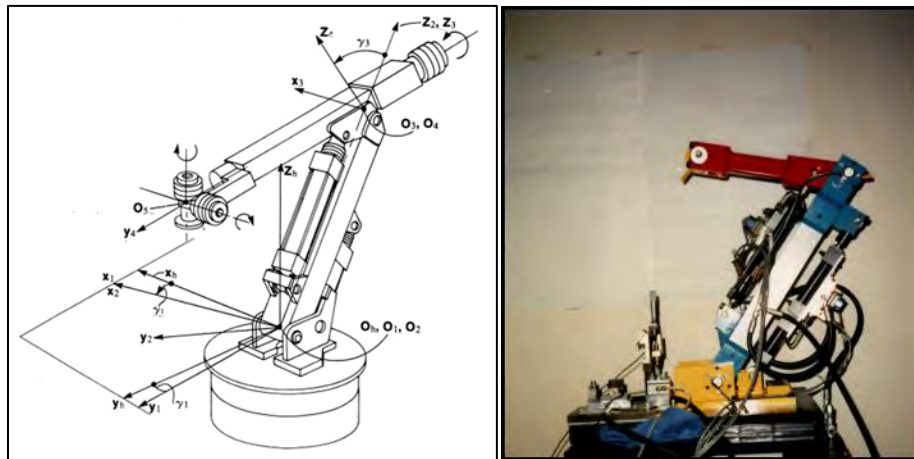
## Design Techniques for a Jointed-Arm Robot

**Keywords:** Actuator position, Control strategy, Cross-coupling, Cubic spline, design, Domain mapping, Instability, Joint DOF, Jacobian method, Jointed-arm robot, Key points, Linear actuator, Matrix of partial derivatives, Parameterization, Robot mechanism, Segment function generator (SFG), Singularity, Task DOF, Trajectory control, Two-arm mechanism.

### INTRODUCTION

An industrial robot is required to position objects within its workspace in a rapid and precise way. The robot designer must consider the control, kinematic, and structural aspects of the robot arm to achieve the best performance.

Robot mechanisms can assume several basic geometric arrangements, which are determined by the type of kinematic joint at each axis. It is common for robot mechanisms to be composed of combinations of only sliding and revolute joints, although spherical joints are used in some wrist designs.



**Fig. (1).** Typical configuration under discussion.

In the anthropomorphic or jointed-arm robot, at least the first three axes are revolute joints (Fig. 1).

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Here we use an anthropomorphic mechanism as an example to present some useful kinematic analysis techniques for designing a robot arm. The techniques described in [1-5, 8-0] can be applied to both the full six-degree-of-freedom robot and to a smaller part of the robot mechanism.

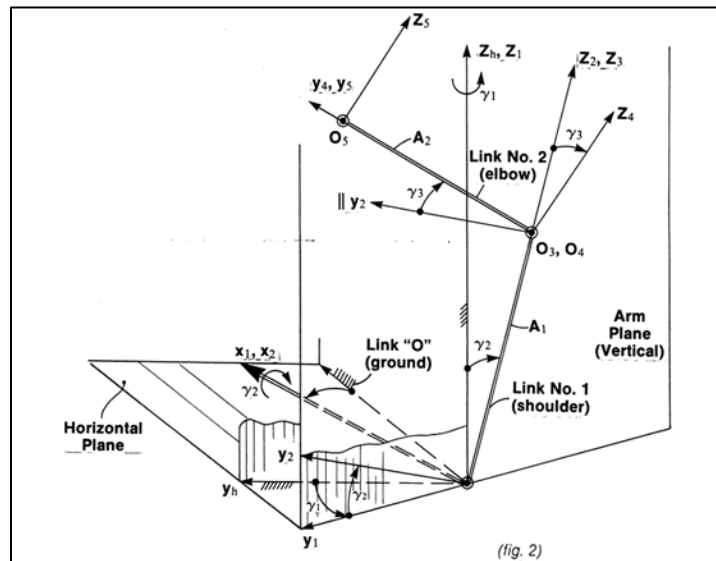
The use of robots can be demonstrated in the following operations:

- load/unload operations.
- assembly.
- manufacturing with the use of a tool carried by the robot (as a flexible CNC machine).

In all these cases, the mechanical system delivers an object in space in a controlled manner. Depending on the required task, the robot must have between one and six *degrees of freedom* (DOF). These DOFs are defined as the maximum number of Cartesian coordinates (including angular orientation) that the robot can change independently, and they are called *Task DOFs* (TDOF). Thus, for our mechanism,  $1 \leq \text{TDOF} \leq 6$ .

Because flexibility is a prime advantage of a system that performs these kinds of tasks, the number of controlled links or the number of joints that constitute the robot may exceed the number of TDOF. For example, the tool in a drilling operation requires motion in only one dimension (excluding the rotation of the drill head itself). However, because the part to be drilled through must be properly positioned, an extra robot DOF to position the drill bit (in addition to the vertical TDOF) is desirable. The number of controlled joints (the Joint DOF or JDOF) must be distinguished from the TDOF.

It is convenient to divide a six degree-of-freedom robot mechanism into three parts: a base with one degree-of-freedom, an arm with two degrees of freedom, and a wrist with three degrees of freedom. Here, the second and third axes of a jointed-arm robot are considered as a submechanism, referred to here as the "two arm" mechanism (Fig. 2) to illustrate the design techniques.

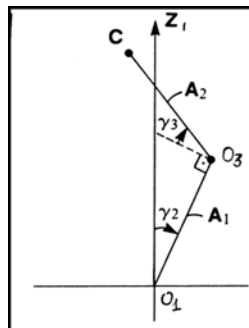


**Fig. (2).** Major coordinate systems.

The first three DOFs move the wrist into position and thus provide "carriage" for the wrist. The kinematic models for the full six degree-of-freedom jointed-arm robot are developed and presented in Chapter 6, "Displacement Model and Kinematics of a Six DOF Robot Mechanism".

### DEGREES OF FREEDOM IN DIFFERENT DOMAINS

The two-arm mechanism can be considered a two-dimensional mechanism that establishes and determines an arm plane, with the center of the wrist attached to the mechanism at the end of the arm at Point C (Fig. 3). Thus, this mechanism has just two DOFs.



**Fig. (3).** Two-arm mechanism.

## CHAPTER 6

# Displacement Model and Kinematics of a Six DOF Robot Mechanism

**Keywords:** Damped least squares method, DLSSM, Direct task, Finite differences method, Gradient method, Inverse task, Iteration, Jacobian, Joint coordinates, Newton method, Objective function, Optimization, Parallel translation matrix, Partial derivatives, Probe function, Probe of derivatives function, Singular value decomposition, Standard least squares method, World coordinates.

## INTRODUCTION

This chapter presents major steps to develop the general-purpose 6 DOF robotic arm, with the main goal to demonstrate the critical aspects of the mechanical arm as a mechanism. Direct and inverse tasks are also presented, with a few alternative solutions, and different methods are analyzed and compared.

## ROBOT ARM MOTION PARAMETERS

The most common configuration of a robotic mechanical arm is shown in Fig. (1). The displacement model can describe the kinematic behavior of this mechanical system with the coordinate systems described below.

System  $S_h$ , is rigidly connected to the fixed part of the robot base "O" (or with the ground), and its orientation is seen in Fig. (2). Angle  $\gamma_1$  reflects the rotation of the *arm plane* around the *base*, which is the same as its rotation around vertical axis  $Z_h$ . The arm plane is rigidly connected to coordinate system  $S_1$ , which generates an angle  $\gamma_1$  relative to the stationary coordinate system  $S_h$ . (Figs. 2 and 3).

The matrices that transform coordinates from  $S_1$  to  $S_h$  can be written as a fourth (4th) order transformation matrix  $M_{h,1}$ :

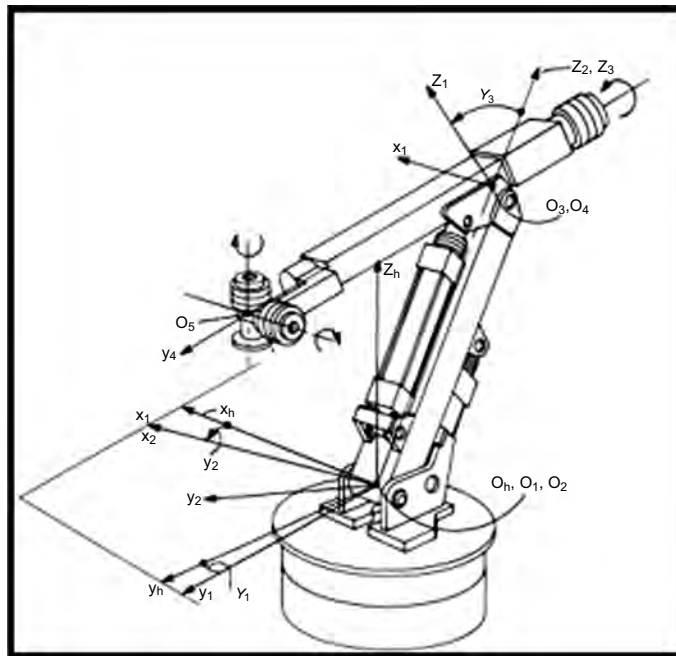


Fig. (1). Common robotic arm.

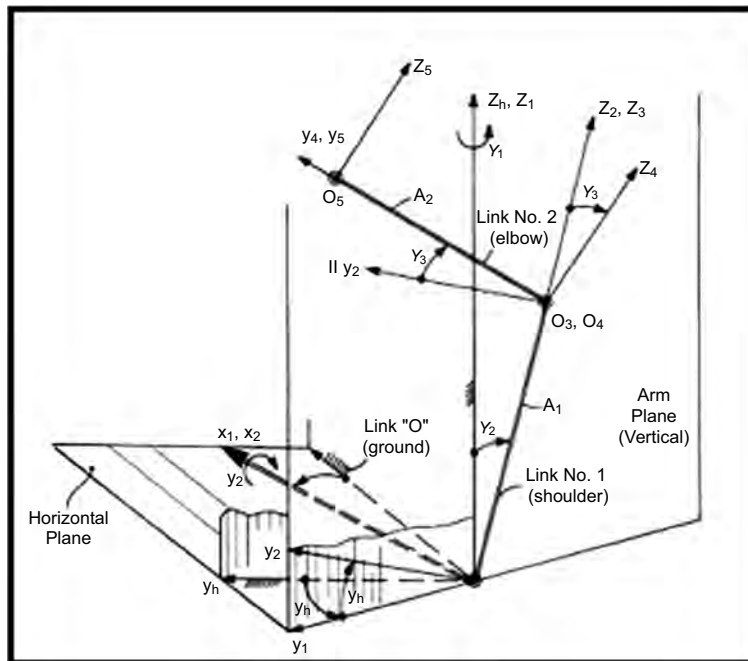


Fig. (2). Arm plane.

$$\mathbf{M}_{h,1} = \begin{bmatrix} \cos \gamma_1 & -\sin \gamma_1 & 0 & 0 \\ \sin \gamma_1 & \cos \gamma_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{M}_{1,h} = \begin{bmatrix} \cos \gamma_1 & \sin \gamma_1 & 0 & 0 \\ -\sin \gamma_1 & \cos \gamma_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The next step is to define matrix  $\mathbf{M}_{2,1}$ , which links systems  $\mathbf{S}_2$  and  $\mathbf{S}_1$ . This new coordinate system  $\mathbf{S}_2$  is rigidly connected to  $\mathbf{S}_1$  with a Link1 (otherwise known as an *arm*), and it differs from  $\mathbf{S}_1$  by rotation through an angle  $\gamma_2$  (Figs. 4 and 5) around axis  $X_1$  in the vertical arm plane, which gives us  $\mathbf{M}_{2,1}$  and  $\mathbf{M}_{1,2}$ :

$$\mathbf{M}_{1,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \gamma_2 & -\sin \gamma_2 & 0 \\ 0 & \sin \gamma_2 & \cos \gamma_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{M}_{2,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \gamma_2 & \sin \gamma_2 & 0 \\ 0 & -\sin \gamma_2 & \cos \gamma_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Until now, we have used only rotation matrices. Now a new type of matrix is required: a parallel translation matrix,  $\mathbf{M}_{3,2}$ .

Consider a new coordinate system  $\mathbf{S}_3$ , which is parallel to  $\mathbf{S}_2$  but has its origin at point  $\mathbf{0}_3$  at distance  $\mathbf{A}_1$  from the  $\mathbf{S}_1$  system origin. The translation matrix in this case is  $\mathbf{M}_{3,2}$  ( $\mathbf{M}_{2,3}$ ) (Figs. 2 and 4).

$$\mathbf{M}_{2,3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathbf{A}_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{M}_{3,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\mathbf{A}_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

**CHAPTER 7****Gantry System as a Light Manufacturing Tool (LMT)**

**Keywords:** 5 DOF system, Control system performance, Extra DOF, Gantry, Gantry system kinematics, Light manufacturing tool, Orientation, Phantom DOF, Performance degradation, Pitch, Position, Singularity, Singularity cone, Singularity issues, Singularity position, Trajectory control, Velocity control, Velocity model, Yaw.

**INTRODUCTION**

It is a common practice to specify a minimum number of kinematic degrees of freedom for a machine to perform a certain job. Unfortunately, in many cases, practical design limitations limit system motion (speed, acceleration) in at least one of these specified degrees of freedom. The result is an overall degradation of system performance, with significant restrictions in motion and the size and shape of the service area. A few approaches that avoid this phenomenon using "phantom" DOF are discussed here.

This chapter is devoted to problems related to the design and control of a gantry system, which is a light manufacturing tool (LMT) with a variety of applications. The material presented in this chapter is based on the author's prior publications and collaborations with other scientists [7,14].

**GENERAL ANALYSIS OF GANTRY SYSTEM KINEMATICS**

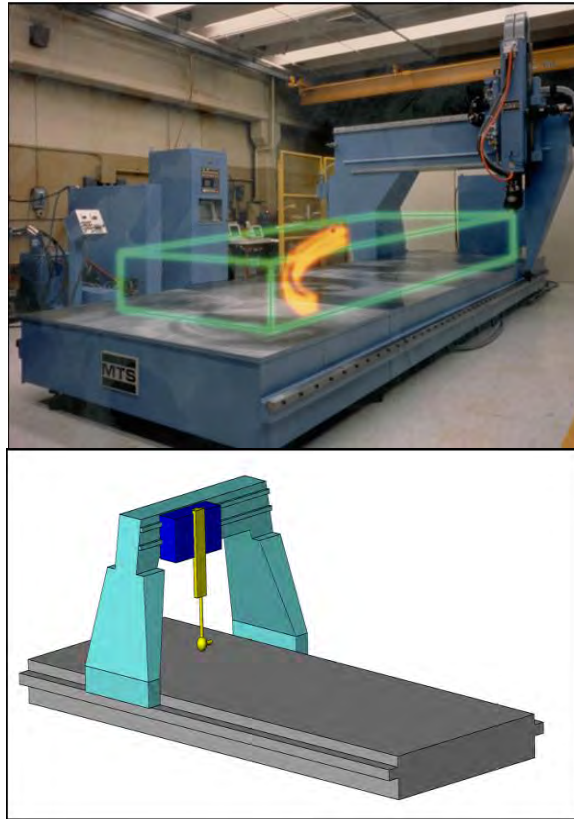
The first topic to discuss is a general analysis of gantry system kinematics, including all coordinate systems and transformations [1-6]. The second is a singularity problem, when control becomes a very important issue [7-10], and two approaches to this singularity problem are described here. The first approach is to use an extra degree of freedom (DOF) with a special algorithm to control it when the system approaches the singularity position [11-13]. The second approach is to use a so-called "*phantom*" degree-of-freedom, which exists only in the control algorithm, not in the actual system. A significant part of this material has been previously discussed and published in detail by this author and colleagues ("Robot Singularity Rate Control with Phantom DOF Strategy" [7]).

Yevsey Gutman

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## COORDINATE SYSTEMS FOR A 5 DOF SYSTEM

The gantry system shown in Fig. (1), often called a “Light Manufacturing Tool” (LMT), can be used for tasks that require from three to six DOF.



**Fig. (1).** Examples of light manufacturing tool configurations.

Fig. (2) shows a side view of the gantry and two coordinate systems,  $S_g$  and  $S_h$ . System  $S_h$  is the main coordinate system, and it is rigidly connected to the base of the LMT. This coordinate system should be established during system calibration using fixed pads on the base and internally as a world coordinate system. System  $S_g$  is rigidly connected to the center of the wrist  $O_g$ . Thus, the position of the gantry coordinate system  $S_g$  relative to the main coordinate system  $S_h$  can be obtained with matrix  $M_{h,g}$  or  $M_{g,h}$ .

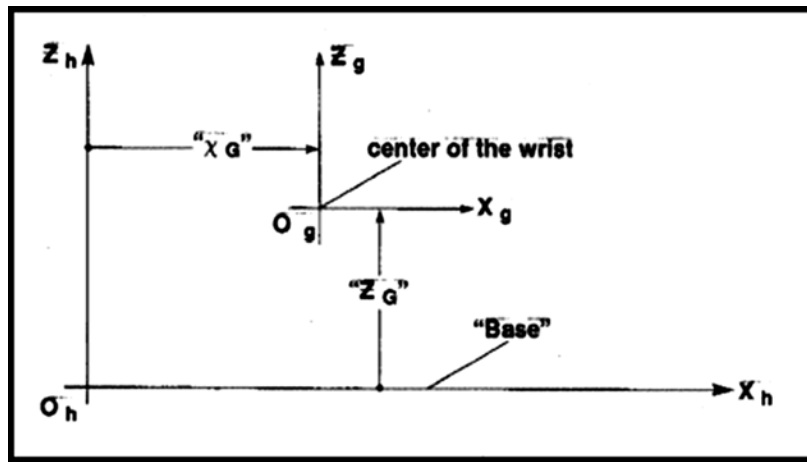


Fig. (2). Location of two-coordinate systems.

$$M_{h,g} = \begin{bmatrix} 1 & 0 & 0 & X_g \\ 0 & 1 & 0 & Y_g \\ 0 & 0 & 1 & Z_g \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad M_{g,h} = \begin{bmatrix} 1 & 0 & 0 & -X_g \\ 0 & 1 & 0 & -Y_g \\ 0 & 0 & 1 & -Z_g \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Here  $X_g$ ,  $Y_g$ , and  $Z_g$  are the joint DOFs as displacements in the gantry’s first translational degrees of freedom.

Fig. (3) illustrates another view of the gantry position relative to the main coordinate system.

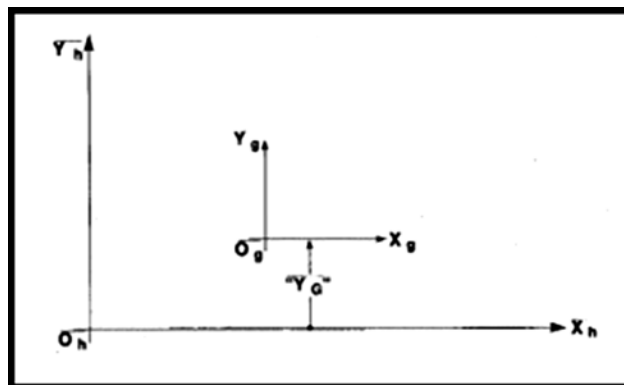


Fig. (3). Position of the gantry in the main coordinate system.



**CHAPTER 8****Kinematics of Testing Platforms**

**Keywords:** Actuator attachment, Actuator degrees of freedom (ADOF), Actuator length, Actuator motion, Actuator position, Actuator stroke, Coordinate transformation, Degrees of freedom, Direct transformation, Displacement model (DM), Feedback, Inverse transformation, Jacobian model, Linear actuator, Pitch, Platform, Sensors, Testing platform, Velocity model (VM), Yaw.

**INTRODUCTION**

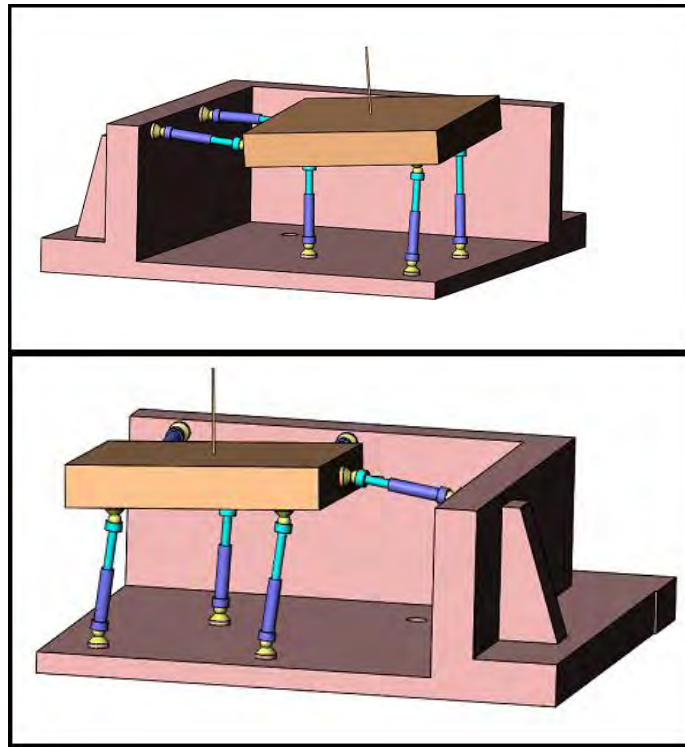
This chapter is devoted to the problem of modeling the motion of a rigid 6 DOF platform suspended on linear actuators by creating mathematical algorithms for the forward and inverse transformations between the platform coordinate system and actuator motions: displacement, velocity, accelerations, and forces. It also describes the main coordinate systems involved in these kinematic transformations. These transformation algorithms are important to establish the relationship between platform-independent DOF and actuator commands for computer analysis and real-time (online) control enhancements.

**TESTING PLATFORMS**

Large testing platforms are widely used in many recent dynamic testing applications [1] that involve real-time structural analysis, and advances in testing technology show promise for wider applications in the future. Application examples include earthquake simulation, environmental simulation for packaging, environmental evaluation systems for automotive components, spacecraft vibration simulation, and other future possibilities.

Control and actuation technology has advanced to where a greater dynamic range in frequency and amplitude and higher accuracies are achievable [2-5]. In general, such platforms can be considered as a moving table suspended on a number of actuators. The number of actuators and the design type of the kinematic support depend on the required degrees of freedom (DOF) of the platform and the nature of the motion, as well as the given mass and elasticity distribution of the specimen on the platform.

A general testing platform system is presented in Fig. (1). It consists of a platform *suspended on* six actuators, a control system, and feedback transducers.



**Fig. (1).** Examples of 6 DOF moving platforms.

It is easy to see that the platform has only 6 rigid-body degrees of freedom (DOF) even though eight or more actuators might be attached to it. This establishes a certain cross-coupling (or linear dependency) between the actuators, which should be addressed in the motion control strategy and technology.

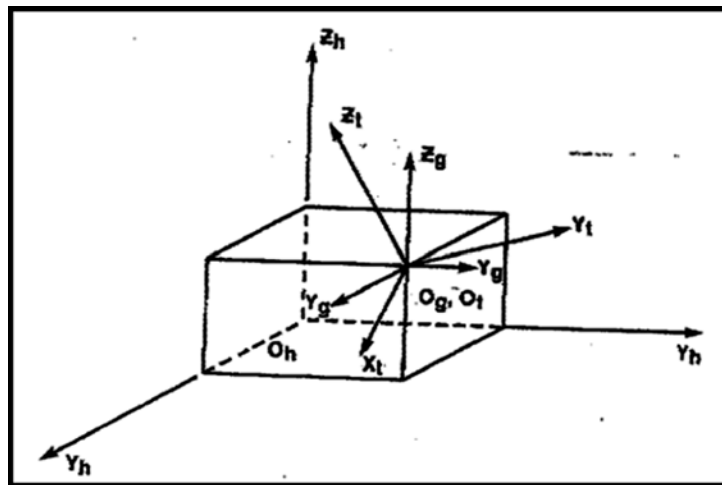
It is very important to distinguish between the platform DOF and the supporting system DOF. The platform DOF can vary from one to six and should be understood as an independent DOF relative to a reference coordinate system that is rigidly connected to the "world" (an inertial base). The supporting system DOF depends on the number of active supporting members (such as actuators). The actuator system degrees of freedom (ADOF) must be equal to or greater than the platform system DOF. If the ADOF is greater than the DOF, some of the actuators must be dependent.

**Note:** This chapter discusses only the platform with six DOFs suspended on six actuators. Three vertical actuators are under the table, and three horizontal actuators are between the table and the “strong wall” attached to one side of the table.

## COORDINATE SYSTEMS

To define the relationship between the platform and actuator motions, we must establish a set of coordinate systems.

Starting with the coordinate system  $S_t$  (which is rigidly connected to the table) and using three independent angles (Euler angles), we can derive the following relationships between all degrees of freedom (Fig. 2).



**Fig. (2).** Coordinate system using three angles.

The desired position and orientation of the table coordinate system  $S_t$  in the reference coordinate system  $S_h$  can be established by the given radius vector of the center (origin  $O_t$ ) of table  $R_h^{(O_t)}$  and three angles  $\varphi_1$  (yaw),  $\varphi_2$  (roll), and  $\varphi_3$  (pitch), or by two unit orthonormal directional vectors  $\bar{C}_x$  and  $\bar{C}_y$ , which represent the orientation of the table's main axis.

Then, using a 4th-order kinematic transformation matrix [1,7], the coordinate transformations between all systems can be obtained. Matrix  $M_{h,g}$  ( $M_{g,h}$ ) (Fig. 3) is used to transfer coordinates between the reference system  $S_h$  and the

**CHAPTER 9****Motion Analysis of a Driving Simulator with a Nine Degrees of Freedom Synergistic System**

**Keywords:** Actuators, Actuator domain, Computer simulation, Coordinate systems, Design tools, Displacement model, Driving simulator, Flight simulator, Home position, Motion equations, Motion parameters, Motion strategy, Pitch, Roll, Rotary table, Stewart table, Stewart-Gough platform, World domain, X-Y table, Yaw.

**INTRODUCTION**

Computer simulation provides an otherwise impossible means for engineers to inspect in detail all the actions and interactions among a system's parts and subsystems to optimize the performance of the total system [1, 2].

For an intricate system such as a driving/flight simulator [3-6] with many complex interrelated subsystems and elements, motion simulation is an ideal way, and sometimes the only way, to observe the behavior of the simulator and the simultaneous motion of its components and subsystems.

Motion analysis of the simulator can be used to quantify and study the detailed motion (velocities, accelerations, and forces) within the simulator and to derive the characteristic handling parameters and dynamic load on the structure during maneuvers.

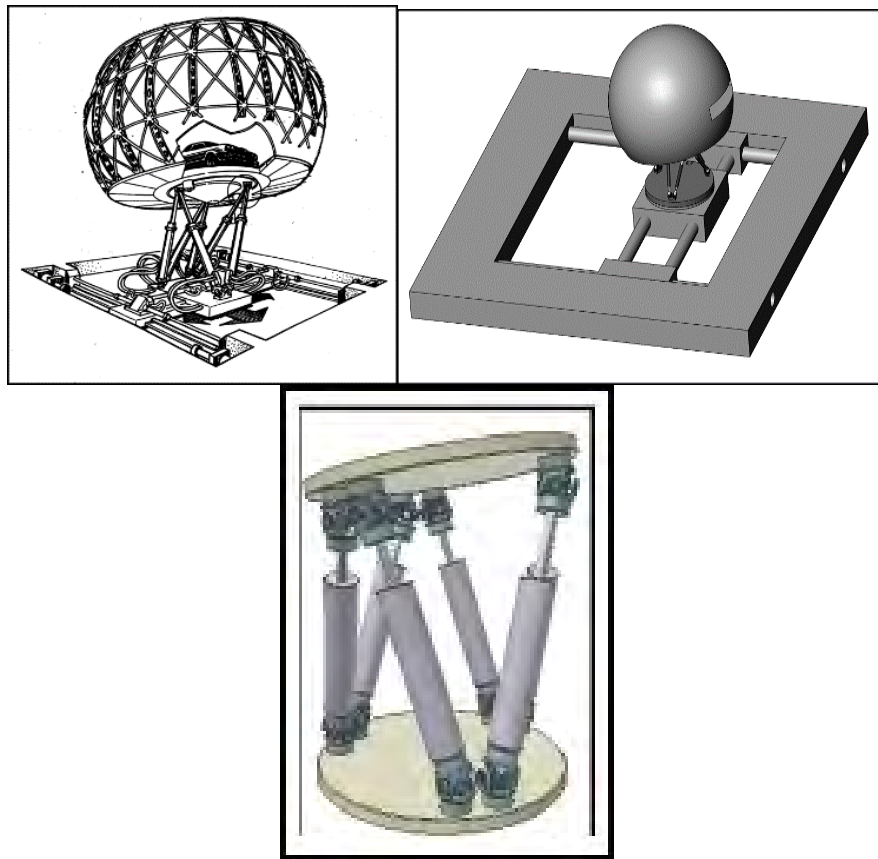
A driving/flight simulator is a complicated mechanical system that has a motion platform with an operator. It is driven by hydraulic actuators under precisely synchronized computer control with computer-generated visual images.

The main purpose of this chapter is to discuss a method that allows an operator to control the position of the payload platform using a simulator with nine degrees of freedom (DOF).

The full six-DOF motion of a synergistic system as applied to flight simulators has been used and studied for many years and is well known [3, 5, 6]. Those systems are more often known as Stewart platforms or Stewart-Gough systems [14,15]. First

invented by V.E. Gough in 1954, its design was published by D. Stewart in 1965. But it was patented by another engineer Klaus Cappel (Franklin Institute Laboratories) involved in the design and later fabrication of “six-legged moving platform” trainers produced by Link.

However, sometimes the traditional six DOF synergistic system does not provide the most realistic cues for a driver in the simulator. The high-performance driving simulator system designed by Evans & Sutherland and MTS Systems Corp. incorporates additional X-Y sliding movement (DOF numbers 7 and 8) that allows a synergistic system to extend acceleration efforts and provide realistic cueing for extended maneuvers. The system platform also has unlimited rotation (DOF number 9), which provides a realistic spin and yaw effect. Fig. (1) shows a conceptual view of this system.



**Fig. (1).** Illustration of a 9 DOF synergistic system.

Clearly, state-of-the-art techniques are required to design, build, and operate such systems. This chapter describes the unique design tools for a computer kinematic or displacement module required to develop a driving simulator.

## ACTUATOR AND WORLD DOMAINS

Previous papers [7, 8] have emphasized that two separate domains of motion variables from the motion control point of view must always be considered for systems such as road, flight, driving, and earthquake simulators.

The first group of parameters comprises the actuator system (prime movers). The actuator system consists of six linear actuators for the synergistic system, two linear actuators for the X-Y table, and one rotary actuator for the rotary platform—a total of nine actuators.

The design of the proposed driving simulator clearly shows that all nine actuators are uncoupled in such a way that each actuator can be moved without creating and storing any statistical internal forces in the system.

This is not the case, for example, when a platform suspended on eight linear actuators is used [8] for earthquake simulation. In that case, the motion for each actuator must be synchronized with all the other actuators. This not only provides the desired platform position, but also avoids any extra forces that could otherwise be applied to the system due to an over-constrained design.

Nine parameters or nine actuators represent a nine DOF actuator system that can be divided into three subsystems: a two DOF X-Y table, a six DOF synergistic system, and a one DOF rotary platform. All three subsystems are designed to *not* have over-constrained conditions and are built in "series." Compare this with the platform example above, which is built on eight actuators in "parallel" [8].

This group of nine variables represents the nine DOF domains of the servo-controlled motors/actuators and is the only source of motion for the entire system.

The other group of parameters comprises the *world* variables or coordinates. These parameters establish or obtain the world position of the specimen/operator during the simulation process. Obviously, there are only six independent DOFs or six coordinates in this domain.

## A Multiple Degree of Freedom Tire Test System

**Keywords:** Calibration, Carriage system, Contact footprint, Contact point, Coordinate system, Coordinate transformation, Displacement model, Force and moment calibration, Force and moment measurement, Footprint, Force model, Load cell, Multi-axis load cell, Normal vector, Roadwheel, Road surface, Tare compensation, Tire surface, Tire tester, Transformation matrix.

### INTRODUCTION

MTS Systems Corporation is known as the world leader in automated servo-hydraulic testing, which in recent years has undergone significant developments in the tire testing industry [1-3]. The Roadwheel tire test system discussed here complements other flat-surface testing systems and significantly extends the overall tire testing capabilities of existing roadwheel tire test systems (Fig. 1).

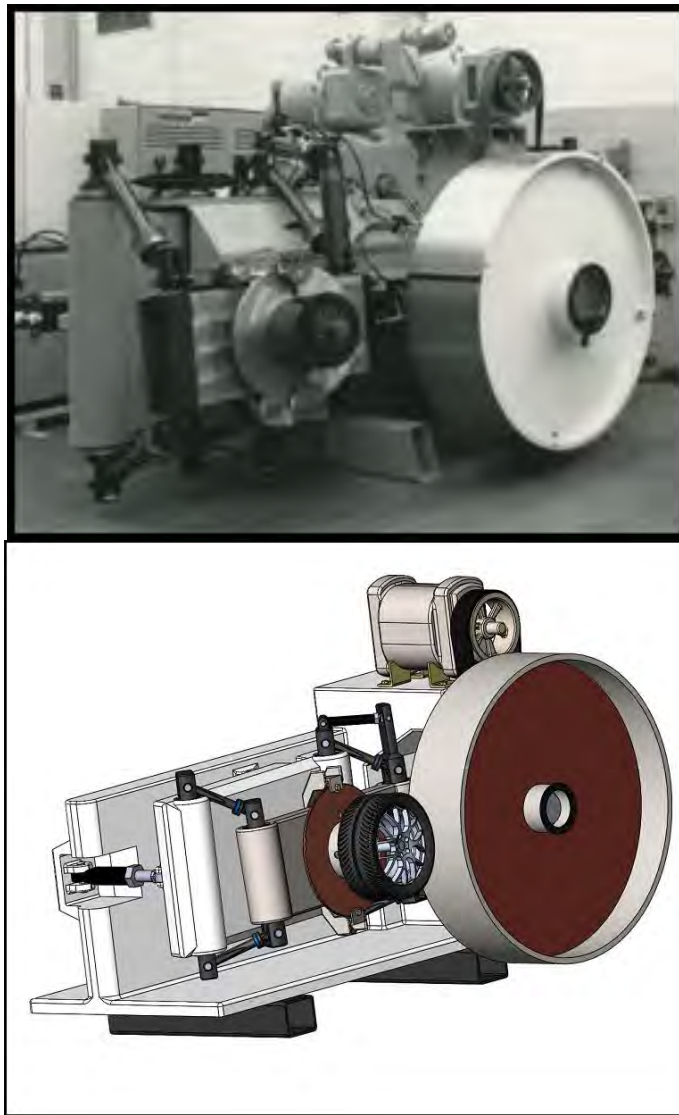
This type of tire test system was designed to simultaneously test four tires. Two stations can perform tire force and moment tests, one station can perform rolling resistance tests, and one station can perform tire durability tests. The force and moment stations are also equipped to introduce dynamic brake torque pulses to the tire. Here we will discuss only the one-station option.

The system operates under computer control with a high-speed real-time interface designed to perform all test commands, data acquisition, data reduction, and test report generation.

An analog control console was used for closed-loop control of the servo hydraulic actuators that position the tire and provide signal conditioning for the system transducers.

### MACHINE CONFIGURATION

The system design depicted in the following figures with its 67-inch diameter roadwheel (Fig. 1) was designed to accommodate different surface treatments to change the coefficient of friction. Additionally, 3/4-inch (19 mm) cleats can be added to perform tests with a periodic disturbance.

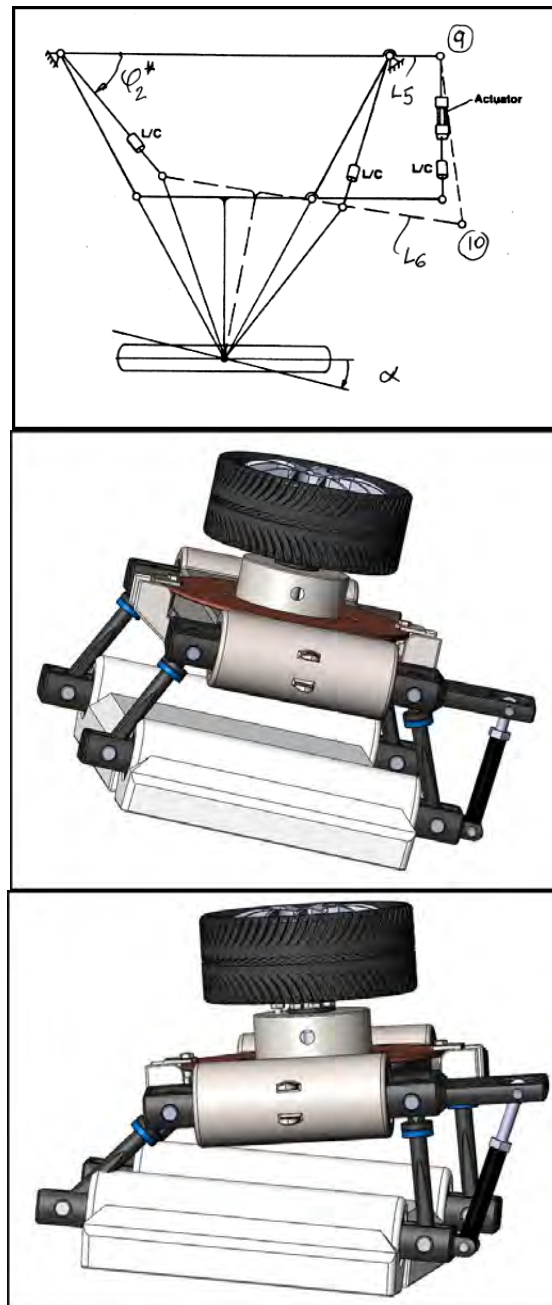


**Fig. (1).** Illustration of roadwheel application to test tires.

The electric drive motors were mounted on top of the frame for a compact, space-efficient design.

A sophisticated mechanism was required to allow the tire to undergo complex *radial*, *steer* and *camber* motions. A multi-degree of freedom linkage, which was designed to perform the tire transport task, has the form of a *focused link* pair, as shown in Fig. (2).





**Fig. (2).** Use of focused linkage to control steer angle.

## Kinematics of Biomechanical Joints

**Keywords:** Biomechanics, Clutches, Coordinate systems, Digital TMJ recorder, Coordinate transformation, Displacement model, Gnath tech dental systems recorder, Hinge motion, Human joint, Jaw motion, Masticatory system, Mechanical joint, Space location system (SLS), Sensor cluster, Temporomandibular joint, Three-sensor cluster, TMJ, TMJ degrees of freedom, Target array, Transfer bar.

### INTRODUCTION

Until this chapter, our discussion of kinematics was confined within the “conventional” structure of multi-DOF spatial mechanisms with well-established joints and pairs. Chapters 1 and 2 covered these topics in more detail.

Another demand for kinematics research arises from the necessity to develop the displacement model for biomechanical applications, such as knee, shoulder, elbow, and other joints in the human body. These investigations are critical when a “mechanical” replacement is required to duplicate a joint’s functionality.

### HUMAN AND MECHANICAL JOINTS

Each biomechanical joint has a different design or different requirements, which in this industry we call *patient-specific information*. This information reflects not only that joints are different in different parts of the body, but the type of joint (for example, a knee) can also vary within the same person (left vs. right).

The other difference between human and mechanical joints lies in their implementation or design characteristics, especially in the material used. While comparing an open-chain robotic arm and a human arm, both would include very similar components: arm, shoulder, wrist, *etc.*, but the main difference can be found in their ability to operate with prime movers. In a robotic arm (see Chapter 5), those joints are controlled by a network of links, cables, and electric motors (or another type of actuator). In most cases, the prime movers that transmit to the robotic arm can be used in both the direct and reverse directions, usually just by switching the rotation of the prime mover “drive” shafts. Thus, these prime movers can push and pull the link attached to these joints.

In a biomechanical system, a system of muscles provides control of the “linkage” through a system of muscles that pull to contract when blood flow and pressure are applied to the muscle system. To stop pulling, the blood supply is “turned” off. When the direction of motion is switched, another network of muscles pulls in the opposite direction (unlike the robotic arm system, where the same prime movers can change direction).

**Note:** Muscles cannot push, they can only pull.

The other distinctive difference between the human muscle system and a mechanical one is how the joints are structurally used. A mechanical system has a network of mechanical links attached to each other through the joints and to the frame to form the mechanism. In a human joint, the kinematic closure is between two links attached by the joint, and joint integrity depends on its suspension, which consists of ligaments and tendons that form the joint’s mechanical “assembly.” The human suspension system is not as stiff as its mechanical equivalent, so it requires a special limit within the field of service (range area).

The system of ligaments and tendons in the human body is an equivalent of strong links in a mechanical system that can precisely control the joint and limit its motion in a range “not to exceed” the maximum muscle range and to prevent muscle interfaces from damaging the bony structure.

To illustrate a biomechanical system, we have selected the temporomandibular joint (TMJ).

### **The Temporomandibular Joint (TMJ) as Part of the Masticatory System**

The rapid development of CAD/CAM packages for dental applications [1] has made virtual replication of the “true” motion of the lower jaw (the TMJ) vs. the upper jaw a significant means to provide an accurate depiction of patient-specific TMJ behavior.

The masticatory system has three major bones:

- the maxilla, also known as the upper jaw,
- the mandible, or lower jaw, with condyles used for articulation, and

- the temporal bone, which is naturally fused to the upper jaw to form a *fossa*, where contact (articulation) between the upper and lower jaws takes place (to act as the TMJ).

The area of the fossa on the temporal bone where the “sliding” process takes place is the *articular eminence*.

This system also has another component: the *articular disc*, which [2, 3] separates the surfaces (fossa and condyles) during movement. The articular disc reduces the friction between the articulated bones and stabilizes the joint.

To “translate” this anatomic terminology into mechanical kinematics, we will use terms more familiar in the engineering world, even though they might elicit a smile or some criticism from the dental community. Nevertheless, terminology does not affect how the engineering world treats the displacement model for the TMJ.

In our kinematic picture, the *cranium* (the skull or the head) represents a *base* for the other components attached to it. A significant part of this “base” is reserved for the *fossa*, which functions as a sub-frame and is rigidly connected to the cranium (base). This fossa has two areas (left and right) designated for joint operation, called the *articular eminence*. In engineering terms, they are a set of “slides” with a specific geometry, form, and shape to accommodate the moving object (condyles) inside. The condyles can be considered (in simplified terms) as a ball, and the fossa with articular eminences as a socket. The critical difference between these images is that condyles have a very complicated shape (not even close to a spherical joint), and eminences are not just simple sockets or slides.

To conclude our kinematic introduction to the TMJ, we need to add one more important element: the articular disc between the eminence surfaces on each side and the condyles. Remember, the condyles on both sides are connected with a bone called the *mandible* to form the lower jaw. The kinematic closure within this joint is provided by a set of muscles and a group of ligaments and tendons. From an engineering point of view, the ligaments are part of a structure made of strong tissues that do not stretch. Comparing the joint to a mechanical mechanism, the ligaments work as a border restraint system as well as a network of links.

### **TMJ Degrees of Freedom**

Consider the number of degrees of freedom that the temporomandibular joint (TMJ) can deliver.

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