

OPTIMIZATION TECHNIQUES FOR DECISION-MAKING AND INFORMATION SECURITY



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Computational Intelligence for Data Analysis

(Volume 3)

Optimization Techniques for Decision-making and Information Security

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Optimization Techniques for Decision-making and Information Security

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CONTENTS

FOREWORD	i
PREFACE	ii
LIST OF CONTRIBUTORS	vii
CHAPTER 1 COST OPTIMISATION ANALYSIS FOR A MARKOVIAN FEEDBACK QUEUEING SYSTEM WITH DISCOURAGEMENT, BREAKDOWN, AND THRESHOLD-BASED RECOVERY POLICY	1
<i>Amit Kumar, Amina Angelika Bouchentouf and Mohamed Boualem</i>	
1. INTRODUCTION	1
2. MODEL DESCRIPTION AND STEADY-STATE PROBABILITY	3
2.1. Assumptions and Notations	3
2.2. Chapman-Kolmogorov Equations	4
2.3. The Steady-state Solution	5
3. SYSTEM CHARACTERISTICS	6
4. COST MODEL	8
5. QUASI-NEWTON METHOD	9
6. NUMERICAL RESULTS	10
CONCLUSION	14
REFERENCES	14
CHAPTER 2 SOLUTIONS OF PARAMETRIC SET-VALUED OPTIMIZATION PROBLEMS OF HIGHER-ORDER	18
<i>Koushik Das and Chandal Nahak</i>	
1. INTRODUCTION	18
2. MOTIVATION	19
3. DEFINITION AND PRELIMINARIES	20
4. MAIN RESULTS	22
4.1. Higher-order Sufficient Optimality Conditions	23
4.2. Higher-order Mond-weir Type Dual	29
CONCLUSIONS	33
REFERENCES	34
CHAPTER 3 DEVELOPMENT AND ASSESSMENT OF DATA IN SOLAR-POWERED SYSTEM FOR SIMULTANEOUS PRODUCTION OF POWER, HEATING, AND COOLING	36
<i>Mohd Parvez, Taufique Ahamad and Osama Khan</i>	
1. INTRODUCTION	36
2. ERROR ESTIMATION	37
2.1. Mathematical Modeling	38
3. RESULTS AND DISCUSSION FOR THE PROPOSED SYSTEM	41
CONCLUSION	44
NOMENCLATURE	44
REFERENCES	46
CHAPTER 4 A NEW ALGORITHM FOR SOLVING FULLY FUZZY LINEAR PROGRAMMING PROBLEMS USING THE LEXICOGRAPHIC METHOD	48
<i>Anil Kumar Nishad, Gunjan Agarwal, S. R. Singh and Gajraj Singh</i>	
1. INTRODUCTION	48
2. PRELIMINARIES	51
3. FULLY FUZZY LINEAR PROGRAMMING PROBLEM (FFLPP)	53
3.1. Algorithm to Solve the System (1) of FFLPP	53
4. NUMERICAL ILLUSTRATION	56
CONCLUSION	59
REFERENCES	59

CHAPTER 5 ANTI-SYNCHRONIZATION BETWEEN DIFFERENT NONLINEAR CHAOTIC SYSTEMS VIA ACTIVE NONLINEAR CONTROL METHOD	62
<i>Sanjay Kumar and Ram Pravesh Prasad</i>	
1. INTRODUCTION	62
2. SYSTEM DESCRIPTION	63
3. ANTI-SYNCHRONIZATION BETWEEN CHAOTIC CHEN-LEE SYSTEM AND CHAOTIC LORENZ-STENFLO SYSTEMS	64
3.1. Numerical Simulation	66
4. ANTI-SYNCHRONIZATION BETWEEN CHAOTIC CHEN-LEE AND LIU-CHEN SYSTEMS	68
4.1 Numerical Simulation	69
CONCLUSION	72
DATA RESPONSIBILITY	73
REFERENCES	73
CHAPTER 6 FIXED CHARGED NONLINEAR SOLID TRANSPORTATION PROBLEM WITH BUDGET CONSTRAINTS USING BOUNDED PARAMETERS	75
<i>A. K. Bhurjee and P. Kumar</i>	
1. INTRODUCTION	75
1.1. Motivation.....	77
2. INTERVAL ANALYSIS	77
2.1. Partial Ordering on $\mathbb{I}(\mathbb{R})$	78
2.2. Interval Valued Function.....	78
3. NOTATIONS AND MODEL FORMULATION.....	79
4. FIXED CHARGED NONLINEAR SOLID TRANSPORTATION PROBLEM IN CRISP ENVIRONMENT	80
4.1. Fixed Charged Nonlinear Interval Solid Transportation Problem (<i>FCNISTP</i>).....	81
4.2. Solution Procedure of <i>FCNISTP</i>	84
5. NUMERICAL PROBLEMS	85
CONCLUSION.....	88
REFERENCES	88
CHAPTER 7 SOFT COMPUTING TECHNIQUES FOR CANCER CLASSIFICATION OF GENE EXPRESSION MICROARRAY DATA: A THREE-PHASE HYBRID APPROACH	92
<i>Amol Avinash Joshi and Rabia Musheer Aziz</i>	
1. INTRODUCTION	93
1.1. Lecture Survey	93
1.2. Recent Related References	95
1.3. Motivation and Contribution.....	96
2. PROPOSED FRAMEWORK	97
2.1. Genes Extraction by ICA.....	97
2.2. Genetic Spider Monkey (GSM) Algorithm.....	98
3. USED CLASSIFIER	98
4. EXPERIMENTAL SETUPS.....	99
5. EXPERIMENTAL RESULT	100
CONCLUSION.....	107
FUTURE WORK	108
REFERENCES	108
CHAPTER 8 A SURVEY OF PAIRING FREE CERTIFICATELESS SIGNATURE SCHEMES FOR SECURE NETWORKING	114
<i>Deepika and Pankaj Kumar</i>	
1. INTRODUCTION	114
1.1. Motivation and Contribution.....	116
1.2. Structure of the Chapter	116
2. RELATED WORK	116
3. PRELIMINARIES	119
3.1. Bilinear Map	119
3.2. Elliptic Curve Cryptography	120

3.2.1. Elliptic Curves.....	120
3.2.2. Elliptic Curve Over Z_q	121
3.2.3. Elliptic Curve Discrete Logarithm Problems (ECDLP).....	122
4. SYSTEM MODEL OF CLS AND VARIOUS ATTACKS IN CLS	122
4.1. System Model in CLS	122
4.2. Security Model of CLS	123
4.3. Some of the Attacks	124
5. PERFORMANCE EVALUATION OF VARIOUS SCHEMES	124
CONCLUSION	128
REFERENCES	128
CHAPTER 9 ANALYSIS AND DEVELOPMENT OF TIE-BREAKING PROCEDURES USING FUZZY TECHNIQUE FOR ORDER PERFORMANCE BY SIMILARITY TO IDEAL SOLUTION	132
<i>Iftikhar Husain</i>	
1. INTRODUCTION	132
1.1. Motivation and Contribution.....	133
1.2. Roadmap of the Chapter.....	134
2. PRELIMINARIES	134
3. MEAN OF REMOVALS APPROACH FOR RANKING FUZZY NUMBERS [4]	135
4. THE WORKING RULE OF FUZZY TOPSIS APPROACH	136
4.1. Combination of the Importance Weights	136
4.2. Combined Ratings of Alternatives Under Decision Criteria	136
4.3. Establishment of the Weighted Decision Matrix.....	136
4.4. Establish the Ideal and Non-perfect Solutions	138
4.5. Determining the Distances of Alternatives from Ideal and Negative-ideal Solutions	138
4.6. Deriving the Closeness Coefficients	139
5. NUMERICAL EXAMPLE	139
5.1. Calculated Entries of the Weighted Decision Matrix	141
CONCLUSION	142
REFERENCES	143
SUBJECT INDEX	146

FOREWORD

It gives me great pleasure to welcome you to this book, which focuses on computational intelligence in data analysis, one of the most active study fields in recent years. The book covers representative data problems and solutions in different directions like cost optimization analysis, Markovian feedback queuing systems, parametric set-valued optimization problems, production of power, fuzzy linear programming problems, nonlinear chaotic systems, nonlinear control methods, solid transportation problems, soft computing techniques, data aggregation optimized technique, wireless sensor network, pairing free certificates signature schemes and tie-breaking procedures using the fuzzy approach. This book provides a thorough analysis without compromising the theoretical complexity of the chosen topics, in contrast to other works. Therefore, I think the book will be an invaluable resource and a thorough reference for students enrolled in a computational data analysis course.

The book's authors belong to different countries and many are well-known data and analysis researchers. Readers of the book can learn a lot from the opinions, ideas, expertise, and experiences of these authors who have worked in this field for a while.

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PREFACE

In the modern era, there is a need for data in every field. To improve the business and related fields, data is an important key for future directions. For this purpose, there are requirements for data analysis *via* computational approaches in every direction. Businesses are now able to efficiently monitor their operations and manage their computationally by gauging customer preferences thanks to the use of data, the implementation of creative strategies, and technology.

We believe that this book will be a pioneering book focusing on research in computational intelligent data analysis. It presents numerous viewpoints on the application of data analysis in various fields, such as cost optimization analysis, Markovian feedback queuing systems, parametric set-valued optimization problems, data aggregation optimized technique, fuzzy linear programming problems, nonlinear chaotic systems, production of power, nonlinear control methods, solid transportation problems, soft computing techniques, wireless sensor network, pairing free certificateless signature schemes and tie-breaking procedures using the fuzzy approach. For professionals, educators, and academics working in the areas of data and its applications, this publication offers an extensive reference source. It is structured in such a way that it begins with data analysis in different domains. We are delighted to learn that this book has produced a significant number of insightful observations and fresh research findings on issues pertaining to data and different applications in many directions.

Chapter 1 deals with a finite-capacity Markovian feedback queue with a threshold-based recovery policy, server breakdown, and customers' impatience. The steady-state probabilities for this system are calculated using the Successive Over-Relaxation (SOR) approach. In addition, characteristics of the queuing system are derived. Following that, a Quasi-Newton approach to cost optimization of the system is performed. To further support the theoretically established results, numerical studies are also conducted.

Chapter 2 deals with a parametric set-valued optimization problem (PSOP), where set-valued functions are used for the constraint and objective functions. We use the idea of higher-order ρ -cone convexity of SVFs as a generalization of cone convex SVFs. Further, the authors provide the Karush-Kuhn-Tucker criteria of sufficiency for the presence of the PSOPs under higher-order ρ -cone convexity assumption. Furthermore, the authors constitute the duality models of Mond-Weir kind and demonstrate the strong, weak, and converse duality theorems under

higher-order convexity epiderivative and higher-order ρ -cone convexity assumptions to a couple of set-valued optimization problems. Additionally, the Chapter provides some examples to justify results. As a special case, our results reduce to the existing ones of scalar-valued parametric optimization problems.

Chapter 3 develops the trigeneration system for the simultaneous production of power, heating, and cooling, driven by solar power towers. The optimal thermodynamic performance of the vapor absorption refrigeration system among the refrigerants (LiNO₃-H₂O and LiBr-H₂O) is assessed by a comparison of the two. In order to deliver far better results with accuracy and remove all human and mechanical faults to 5.34% which is to be discovered in the intended range, a new idea of uncertainty analysis is proposed, which is a prime instance in this research area. To explore variable inefficiencies when changing different performance parameters for the trigeneration system, combined energy and exergy evaluations are carried out. The central receiver, heliostat, and heat recovery steam generators were found to have the highest rates of exergy destruction, at 33.6%, 24.9%, and 7.8% respectively. LiBr-H₂O has the highest energy and exergy efficiency of 62.6% and 20.6%, while LiNO₃-H₂O refrigerant only managed to achieve 60.9% and 19.6%.

A general fuzzy number must currently be ranked, which is a challenging task. Various ranking methods have been devised, but there is still room for improvement.

Chapter 4 solves fully fuzzy linear programming problems, and many ranking functions have been developed and implemented in past research. All of these techniques have certain drawbacks, though. In this study, the authors present a novel generalized approach for comparing two triangular fuzzy numbers. With the provided solution, Ezzati's method has been improved to address fully fuzzy linear programming problems (FFLPP). Through numerical examples, the developed algorithm's implementation has been demonstrated. After rigorous testing, the suggested approach has been used to solve a transportation problem, and it has been found to be efficient and generally provides better results.

Chapter 5 establishes the anti-synchronization between the three-dimensional non-identical nonlinear Chen-Lee, Lorenz-Stenflo, and Liu-Chen chaotic systems *via* active nonlinear control techniques. It explores how anti-synchronization presents phase pictures of master and slave systems. The Lyapunov function stability theory discusses the results of stability. Active control approaches have been used to establish anti-synchronization of the chaotic Chen-Lee, Lorenz-stifle,

and Liu-Chen systems as well as anti-synchronization of the chaotic Chen-Lee and Liu-Chen systems. Obtaining the anti-synchronization between various chaotic systems is more effective with the active control strategy. The suggested strategy is explained together with numerical outcomes *via* data analysis.

Chapter 6 considers a fixed-charged solid transportation problem where all parameters of the problem have uncertainty due to ambiguity in the data set. These uncertain parameters (destination budget, accessibility, necessity, conveyance capacity, and transportation cost) are thought of as bounded. By converting the interval-based model into a typical nonlinear programming problem, the issue's solution is attained. Any nonlinear programming technique is used to solve the changed model. The result of the deterministic model is a workable solution to the primary model. The developed concept and techniques are validated using instances with fictitious data.

Many soft computing techniques have recently been used to extract information from large amounts of data. DNA microarray technology makes it possible to evaluate the levels of gene expression for thousands of different genes in a uniform fashion. The patterns created by gene expressions in microarray technology can be used to identify malignancies in a variety of anatomical areas. Due to the curse of the dimensionality problem, the microarray data is too large to process.

Chapter 7 deals with a hybrid machine learning framework using soft computing techniques for feature selection, which is designed and executed to eliminate unnecessary genes and identify important genes for the identification of cancer. The higher-order Independent Component Analysis (ICA) technique is used in the first stage to remove the genes or characteristics. Then, a wrapper approach built on Spider Monkey Optimization (SMO) and Genetic Algorithm (GA) is utilized to identify the genes that enhance the classification accuracy of the Naive Bayes (NB) classifiers and Support Vector Machines (SVM). Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), and Genetic Algorithm are the other three optimization methods that are compared in this chapter (GA). The most popular classifiers, Naive Bayes (NB) and Support Vector Machine (SVM), are trained with the chosen genes once the relevant expressed genes have been chosen, and the classification accuracy is then assessed using data analysis.

Chapter 8 deals with the technology that has enhanced the jeopardy of the misuse of individual confidential data and identity theft attacks, where securing the content of communication and confidentiality has become one of the chief

requirements. The cornerstone of computer and communications security in this case is cryptography. For establishing core securities, such as secrecy, data integrity, non-repudiation, and authentication, certificateless cryptography in conjunction with digital signature is the best option. Public-key cryptography (PKC) and identity-based cryptography's certificate management issues and key escrow problems are eliminated by certificateless cryptography (CLS). CLS is more effective since it is the ideal fusion of PKC and ID-based PKC. Bilinear pairing and the MTP hash functions are used to create many of the CLS that are dependent upon for signatures. Both the elliptic curve bilinear pairing and the MTP (map-to-point) hash function are extravagant schemes that increase the computation cost and time consumption. Comparatively less computationally expensive, Pairing-free CLS techniques, on the other hand, rely on Elliptic Curve Cryptography (ECC) and offer the desired time requirement, making the scheme more effective. We conducted a study of different pairing-free CLS methods and compared them in this work. The security and effectiveness of various methods are analyzed by the writers. Some have even been proven to be vulnerable to different attacks.

Chapter 9 focuses on tie-breaking procedures that employ the fuzzy technique for order performance by similarity to the ideal solution. A tie occurs when two or more competitors receive the same score in a competition. The fuzzy technique for ordering performance by similarity to the ideal solution offers an effective and efficient decision-making procedure. The fuzzy technique for ranking performance by the resemblance to the ideal response is used to evaluate precise and methodical decision-making based on multiple criteria among a small number of viable options. The proposed tie-breaking technique generates ranking and ordering relationships among a group of alternatives by combining objective and discretionary decision criteria. A numerical example is used to demonstrate how to compute data in this system.

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CHAPTER 1**Cost Optimisation Analysis for a Markovian Feedback Queueing System with Discouragement, Breakdown, and Threshold-based Recovery Policy****Amit Kumar¹, Amina Angelika Bouchentouf² and Mohamed Boualem^{3,*}**¹ *Department of Mathematics, Chandigarh University, Mohali (Punjab), Pin No- 140413, India*² *Laboratory of Mathematics, Djillali Liabes University of Sidi Bel Abbes, Sidi Bel Abbes 22000, Algeria*³ *Research Unit LaMOS (Modeling and Optimization of Systems), Faculty of Technology, University of Bejaia, 06000 Bejaia, Algeria*

Abstract: In this investigation, we deal with a finite capacity Markovian feedback queue with threshold-based recovery policy, server breakdown, and customers' impatience. For this system, the Successive Over-Relaxation (SOR) method is used to obtain the steady-state probabilities. Besides, the queueing system's characteristics are also derived. Then, cost optimization of the system is carried out using the Quasi-Newton method. Further, numerical studies are performed to support the theoretically obtained results.

Keywords: Feedback queue with server breakdown; Balking and threshold-based recovery policy; Reneging and retention of renege customers; SOR method; Quasi-Newton method.

1. INTRODUCTION

In recent times, the queueing principle gained great importance in studying scheduling and system performance. It is an omnipotent tool for solving various complex systems problems including Job-Shop Problems, computer/telecommunications systems, and production/manufacturing systems.

This work deals with a feedback queueing model with an unreliable server, threshold-based recovery policy, and customers' impatience. The concept of threshold-based recovery policy has attracted the attention of many researchers [1-6].

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Queueing models with customers' impatience play a vital role in computer networks with packet loss, call centers, railways ticket windows, banks line, transport problems, congestion problems in networks, *etc.* The concepts of balking and renegeing were first introduced by Haight [7, 8]. In recent years, the subject attracted increasing interest; see for instance [9-19].

The finite-buffer queueing model analysis is instrumental. Abou-El-Ata and Hariri [20] presented a multi-server queue with finite space, balking, and renegeing. Ammar *et al.* [21] investigated an $M/M/1/N$ queueing system with discouraged arrival and renegeing. Yang [2] examined a repairable $M/M/1/N$ queueing model under a threshold-based recovery policy. Then, Kumar [22] considered customers' impatience in the finite capacity multi-server queueing model. Using the matrix method, transient and steady-state probabilities were derived. Recently, both finite-capacity single and multi-servers queueing models with vacation, balking, renegeing, and retention of renegeed customers were studied in [23] and [24], respectively.

Controllable queues have gained considerable attention. A particular importance is devoted to the N -policy problems characterized by the fact that an idle server is active once the number of waiting customers in the queue reaches a certain threshold; $N(N \geq 1)$. Then, the server turns off whenever the queue gets empty. This policy was introduced by Yadin and Naor [25]. Since then, extensive research works on the subject have been done; see for instance [26, 27, 28, 29, 30]. On the other hand, unreliable queueing models with N -policy have been greatly considered because of their wide practical situations. Excellent papers on different queues with breakdowns under N -policy can be found in some studies [29, 31, 32, 33].

The concept of feedback has a vital importance in many real-world applications like mobile service, IT communication, banking service, and printed circuit board (PCB) as well as in improving the quality of service (QoS). The pioneer work on this subject was given by Takacs [34]. Then, multiple research works have been consecrated to the study of different queues with feedback, the reader may refer to [35–39] and the references therein.

Motivated by the paucity of research in the field of customer impatience in unreliable queueing systems as well as by the multitudinous applications in communication and production systems, this chapter proposes to analyze a finite-capacity Markovian feedback queueing system with server breakdowns, threshold-based recovery policy, and impatience. Our contribution consists of including a threshold-based recovery policy to make the problem more realistic. The suggested

queueing model has important practical applications in various real-life problems, such as manufacturing, inventory systems, and computer systems, where we often encounter a situation where the service may break down, and the system is distinguished by the fact that the broken server is repaired whenever there are L ($L \geq 1$) or more customers in the system. The key contributions of this chapter are the steady-state solution of the considered queueing system, as well as the cost-parameter optimization. Moreover, the numerical examples performed for the cost model shed new light on the numerical section. Using the GEE (Gauss Elimination Extended) numerical technique and the SOR (Successive Over-Relaxation) method, the system steady-state solution is obtained. Diverse performance metrics are discussed. To economize operating costs, a cost optimization problem using the Quasi-Newton method has been considered.

The structure of the present chapter is as follows. In Section 2, we describe the model and present Chapman-Kolmogorov equations as well as the steady-state solution of the suggested queueing model. In Section 3, we derive some essential characteristics of the model. The cost function is developed in Section 4. Then, in Section 5, we establish a cost optimization parameter using the Quasi-Newton method. Section 6 is dedicated to a numerical study. Finally, Section 7 concludes the chapter.

2. MODEL DESCRIPTION AND STEADY-STATE PROBABILITY

We develop an $M/M/1/K$ Markovian queueing model with a server breakdown, threshold-recovery policy, balking, reneging, retention, and Bernoulli feedback. The Fig. (1) shows the state diagram.

2.1. Assumptions and Notations

- Customers arrive at the system, according to a Poisson process, with parameter λ . The system can adapt to most K customers.
- During the busy period, the service time follows an exponential distribution with rate μ . The service discipline is First-Come-First-Served (FCFS).
- The server may fail, if there exists at least one customer in the system. The breakdown time follows an exponential distribution with rate ϕ .
- The server can not be repaired until the number of customers achieves a determined threshold value L ($1 \leq L \leq K$). The repair time follows an exponential distribution with rate φ .
- Once the server is repaired, it returns to the busy period in order to offer service to the waiting customers.

Solutions of Parametric Set-Valued Optimization Problems of Higher-Order

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Abstract: We deal with a parametric set-valued optimization problem (in brief, PSOP), where set-valued functions (in brief, SVFs) are used for the constraint and objective functions. We use the idea of higher-order ρ -cone convexity of SVFs (introduced by Das and Nahak [1] as a generalization of cone convex SVFs. We provide the Karush-Kuhn-Tucker (in brief, KKT) criteria of sufficiency for the presence of the minimizers of the PSOPs under higher-order ρ -cone convexity assumption. Further, we constitute the duality models of Mond-Weir kind and demonstrate the strong, weak, and converse duality theorems under higher-order contingent epi-derivative and higher-order ρ -cone convexity assumption to a couple of set-valued optimization problems (in brief, SOPs). We provide some examples to justify our results. As a special case, our results reduce to the existing ones of scalar-valued parametric optimization problems.

Keywords: Convex cone, Duality, Set-valued function, Contingent epi-derivative.

1. INTRODUCTION

Many vector optimization problem results over the past two decades have seen a large growth in extension to SOPs. In the study of SOPs, the concepts of cone convexities and differentiability of SVFs are crucial. Various kinds of cone convexities and differentiability of SVFs have been developed to investigate SOPs. SOPs and optimum control problems with differential inclusions are closely connected. SOPs can be used to evaluate several problems in image processing, viability theory, mathematical economics, and many other fields.

The concept of cone convexity is essential in the development of the efficient point existence theorem for SOPs. For SVFs, the cone convexity was proposed by Borwein [2]. Aubin [3] proposed the contingent derivative of SVFs. It is a SVF extension of the Fréchet derivative idea. Corley [4] proved the existence of set-

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valued maximizing problems and developed Lagrangian duality theory *via* contingent derivative. The idea of contingent epi-derivative, which is an expansion of the directional derivative idea to the set-valued structure, was constituted by Jahn and Rauh [5] as an additional idea of differentiability of SVFs. Sheng and Liu [6] developed the KKT conditions of SOPs *via* generalized contingent epiderivative. Das and Treantă [7] obtained the sufficient conditions of KKT type and constituted the theorems of duality of semi-infinite SOPs in terms of ρ -cone arcwise connectedness and contingent epi-derivative.

The term “parametric optimization problem” refers to a specific kind of optimization problem. It is mostly investigated in the domains of mathematics, operations research, and economics. Optimization problems with parameters for single-valued case were studied by various authors like Ioffe [8], Khanh [9-11], Khanh and Nuong [12, 13], and Nuong [14]. In the set-valued structure, this type of problem was investigated by Khanh and Luu [15, 16]. Under weak differentiability suppositions and convexlikeness suppositions, they provided the requisite conditions of optimality for Fritz John and Kuhn-Tucker types. The PSOP arises in a variety of circumstances where SVFs are involved and equations such as differential equations and initial conditions are represented by the equality constraint. A PSOP can be defined as one in which differential inclusions substitute differential equations to represent the system under study. Using a higher-order contingent derivative, Li *et al.* [17, 18] were able to define the sufficient and necessary conditions of optimality. For SOPs, they also developed the Mond-Weir dual of higher-order.

In this work, we explore a PSOP including the objective function as well as the constraints as SVFs. The KKT conditions for sufficiency of higher-order for the problem are developed using the suppositions of generalized cone convexity and contingent epi-derivative of higher-order. Finally, the Mond-Weir kind of duality is described, and the relationship between primary and dual problems is examined.

2. MOTIVATION

Higher-order ρ -cone convexity of SVFs was developed by Das and Nahak [1] as a generalization of higher-order cone convexity of SVFs. We get the conventional idea of higher-order cone convex SVFs for $\rho = 0$. In addition, they formulated a higher-order ρ -cone convex SVF that is not higher-order cone convex. Using contingent epi-derivative and ρ -cone convexity of higher-order suppositions on the objective functions and constraints, we are primarily concerned with in determining the sufficient conditions of optimality of SOPs in a more generalized manner.

Under contingent epi-derivative and ρ -cone convexity of higher-order suppositions, we investigate the strong, weak, and converse theorems of duality for Mond-Weir kind. Our findings for $\rho = 0$ are more conservative than those found in the literature.

3. DEFINITION AND PRELIMINARIES

Suppose that Z is a real normed space (in brief, RNS) and L is a subset of Z , that is not empty. L is stated to be a cone if $\lambda z \in L$, for all $z \in L$ and $\lambda \geq 0$. In addition, L is stated to be pointed when $L \cap (-L) = \{\theta_Z\}$, solid when $\text{int}(L) \neq \emptyset$, and convex when $\lambda L + (1 - \lambda)L \subseteq L, \forall \lambda \in [0, 1]$, $\text{int}(L)$ represents the set of interior points of L and θ_Z stands for the zero of Z .

Let Z^* be the set of all continuous linear functionals on Z and L be a convex cone of Z that is also pointed as well as solid. The dual cone of L , denoted by L^+ , is therefore defined in the following way:

$$L^+ = \{l^* \in Z^* : \langle l^*, l \rangle \geq 0, \forall l \in L\},$$

Where $\langle \cdot, \cdot \rangle$ is the most common bilinear form in terms of the dual between Z^* and Z .

Suppose that C is a subset of a RNS Z , which is not empty and $z^* \in Z^*$. Define a subset $\langle z^*, C \rangle$ of \mathbb{R} by;

$$\langle z^*, C \rangle = \bigcup_{c \in C} \{\langle z^*, c \rangle\}.$$

Suppose that L is a convex cone of Z that is also pointed as well as solid. Cone-orderings can be mainly divided into two categories in Z with respect to (in brief, w.r.t.) L . For any two elements $l_1, l_2 \in Z$,

$$l_1 \leq l_2 \text{ if } l_2 - l_1 \in L$$

and;

$$l_1 < l_2 \text{ if } l_2 - l_1 \in \text{int}(L).$$

For any subset D of Z , that is not empty, the notations we use are as follows:

$$D \leq \theta_Z \text{ if and only if } d \leq \theta_Z, \forall d \in D$$

Development and Assessment of Data in Solar-powered System for Simultaneous Production of Power, Heating, and Cooling

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Abstract: The primary objective of the study is to investigate the trigeneration system for the simultaneous production of power, heating, and cooling, driven by solar power towers employing molten salt as the heat transfer fluid. A comparative analysis is provided between refrigerants (LiNO₃-H₂O and LiBr-H₂O), so as to evaluate the best thermodynamic performance for the vapor absorption refrigeration system among them. A novel concept of uncertainty analysis is introduced which is a prime instance in this research area so as to provide much better results with precision removing all human and machine errors stipulated to be 5.34 % which is to be found in the desired range. Combined energy and exergy analyses are performed to investigate the variation in efficiencies while altering various performance parameters for the trigeneration system. The highest exergy destruction was found to be 33.6% by the central receiver, 24.9% by heliostat, and 7.8% by heat recovery steam generators. The highest energy and exergy efficiencies (62.6% and 20.6%) were obtained by LiBr-H₂O, whereas (60.9% and 19.6%) were obtained from LiNO₃-H₂O refrigerant.

Keywords: Cycle, energy efficiency, exergy efficiency, irreversibility, molten salt, central receiver, direct normal irradiance.

1. INTRODUCTION

The conversion of solar energy into thermal energy is primarily fulfilled by employing special types of solar collectors such as a flat plate, evacuated tube, parabolic trough, and solar power tower plants [1, 2]. Among all these solar thermal technologies, solar power tower plants (Heliostat field) technology is quite popular and has been receiving considerable attention due to its sustainable merits of producing power [3]. The thermal heat energy is absorbed during the radiation process by the central receiver which is further transferred to a circulating fluid usually air, water, Duratherm oil 600, or molten salt, flowing through the collector to the heat recovery steam generator (HRSG) so as to generate steam [4]. In recent

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years, several types of research have been carried out to estimate the performance evaluation of the solar-operated cogeneration and trigeneration system from the energy and exergy point of view [5].

The concept of exergy analysis is broadly based on the “second law of thermodynamics” which is seen as an essential tool to contemplate, identify the magnitudes, and locate the thermodynamic losses thereby pinpointing the locations of imperfection within the system [6].

Often utilization of solar thermal energy for power generation is perceived to be not competitive since conventional power generation technology has a tendency of obtaining low electric efficiency at a substantial cost per MW [7]. However, in search of continuous pursuit of enhancement, solar thermal power can be modified to obtain higher efficiency or lower emissions. In recent times, waste heat is often released into the environment without complete utilization of it, thereby resulting in substantial damages to the current environment strategies specified by governments of various countries. To utilize this waste heat based on their potential and thereby alleviating environmental problems, cogeneration, and trigeneration power cycles for solar power generation are receiving considerable attention from researchers of various renewable energy backgrounds [8]. Among many popular applications, combined power, heating, and cooling cycles are the vapor absorption refrigeration cycle which is integrated at the exit of the heat user. This cycle utilizes working fluids such as $\text{LiNO}_3\text{-H}_2\text{O}$ and $\text{LiBr-H}_2\text{O}$, which have zero ozone depletion potential using solar energy as primary input [9]. Therefore, a system that produces combined electrical power, process heat, and cooling simultaneously from a single source of energy supply is known as a trigeneration system and has drawn the keen interest of many researchers worldwide. There are several advantages associated with the application of trigeneration systems such as reduced greenhouse gas and emissions, lower cost of fuel, and lower electricity usage during peak winter or summer demand [10]. The proposed system is a solar-operated trigeneration system as depicted in Fig. (1).

2. ERROR ESTIMATION

The total percentage of uncertainty is determined in this study by applying the Holmanns principle which is an exceptional tool in the estimation of any inherent device error present as evident from previous research [11]. Table 1 displays the errors that might be encountered while measuring various parameters in the proposed cycle.

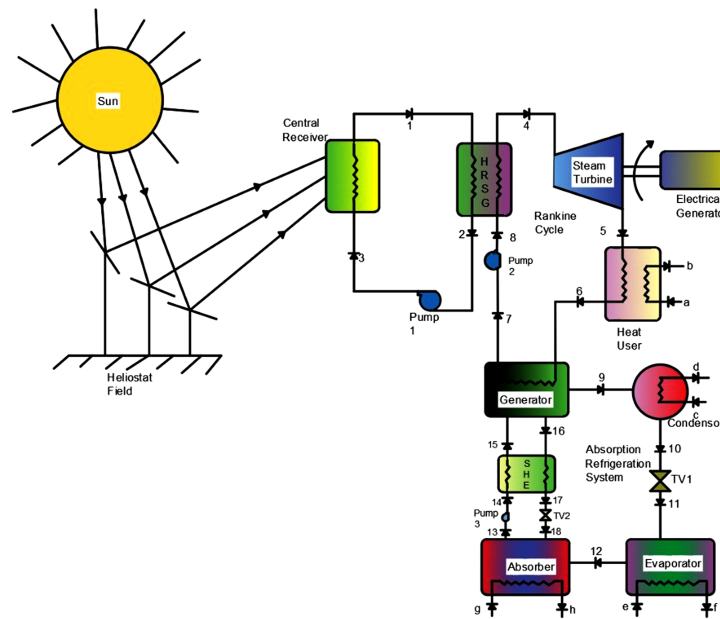


Fig. (1). Shows the solar operated trigeneration system.

Table 1. The measurement accuracies and analytical uncertainties associated with sensors and parameters.

Sensors and Parameters	Accuracies and Uncertainties Measurement
T-type Thermocouples	± 0.5 C
Flow Meter	± 5 ml
Pressure Transducer	± 1.8 m bar
Voltage Measurement	± 0.06 V
Current Measurement	± 0.15 A
Silicon Irradiance Sensor (SIS sensor)	± 5 W/m ² \pm 3.5% of measurement value
Power Temperature Coefficient	-0.29%/C

2.1. Mathematical Modeling

The mathematical model primarily aids in the evaluation of the energy and exergy methods where mass, energy, and exergy balances are written for each component below:

A New Algorithm for Solving Fully Fuzzy Linear Programming Problems using the Lexicographic Method

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Abstract: In the current situation, ranking a general fuzzy number is a difficult task, and various ranking methods have been developed, but no perfect ranking method exists. To solve fully fuzzy linear programming problems, many ranking functions have been developed and implemented in the literature. However, all of these methods have some limitations. In this chapter, we propose a new method for comparing two triangular fuzzy numbers in a generalised form. The Ezzati method [1] has been expanded upon using the suggested approach to handle fully fuzzy linear programming issues (FFLPP). The implementation of the developed algorithm has been illustrated through numerical illustrations. The proposed algorithm has been applied to a transportation problem in light of extensive testing, and it has been discovered that it is effective and generally offers a better solution.

Keywords: Fully fuzzy linear programming problem (FFLPP), Triangular fuzzy number (TFN), Ranking function, Dummy fuzzy variable.

1. INTRODUCTION

During the stages of modelling and optimizing decision-making problems, linear programming emerges as a pivotal technique for attaining the optimal solutions to decision variables. The modelling process involves the assimilation of information pertinent to the system or process under consideration. This information is often

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sourced from experts or through data collection techniques, but frequently, it lacks exactness or quantitative precision. Instead, it may manifest as linguistic variables, a spectrum of approximations, or even values that bear a degree of error. In light of these uncertainties, it becomes imperative to introduce approximations to account for the inherent imprecision associated with quantitative values in the modelling of such problems. Moreover, it becomes evident that the conventional linear programming approach is ill-suited to tackle and resolve these intricacies.

In this study, Bellman and Zadeh [2] introduced the idea of making decisions in a fuzzy environment, a new dimension in the study of decision sciences with ambiguous parameters has developed. The idea motivated Zimmermann [3] to develop a fuzzy linear programming method for solving such problems. Initially, fuzzy linear programming mainly addressed issues involving constraints and/or resource vectors with erroneous parameters; these quantities were then approximated as fuzzy numbers. Therefore, in the process of solving fuzzy linear programming problems, a comparison of these fuzzy numbers was taken into consideration as the ranking of fuzzy numbers. In the literature, there exist many ranking techniques for making such a comparison between fuzzy numbers. One such approach was given by Liou and Wang [4], who proposed a method for ranking fuzzy numbers based on their integral values. The various approaches for measurement of the distance between fuzzy numbers produced a variety of ranking methods as given by [5, 9].

There are further methods that have been explored for producing ranking functions for fuzzy numbers by Grzegorzewski [10]. A ranking on epsilon deviation degree is additionally suggested by Yu *et al.* [11], and a comparison of fuzzy numbers based on preference measure is proposed by Tsai [12]. A fascinating investigation into the fundamental distinctions between a few ranking techniques was conducted by Brunelli and Mezei [4]. Fully fuzzy linear programming problems had to be modeled due to the complexity of optimization problems in a fuzzy environment and their increased applicability to dealing with real-world scenarios. It was Allahviranloo *et al.* [13], who studied the solution procedure for solving a fully fuzzy linear programming problem using a ranking function. Lotfi *et al.* [14], considered a fuzzy approximate solution for a fully fuzzy linear programming problem using the lexicography method given by Isermann [15]. Kumar *et al.* [16], further studied the fuzzy approximate solution and proposed a method of solution for a fully fuzzy linear programming problem. Several studies are available in the literature for solving fuzzy linear programming problems by developing improved fuzzy ranking methods such as Cheng *et al.* [17], and Ebrahimnejad and Tavana [18]. The ranking approach for trapezoidal fuzzy numbers provided by

Abbasbandy and Hijjari [18] was updated by Ezzati *et al.* [19] to provide a new ranking method for triangular fuzzy numbers, which was inspired by the concept of ranking functions provided by Liou and Wang [4].

The methodology employed for dealing with the Fully Fuzzy Linear Programming Problem (FFLPP), initially formulated by Kumar *et al.* [16], underwent significant modification [20]. These enhancements, in turn, served as a catalyst for the ingenuity displayed by Ezzati *et al.* [1], who devised a groundbreaking algorithm predicated on a distinctive triangular lexicographic ordering on Triangular Fuzzy Numbers (TFN). However, Bhardwaj and Kumar [21], appended a noteworthy caveat, to Ezzati *et al.* [1] asserting that the prescribed procedure might not be suited for addressing FFLPP instances characterized by inequality constraints. Motivated by the insights gleaned from the aforementioned research, this study embarks on a comprehensive exploration of both the methods by Kumar *et al.* [16] and Ezzati *et al.* [1] as applied to FFLPP. In a proactive stance toward refining existing methodologies, our endeavor extends to enhancing the Ezzati method specifically for the comparison of triangular fuzzy numbers. Notably, we introduce an innovative approach to comparison of general triangular fuzzy numbers. This newly proposed definition serves as the hub of the adaptation and optimization of the Ezzati method. The outcome is an efficacious algorithm aim to efficiently resolve instances of FFLPP, thereby advancing the state-of-the-art in this domain.

The organization of this study unfolds in the following manner:

Section 2: This pivotal section lays the groundwork, furnishing essential preliminaries crucial for a nuanced understanding of the ensuing chapters. Additionally, it introduces cutting-edge techniques for the comparison of fuzzy numbers.

Section 3: Within this section, we unveil the evolution of an enhanced algorithm meticulously crafted to address the challenges posed by the Fully Fuzzy Linear Programming Problem (FFLPP). To substantiate and validate the prowess of this algorithm, we present numerical examples, inviting a comparative analysis with existing results.

Section 4: Here, we include our findings into comprehensive conclusions, synthesizing insights gleaned from the study. This section encapsulates the outcomes of our exploration, offering a panoramic view of the implications and potential avenues for further research.

Anti-synchronization Between Different Nonlinear Chaotic Systems Via Active Nonlinear Control Method

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Abstract: This research article establishes anti-synchronization between the three-dimensional non-identical nonlinear Chen-Lee, Lorenz-Stenflo and Liu-Chen chaotic systems *via* active nonlinear control techniques. Phase portraits of master and slave systems in the form of anti-synchronization are investigated. The stability results are discussed by the stability theory of Lyapunov function. Anti-synchronization of chaotic Chen-Lee system and chaotic Lorenz-stenflo systems as well as anti-synchronization of chaotic Chen-Lee and Liu-Chen systems have been established using active control methodologies. The active control method is more efficient to obtain the anti-synchronization between different chaotic systems. Numerical results are also discussed by the proposed method.

Keywords: Chen-Lee systems; Lorenz-Stenflo systems; Liu-Chen systems; Active control; Stability theory.

1. INTRODUCTION

Pecora and Carroll [1] developed the concept of chaos synchronization in 1990 since then chaos synchronization theory has become a hot topic for researchers and scientists in nonlinear dynamical systems. Chaos synchronization exists between two or more coupling systems in which one is known as the master or drive and the other is a slave or response system. Different types of chaos synchronization have been discussed such as complete synchronization, projective synchronization, modified projective synchronization, and anti-synchronization. [2-4]. They are applied in different branches of sciences and technologies- electrical signals, network theory chemical reactions, satellite systems, *etc.* Many different methods have been applied successfully for the chaos synchronization of coupling nonlinear chaotic systems, such as adaptive control, active control, back-stepping design and nonlinear control. With these methods, the synchronization problems of chaotic

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systems have been dealt with by many researchers and scientists so far [5-8]. Many researchers have addressed the different techniques for anti-synchronization and synchronization of chaotic systems such as anti-synchronization using fuzzy logic constant controllers, active control techniques, adaptive control techniques, predictive control, sliding mode control techniques, and adaptive sliding mode control techniques feedback control techniques [9-13].

Motivated by the above discussion, authors have addressed the anti-synchronization of chaotic non-identical systems using active control techniques in this chapter. Further, a lot of research being going on in these fields and new trends in the synchronization of chaotic systems could be seen in this contemporary world due to new evolution in sciences and technologies, we need more robustness and efficient work for coming generations.

2. SYSTEM DESCRIPTION

The drive nonlinear system is defined as,

$$\dot{x} = g(x),$$

And the response system is also defined as,

$$\dot{y} = f(y) + u(x, y).$$

Where x, y are states variables of the systems and “ $g, f : R^n \rightarrow R^n$ ” are nonlinear parts of the systems and $u(x, y)$ are taken as controllers.

To obtain the anti-synchronization between the systems, we define the error dynamics, as $e = x + y$, and design a controller $u(x, y)$ for which the trajectories of the response system with the initial condition y_0 can asymptotically approach the drive system with the initial condition x_0 in the opposite direction *i.e.* “ $\lim_{t \rightarrow \infty} \|y(t) + x(t)\| = 0$ ”, where $\|\cdot\|$ is the Euclidean norm.

Defining the chaotic Chen-Lee system [7, 8] as follows:

$$\begin{aligned}\dot{x}_1 &= -x_2x_3 + a_1x_1 \\ \dot{x}_2 &= x_1x_3 + b_1x_2 \\ \dot{x}_3 &= \frac{1}{3}x_1x_2 + c_1x_3\end{aligned}$$

Where " $x_1, x_2, x_3 \in R^n$ " are state vectors and a_1, b_1, c_1 are constant parameters. The system has chaotic behaviour for the parameter values $a_1 = 5.1, b_1 = -11, c_1 = -3.9$.

The chaotic Lorenz-Stenflo system [9] is defined as:

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= x_1(c - x_3) - x_2 \\ \dot{x}_3 &= x_1x_2 - dx_3\end{aligned}$$

Where " $x_1, x_2, x_3 \in R^n$ " are state vectors and a, b, c, d are constant parameters. The system has chaotic behaviour for the parameter values $a=1.1, b=1.4, c=26.1, d=0.70$

The chaotic Liu- Chen system [10] is defined as:

$$\begin{aligned}\dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= -bx_2 + x_1x_3 \\ \dot{x}_3 &= x_1x_2 - cx_3\end{aligned}$$

Where " $x_1, x_2, x_3 \in R^n$ " are state vectors and a, b, c are constant parameters. The system has chaotic behaviour for the parameter values $a = 0.40, b = 12.0, c = 5.0$.

3. ANTI-SYNCHRONIZATION BETWEEN CHAOTIC CHEN-LEE SYSTEM AND CHAOTIC LORENZ-STENFLO SYSTEMS

Chen-Lee system is taken as a master (drive) system and Lorenz-Stenflo is taken as a slave (response) system.

$$\begin{aligned}\dot{x}_1 &= -x_2x_3 + a_1x_1 \\ \dot{x}_2 &= x_1x_3 + b_1x_2 \\ \dot{x}_3 &= \frac{1}{3}x_1x_2 + c_1x_3\end{aligned}\tag{1}$$

CHAPTER 6

Fixed Charged Nonlinear Solid Transportation Problem with Budget Constraints Using Bounded Parameters

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Abstract: This chapter considers a fixed charged solid transportation problem where all parameters of the problem have uncertainty due to ambiguity in the data set. These uncertain parameters (destination budget, accessibility, necessity, conveyance capacity, and transportation cost) are considered bounded. The problem's solution is obtained by applying a transformation of the interval-based model into a conventional nonlinear programming problem. The transformed model is solved using any nonlinear programming technique. The solution obtained by the deterministic model is an acceptable solution to the main model. An examples with hypothetical data are explained to validate the developed model and methodology.

Keywords: Fixed charge, Solid transportation problem, Interval analysis, Partial ordering, Nonlinear programming problem, Mathematics.

1. INTRODUCTION

A conventional problem of transportation (in short (*TP*)) deals with a uniform product's minimum total transportation cost from sources to destinations. It can formulate a linear programming problem model with two types of equality constraints: source and destination constraints. These two constraints play a vital role in reducing transportation costs and improving service quality in logistics, industrial, and supply chain management. In various supply chain problems, an identical item manufactured is transported from a producer to a consumer by different conveyance like ships, trucks, cargo flights, goods trains, *etc.* A solid transportation problem (*STP*) is improved from a traditional *TP* by taking into the

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considering of several types of conveyance modes. It is also designed as a nonlinear programming problem by squaring the number of supplies transported relative to the variable cost. For example, a fixed charge nonlinear solid transportation problem (*FCNSTP*) is one of the essential extensions of the conventional *TP*. There is another cost include in the *STP* known as fixed cost apart from the variable transportation cost. This cost is associated with the track and free from the number of supplies elated.

In a real-life supply chain problem, estimating the crisp value of parameters is not easy because of the uncertainty in the data set, such as shortage of raw materials, nasty weather conditions, availability of the conveyances, price rise of fuel, traffic jam, *etc.* The uncertainty avail in the data set may be handled by considering these parameters as random numbers or fuzzy numbers through an appropriate probability distribution function or membership function, respectively. Therefore, it is not easy for the DM to define appropriate such types of functions. Consequently, we consider these parameters as closed intervals. Then the *FCNSTP* with interval parameters is said to be fixed charged nonlinear interval solid transportation problem (*FCNSTP*).

The classical *TP* is defined as a particular type of linear programming problem where demand and supply considered as equality constraints by Hitchcock [1] and obtained a solution of the problem. K. Haley [2-4] established the relationship between the *STP* and the *TP*, and developed a methodology to solve the *STP*. Many researchers have studied the *STP* under uncertain environment in various directions and obtained the solution of the problem (see [5-10]). An important extension of *TP* known as fixed charged *TP* is considered by many researchers during last half century and develop several ways to solve this problem (see [11-22]).

Due to uncertainty in the data set for the real-life *TP*'s, the fixed charged solid transportation problem with an uncertain environment is one of the attractive research areas for researchers in the last decade. In the beginning, many authors have considered fuzzy theory to deal with the uncertainty of the problem. As one can see that Yang *et al.* [23] is examined a solid *TP* with a constant charge under fuzzy uncertainty and obtained the solution of the model using a fuzzy simulation method and search algorithm. *M-A-Zavardehi et al.* [24] have formulated a fixed charged *TP* as a two-stage supply chain model, where likely stations are contenders to be supply hubs and customers with a specific requirement. Further, a fixed charged *TP* under a fuzzy uncertainty by assuming the direct and fixed costs as fuzzy numbers [25] is studied. While *Safi and Razmjoo* [26] have presented a fixed-charged transportation problem with bounded parameters and solved using a bi-

objective deterministic linear programming problem. P.K. Giri *et al.* [27] considered fixed charged multi-item solid transportation problems with fuzzy parameters and proposed an approach to solving this problem. Midya and Roy [28] considered a fixed-charge *TP* in an interval & rough interval setting and studied the solution of the problem by interval programming.

1.1. Motivation

As mentioned above, most of the authors have studied the fixed charged solid *TP* with parameters as fuzzy numbers. However, some of the authors mentioned above have also considered the issue with interval parameters [29-31] and have developed the problem's solution using a genetic algorithm. Pararily. Nevertheless, none of them thought a fixed charged nonlinear solid transportation problem under budget constraints for destinations, conveyance capacity, transportation cost, availability, requirement are represented as closed intervals. Therefore, a general fixed charged nonlinear solid transportation problem with interval parameters designs and obtains the solution to this problem using interval analysis and parametric concepts for the interval. The developed methodology illustrates through a real-life numerical *TP*.

The chapter is organized as follows, Section 2 defines some fundamentals in interval analysis, these are employed for developing the outcomes of the chapter. In Section 4, a fixed charged nonlinear solid transportation problem is defined for crisp and interval parameters and develops the interval problem's solution procedure. Further, develop solution procedure of the problem is verified in a real life numerical problem in Section 5.

2. INTERVAL ANALYSIS

An interval is denoted by $[x^L, x^R]$ with $x^L \leq x^R$; $x^L, x^R \in \mathbb{R}$. If $x^L = x^R = x$, then $[x, x]$ is said to be a degenerate interval or a real number x . Interval's set on \mathbb{R} is denoted by $\mathbb{I}(\mathbb{R})$, and $\mathbb{I}(\mathbb{R}) = \{[x^L, x^R] | x^L \leq x^R; x^L, x^R \in \mathbb{R}\}$. For a binary operation $*$ $\in \{+, -, \cdot, /\}$ on the set of real numbers, The binary operation \circ on $\mathbb{I}(\mathbb{R})$ can be expressed as:

$$[x^L, x^R] \circ [y^L, y^R] = \{x * y \mid x \in [x^L, x^R], y \in [y^L, y^R]\},$$

Where $*$ $\in \{+, -, \cdot, /\}$. For division on $\mathbb{I}(\mathbb{R})$, it is supposed that $0 \notin [b^L, b^R]$.

Soft Computing Techniques for Cancer Classification of Gene Expression Microarray Data: A Three-Phase Hybrid Approach

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Abstract: Recently, many soft computing methods have been implemented to extract information from big data. A standardized format for evaluating the expression levels of thousands of genes is made available by DNA microarray technology. Cancers of several anatomical regions can be identified with the help of patterns developed by gene expressions in microarray technology. Since the microarray data is too huge to process due to the curse of dimensionality problem.

Methodology: Therefore, in this chapter, a setup based on a hybrid machine learning framework using soft computing techniques for feature selection is designed and executed to eliminate unnecessary genes and identify important genes for the identification of cancer. In the first stage, the genes or the features are taken out with the aid of the higher-order Independent Component Analysis (ICA) technique. Then, a wrapper algorithm that is based on Spider Monkey Optimization (SMO) with Genetic Algorithm (GA) is used to find the set of genes that improve the classification accuracy of Naïve Bayes (NB) classifiers and Support Vector Machine (SVM). For comparison purposes, three other optimization techniques considered in this chapter are Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), and Genetic Algorithm (GA). After the selection of relevant expressed genes, the most popular classifiers namely Naïve Bayes (NB) and Support Vector Machine (SVM) are trained with selected genes, and in the end, the accuracy of classification is determined using test data.

Result: The experimental results with five benchmark microarray datasets of cancer prove that Genetic Spider Monkey (GSM) is a more efficient approach to improve the classification performance with ICA for both classifiers.

Keywords: Genetic spider monkey (GSM); Independent component analysis (ICA); Genetic algorithm (GA); Particle swarm optimization (PSO); Support vector machine (SVM) and Naïve Bayes (NB).

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1. INTRODUCTION

Cancer is an unnatural growth of the cells in the affected area. These abnormally grown cells can reach other vital organs of the body [1-3]. The classification of the types of cancer relies on the expression profiles of the genes and has shown a path for designing an independent approach toward designing a possible medical treatment method. Recently, due to the discoveries in data analytics and enhanced storage capacities, numerous big data applications gained attention in the development of applications, for a wide spectrum of biological problems [1,2,4]. The identification of related genes plays an important role in the detection of cancer and has a severe impact on the predictions of cancer based on microarray data [5, 6]. As a result, dimension reduction and classification serve as the fundamental steps in further analysis. Currently, due to time complexity in current data generation techniques, the dimensions of the generated data create many challenges in the reduction of the same. Microarray data of cancer is required to be processed through machine learning algorithms and soft computing techniques in the extraction of informative genes expressed significantly in the given sample. A large number of datasets create complications for researchers in identifying useful information through the applications of statistical and bioinformatics tools. This research has moved to a new age of molecular classification [7-9]. Microarray data analysis is among the widely utilized methods in the detection of several types of cancers [10].

1.1. Lecture Survey

To observe the spectrum of genome-wide expression, one of the vital tools that many biologists rely on is microarray technology. The gene expression data classification is an important task that identifies the linked genes among huge no of genes reflected through samples. Many genes will not be more informative, as they may be irrelevant or redundant. For this reason, the number of expressed genes needs to be reduced to informative related genes to increase the accuracy of the classification task. So, classification with this large amount of data is difficult as it increases computational costs and degrades the speed and accuracy of the classifier. In such cases, it is difficult to apply traditional classifiers to asymmetric data, hence for further scientific analysis, reduction in the dimension of microarray data is highly recommended [11]. The main aim of applying the dimension reduction approach is to find the most informative, relevant and highly correlated features from the large datasets using feature selection and feature extraction methods [12]. Identification and drug discovery have benefited from the classification of several forms of predominate gene expression when counting genes for biological

concerns. [9, 13]. Recently, Gene expression analysis techniques were implemented to find out the risk analysis of cancer through suitable data mining with soft computing techniques. Many researchers used multiple soft computing paradigms, the statistical characterization for feature selection, and cancer classification of microarray. Various supervised and unsupervised techniques with soft computing methods are useful tools that generally yield high accuracy in the classification and prediction process. In a study [14], the author increased the efficiency of the Artificial Bee Colony Algorithm by using a genetic algorithm for the classification of support vector machine. The suggested algorithm has been operated with several microarray datasets for choosing the most informative genes responsible for the classification of the type of cancer. The proposed technique shows improvements in the accuracy of the classification with very few average numbers of selected genes. For the purpose of diagnosis and prediction of survival of cancer patients, most researchers apply intelligent techniques of gene selection and classify it using different classifiers. In another study [15], the author reviewed algorithms for selecting genes based on machine learning. In order to increase the classification accuracy of various machine learning classifiers, the suggested approach demonstrates that gene selection selects the pertinent and instructive genes and eliminates irrelevant genes. Recently, the use of a hybrid strategy has improved the overall accuracy of the classifiers by cutting down calculation time and maximising the benefits of combining various dimension reduction strategies. [16, 17]. The mixed approach reduces the amount of the dimension of created data by taking into account the various beneficial characteristics of separate algorithms in feature selection and extraction procedures. Depending on the need for the data sets and available features, researchers implemented many combinations of the algorithms. In another study [18], in order to improve the performance of the KNN classifier, the author proposed an efficient ensemble strategy for three benchmark data sets, including the leukaemia dataset, colon dataset, and breast cancer dataset. When the author coupled PSO and Artificial Bee Colon to SVM to classify nine cancer datasets, the results were better compared to when PSO and ABC were used separately [19]. The strategies for feature selection in gene expression have been studied by scholars [20]. They focus on the filter, wrapper, and embedding approaches and discovered that a hybrid strategy based on filter and wrapper produces better results when used for pattern rearrangement or machine learning challenges. Using a combination of hybrid gene selection algorithms that are based on various sorts of nature-inspired metaheuristic techniques, a few authors constructed a classification setup and utilised it to find and examine the gene expression profiles that cause cancer [16-21]. A classification model was proposed that estimates the reduced gene sets quality with a hybrid approach based on optimum path forest and bat algorithm. To overcome issues with advanced feature

A Survey of Pairing Free Certificateless Signature Schemes for Secure Networking

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Abstract: Technology has enhanced the jeopardy of misuse of individual confidential data and identity theft attacks, where securing the content of communication and confidentiality has become one of the chief requirements. Here Cryptography operates as the cornerstone of computer and communications security. Certificateless cryptography, together with digital signatures, is the best solution for achieving fundamental security, *i.e.*, confidentiality, data integrity, non-repudiation, and authentication. Correspondingly, certificateless cryptography (CLS) abolishes certificate management problems and the key escrow problems of public-key cryptography (PKC) and identity-based cryptography (IBC). CLS is the optimum combination of PKC and ID-based PKC, hence making it more efficient. Many signature schemes rely on certificateless schemes (CLS) and are formed using bilinear pairing and the MTP hash functions. An MTP (map-to-point) hash function and elliptic curve bilinear pairing are lavish procedure schemes, and they enhance computation expenses in the schemes and are time-consuming. In contrast, pairing-free CLS schemes rely on elliptic curve cryptography (ECC) and have comparatively less computational cost, providing the desirable time requirement and making the scheme more efficient. In this chapter, we have done a survey of various pairing-free CLS schemes and compared them with each other. We have analyzed the performance evaluation of various schemes' security and efficiency. Some are even observed to be precarious under various attacks.

Keywords: Cryptography, Certificateless Cryptography, Security, Identity Based Cryptography, Public-key cryptography, Elliptic curve cryptography.

1. INTRODUCTION

With the rapid blooming of human civilization towards metropolitan cities and the IoT (Internet of Things), secrecy in communication and information security has become very important. When it concerns the content of communication, confidentiality becomes one of the chief requirements for each communication, particularly for diplomatic purposes and the military. Intended to ensure data secrecy and integrity, customary data defense etiquette may not be entirely fitting

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during the execution of online services [1]. Technology enhances the jeopardy of misuse of individual confidential data and identity theft attacks. In the open environment of communications, unauthorized entities can effortlessly instigate numerous attacks, for instance, replay, data modification, denial of service, impersonation, and other attacks. Cryptography operates as the cornerstone of computer and communications security. Cryptography provides the main security services, *i.e.*, confidentiality, data integrity, non-repudiation, and authentication. Digital signature and cryptography are esteemed as essential and valuable tools for achieving information security. A digital signature is a new substitute designed for signing documents with a pen and chapter. It mainly uses elementary methodology together with cryptography. The digital signature is a pathway with the purpose of uniting our entity with digital data that can be validated individually *via* a receiver besides any third party. Recently, many works have been presented based on PKC, but PKC has an issue with certificate management and storage. In 2003, Shamir proposed an extra cryptographic primitive named identity-based cryptography (IBC) to rectify the certificate supervision crisis inherited in the PKC. The ID-based scheme's superiority is that a unique ID is used to recognize the individual's identity as the user's public key, while PKG (private key generator), a trusted third party, is accountable for generating the corresponding private key of the user with his master secret key [2]. With this approach, the limitation of PKC (public-key cryptography), *i.e.*, the requirement of the certificate, is eliminated. However, PKG is able to falsify anyone's signatures easily as PKG knows the private key, and this results in the key escrow problem. Al-Riyami and Peterson were the first to broach the concept of certificateless cryptography in 2003. In the CLC, the individual himself computes the private and public key duos. The unification of a randomly selected secretive value and the partial private key of the user generates the private key of the user. At CLC, for calculating the partial private key, a trusted third party, *i.e.*, the key generation center (KGC), is use his private or master secret key. Therefore, the KGC cannot get hold of the private keys of the users; as a result, the CLC abolishes certificate supervision and the key escrow problem. Recently, many works have been done based on the elliptic curve bilinear pairing and the map-to-point hash function. However, in opposition to diverse security attacks, MTP (map to point) hash functions and elliptic curve bilinear pairing are lavish procedure schemes that are not safe and sound [3-5]. They enhance computation expenses in the schemes. In view of the fact that roughly two-three times more than an ECPM (elliptic curve scalar point multiplication) necessitates the desirable time to achieve the requirement of one bilinear pairing operation [6], Besides comparatively, the computation cost of an ECPM is much less than that of the MTP hash function [7]. In addition, the execution of such schemes of bilinear pairing is a complex operation

as it requires a supersingular elliptic curve group with a huge size component [8]. For strong resistance and low computation costs, building an effective CLS scheme becomes necessary. Ballare *et al.* [9] have demonstrated the practical efficiency of their computational approach while also maintaining strong security measures. They conducted a security evaluation using the random oracle model, considering various adversaries with different attack capabilities and adaptive message selection. The certificateless signature scheme provides strong security against chosen message identity attacks within the random oracle model.

1.1. Motivation and Contribution

A comprehensive literature review of the existing pairing-free CLS scheme was carried out. Pairing free CLC conjointly with a digital signature provides the required securities and subdues the PKC and ID-BC inadequacies. It was found that pairing-free CLS schemes are based on the hard problem of the elliptic curve discrete logarithm problems. Pairing free CLS scheme provides better efficiency and security than the pairing-based signature schemes.

After an extensive examination, we have observed that some of these schemes are precarious under different adversary attacks. This motivates a data survey on the various pairing-free CLSs' efficiency and security. This data survey highlights the robustness of the various schemes and their performance evaluations regarding computational cost and security.

1.2. Structure of the Chapter

The chapter is systematized as follows. Section 2 highlights some of the related works. Section 3 describes basic conceptions about the bilinear map, complexity assumption, and the elliptic curve cryptography; section 4 describes the system model of CLS and various attacks. In section 5, we have discussed the performance evaluation of various pairing-free CLSs. At last, we given a conclusion and future scope of the chapter.

2. RELATED WORK

An identity-based cryptosystem conception was brought up by Shamir in 1984 [10]. Identity-based cryptosystem has one flaw, *i.e.*, the key escrow difficulty and abolishing this generalization on certificate-less cryptography is presented by Al-Riyami and Paterson [11]. In contrast, a study by Huang *et al* [12] revealed that their scheme is not secure against adversary A_1 . The initial generic construction of

Analysis and Development of Tie-breaking Procedures Using Fuzzy Technique for Order Performance by Similarity to Ideal Solution

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Abstract: This work focuses on tie-breaking procedures that employ the fuzzy technique for order performance by similarity to the ideal solution. A tie situation occurs when two or more competitors receive the same score in a competition. Fuzzy technique for order performance by similarity to ideal solution provides efficient and effective decision making. The fuzzy technique for order performance by similarity to ideal solution assesses accurate and systematic decision-making based on multiple criteria among a finite set of possible alternatives. The proposed tie-breaking approach establishes an ordering relationship and a ranking among a set of alternatives under certain objective and subjective decision criteria. A numerical example is considered to demonstrate the computing process.

Keywords: Tie-breaking procedures; Fuzzy technique; Set of alternatives; Decision criteria.

1. INTRODUCTION

Fuzzy technique for order performance by similarity to ideal solution (Fuzzy TOPSIS) is introduced by Hwang and Yoon [1, 2]. Fuzzy TOPSIS is a modern approach for multiple criterion decision-making (MCDM) under the consideration of uncertainties. Fuzzy TOPSIS derives the priorities among a set of alternatives based on certain decision criteria. Fuzzy TOPSIS obtains an aggregate ranking among a set of alternatives on the basis of pre-assumed decision criteria. Fuzzy TOPSIS deals with fuzzy judgments and provides more exact and accurate results as compared to crisp judgments. Fuzzy TOPSIS facilitates the decompositions, reduces the inconsistency, provides pair-wise comparisons, and develops the priority vectors.

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Many authors, academicians, and researchers have worked on fuzzy TOPSIS in order to deal with multiple criteria decision-making problems. Sengulf *et al.* [3] ranked renewable energy supply systems in Turkey using the fuzzy TOPSIS method. Chu and Lin [4] used the fuzzy TOPSIS for the robot selection. Torlak *et al.* [5] used fuzzy TOPSIS to analyze the business competition in the local airline industry of Turkey. Sun [6] applied fuzzy TOPSIS and fuzzy analytical hierarchy process (fuzzy AHP) for the evaluation of the performance model. Sun and Lin [7] composed fuzzy TOPSIS to evaluate the competitive advantages of shopping websites. Torfi *et al.* [8] employed the fuzzy TOPSIS and the fuzzy AHP methods to establish the relative weights of assessment criteria. Hatami-Marbini and Kangi [9] applied fuzzy TOPSIS for group decision-making in the Tehran stock exchange. Shen *et al.* [10] developed an expanded intuitionistic fuzzy TOPSIS method based on a new distance measure to evaluate credit risk. Onu *et al.* [11] created a fuzzy TOPSIS multi-criteria decision analysis model framework for the evaluation of sustainable acid rain control alternatives. Zyoud *et al.* [12] used fuzzy TOPSIS and fuzzy AHP for water loss management in underdeveloped nations. Chen [13] suggested an inclusion-based TOPSIS technique with interval-valued intuitionistic fuzzy sets. Wu *et al.* [14] created an interval Type-2 fuzzy TOPSIS model integrating social network information for large-scale group decision-making challenges. Junior *et al.* [15] studied supplier selection problems by comparisons among AHP and TOPSIS methods in fuzzy environments. Taylan *et al.* [16] used fuzzy AHP and fuzzy TOPSIS techniques to pick construction projects and estimate risk. Kannan *et al.* [17] used fuzzy TOPSIS to pick green suppliers based on GSCM practices for a Brazilian electronics business. In this chapter, we employ the fuzzy TOPSIS method in order to break the tie-situations using multiple criteria decision-makings.

1.1. Motivation and Contribution

As previously mentioned, while various methods [18-24] have utilized the fuzzy TOPSIS approach for various decision-making scenarios, none of them have, as far as we are aware, utilized it for tie-breaks, or instances in which two or more players in a match or exam receive identical results. These facts motivated us to employ a fuzzy TOPSIS method in order to break tie-situations among a set of alternatives based on certain decision criteria. For achieving this goal, we proposed fuzzy TOPSIS-based tie-breaking procedure, established an order relationship, and ranked among a set of alternatives with the help of a finite set of decision criteria. To show the feasibility and computational process of the proposed method, a numerical example is considered from Iftikhar *et al.* [25].

1.2. Roadmap of the Chapter

The rest of the chapter is organized as follows: In Section 2, we give the basic preliminary on fuzzy sets and fuzzy logic. In section 3, we propose the basic idea of fuzzy TOPSIS for multiple criteria decision-making. In Section 4, we illustrate the computing process of the proposed method. Finally, we reached a conclusion and provided remarks.

2. PRELIMINARIES

The basic definitions of fuzzy sets, fuzzy logic, and fuzzy TOPSIS are presented in this section.

Definition1. A fuzzy set of the real line $\tilde{M}: R \rightarrow [0, 1]$ is said to be a fuzzy number if it satisfies the following properties [4]:

1. \tilde{M} is a convex fuzzy set, *i.e.* $\exists a_0 \in R$ such that $\mu_{\tilde{M}}(a_0) = 1$.
2. \tilde{M} is a normalized fuzzy set, *i.e.* $\mu_{\tilde{M}}(\theta x + (1 - \theta)y) \geq \min\{\mu_{\tilde{M}}(x), \mu_{\tilde{M}}(y)\}$
3. \tilde{M} has a piecewise continuous membership function.

The class of all fuzzy numbers defined on the real line is denoted by $F(R)$.

Definition2. A fuzzy number $\tilde{M}: R \rightarrow [0, 1]$ is said to be a triangular fuzzy number if its degree of belongingness is defined by [4]:

$$\mu_{\tilde{M}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \frac{-x + a_3}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

The degree of belongingness of a triangular fuzzy number is of triangular shape, and it is represented by three points $\tilde{M} = (a_1, a_2, a_3)$.

Definition3. The alpha-cut of triangular fuzzy number \tilde{M} is a closed interval $[\tilde{M}_\alpha^L, \tilde{M}_\alpha^U]$, where \tilde{M}_α^L is the lower bound and \tilde{M}_α^U is the upper bounds of the closed interval. For instance, if triangular fuzzy number $\tilde{M} = (a_1, a_2, a_3)$, then the alpha-cut of \tilde{M} can be expressed as [4]:

$$\tilde{M}_\alpha = [a_1 + (a_2 - a_1)\alpha, \quad a_3 - (a_3 - a_2)\alpha]$$

SUBJECT INDEX

A

Absorption refrigeration 43
 Active control 62, 65, 69
 method 62
 theory 65, 69
 Acute leukaemia 100
 Ad hoc networks 117, 119
 Addition, elliptic 124, 125
 Adversary attacks 116, 117, 118
 Algorithm 48, 50, 53, 58, 59, 94, 95, 96, 98,
 100, 104, 105, 107, 108
 developed 48
 hybrid 104
 nature-expired 98
 nature-inspired 96
 Anti-synchronization 62, 65, 68, 72
 error 65, 68
 of chaotic Chen-Lee system and chaotic
 Lorenz-stenflo systems 62
 of error dynamics of master and slave
 systems 72
 Applications 2, 3, 37, 59, 93
 real-world 2
 Approach 49, 88, 96, 116
 computational 116
 conventional linear programming 49
 machine learning 96
 parametric 88
 Arithmetic operations 51
 Artificial bee colony (ABC) 92, 94
 algorithm 94

B

Balking and threshold-based recovery policy 1
 Bat algorithm 94, 96
 binary 96
 Bayes 98
 rule 98
 theorem 98
 theory 98

Bernoulli feedback 3, 14
 Bilinear pairing multiplication 125
 Binary 77, 96
 operation 77
 particle swarm optimization algorithm 96
 Brazilian electronics business 133
 Breast cancer dataset 94

C

Calculation, time-consuming 138
 Cancer 92, 93, 94, 96, 99, 100, 108
 diagnosis 108
 microarray 99
 prostate 100
 Cancer microarray 93, 96
 data 93, 96
 profiles 96
 Cargo flights 75
 Chaos synchronization theory 62
 Chaotic 62, 63, 64
 behaviour 64
 Chen-Lee system 62, 63, 64
 Lorenz-stenflo systems 62, 64
 systems 62, 63
 Chen-Lee 62, 64
 systems 62, 64
 three-dimensional non-identical nonlinear
 62
 CLC cryptography 124
 CLS, pairing-based 125
 Colon cancer 100
 Communication, protected 119
 Competition, business 133
 Complex systems problems 1
 Cone convexity 18, 22, 33
 assumption 18
 suppositions 22, 33
 Congestion problems 2
 Consumption, real-time 83
 Controllers, fuzzy logic constant 63

Convex 13, 18, 19, 20, 21, 22, 24, 26, 29, 30, 32, 51, 134
 higher-order cone 19
 fuzzy set 51, 134
 cone 18, 20, 21, 22
 Cost, flexible carriage 83
 Cryptographic operations 124, 125
 Cryptography 114, 115, 120
 identity-based 114, 115
 public-key 114, 115, 120
 Cuckoo search 95, 96
 binary 96
 binomial 96
 hybrid 95
 Cycle, vapor absorption refrigeration 37

D

Data 75, 82, 88, 93, 94, 97, 100, 102, 114, 115
 asymmetric 93
 crisp 82
 defense etiquette, customary 114
 digital 115
 high-grade glioma 102
 hypothetical 75, 88
 microarray cancer 100
 mining 94
 multidimensional 97
 Datasets, five benchmark microarray gene
 expression 100
 Decision maker (DM) 76, 84, 136
 pooling 136
 Decision-making problems 59, 133
 Design, back-stepping 62
 Destination constraints 75
 Deterministic programming problem 87
 Digital signature and cryptography 115
 Distinctive triangular lexicographic ordering
 50
 DNA microarray technology 92
 Duratherm oil 36

E

Economics, mathematical 18
 Efficiency 37, 39, 41, 43, 44, 46, 94, 97, 98, 108, 114, 116, 117, 128
 computational 108, 117
 electrical 46
 electrical generation 39

Electrical power 46
 Elite hybrid binary cuckoo search (EHBCS)
 95
 Elliptic curve(s) 114, 115, 116, 117, 120, 121, 122, 124, 125, 128
 bilinear pairing 114, 115, 128
 cryptography (ECC) 114, 116, 117, 120, 124
 discrete logarithm problems (ECDLP) 122, 128
 nonsingular 124
 Elliptic scalar multiplication 125
 Energy 36, 37, 39, 40, 41, 42, 43, 44, 45, 46
 balance equations 40
 efficiency of heliostat 39, 46
 solar 36, 37, 39
 thermal 36, 37, 39
 Energy efficiency 36, 39, 42, 43, 46
 trigeneration 39
 Environment, resource-constrained 118
 Evolutionary operators 96
 Exergy 36, 37, 39, 40, 43, 44, 45
 analyses 36, 37
 balances 39
 efficiencies efficiencies 43
 input 44, 45
 methods 39
 rate 45
 Exergy destruction 44, 45
 rate 45
 Exergy efficiencies 36, 40, 41, 42, 43, 44, 46
 trigeneration 40
 Expense, computational 128
 Ezzati 48, 50, 52, 59
 method 48, 50, 59
 ranking function 52

F

Feature selection techniques 100, 101, 102, 103, 108
 Feedback queue 1
 finite capacity Markovian 1
 Findings, post-clustering 95
 Five hybrid feature selection methods 104
 Fixed-charge TP 77, 81
 nonlinear interval solid transportation
 problem 81
 Fluids, working 37, 41, 42, 44
 Form 78, 121

- addition operation 121
 - parametric 78
- Framework 92, 97, 133
 - fuzzy TOPSIS multi-criteria decision analysis model 133
 - hybrid machine learning 92
- Fuel charges 81
- Fully fuzzy linear programming problem 50
- Function 21, 22, 28, 49, 69, 76, 78, 99, 119
 - interval-valued 78
 - kernel 99
 - probability density 99
 - probability distribution 76
 - producing raking 49
- Fuzzy 48, 49, 51, 53, 76, 132, 133, 142, 143
 - AHP method 133, 142, 143
 - environments 49, 133
 - judgments 132
 - number foundations 51
 - simulation method 76
 - theory 76
 - vector variable 53
- Fuzzy numbers 49, 50, 51, 52, 53, 56, 57, 59, 76, 77, 134, 135
 - arbitrary 52
 - arbitrary triangle 59
 - comparison of 49, 50
 - intuitionistic 57
- Fuzzy TOPSIS 132, 133, 134, 136, 142, 143
 - method 133, 136
 - techniques 133

G

- GA algorithm 98
- Gene(s) 92, 93, 94, 95, 96, 97, 98, 100, 101, 104, 105, 107, 108
 - counting 93
 - expressed 92, 93
 - ontology 95
 - informative 93, 94
 - linked 93
 - redundant 104
- Gene expression 92, 93, 94, 108
 - microarrays 108
 - profiles 94
- Gene expression data 93, 108
 - classification 93
- Genetic algorithm (GA) 77, 92, 94, 96, 98
 - binary 96

- Genome-wide expression 93
- Greenhouse gas, reduced 37
- GSM 98, 100, 104
 - algorithm 98, 100
 - new improved optimization technique 100
 - wrapper approach 104

H

- Hash function 114, 115, 124, 125
 - map-to-point 115
- Healthcare wireless 118
 - medical sensor networks (HWMSNs) 118
 - network 118
- Heat 36, 37, 40, 43, 44
 - input 43
 - recovery steam generator (HRSG) 36, 40, 44
 - transfer fluid 36
 - waste 37
- Heat energy 36, 45, 46
 - loss of 46
 - thermal 36
- Heliostat field 36, 44
- High-grade glioma 100
- Hybrid approach 94, 95, 96, 100, 108
 - methods-based 96

I

- IBC cryptography 128
- ICA 95, 96, 104
 - feature vectors 96, 104
 - Technique 95, 96
- Identity-based 114, 115, 116
 - cryptography (IBC) 114, 115
 - cryptosystem 116
- Image processing 18
- Independent component analysis (ICA) 92, 96, 97, 98, 100, 104
- Information, social network 133

K

- Kernel function maps 99
- Key 115, 117, 119, 122, 123, 124
 - generation center (KGC) 115, 122, 123
 - replacement attacks 117, 119, 124
- KGC, malicious 123

L

Leukaemia dataset 94
 Linear kernel 99
 Linear programming 48, 49, 53, 56, 76
 method 49
 problems 49, 53, 56, 76
 LINGO software 88
 Linguistic variables 49
 Lorenz-Stenflo 62, 66
 chaotic systems 66
 systems 62
 Low-temperature meals 56
 Lyapunov function 62, 66, 69, 72

M

Machine learning 93, 94, 108
 algorithms 93, 108
 challenges 94
 Markovian queueing model 3
 Mathematica software 66, 70
 Medical treatment method 93
 Memory-based mechanism 95
 Methods 49, 104
 hybrid 104
 lexicography 49
 Microarray 92, 93, 96
 techniques 96
 technology 92, 93
 Microarray data 92, 93, 96, 99, 101, 104
 analysis 93
 Microarray datasets 92, 94, 95, 99, 104, 108
 five 99
 five benchmark 92, 95, 108
 five cancer 95, 104
 MTP hash function 114, 115, 125, 128

N

Naïve Bayes classifier (NBC) 98
 Nature-inspired metaheuristic 96
 algorithms 96
 strategy 96
 Networks, medical sensor 118
 Nonlinear programming problem 75, 76
 conventional 75
 Nonparametric kernel density estimation
 method 99

O

Online services 115
 Optimization 85, 104
 methods 104
 software 85
 Optimization problems 18, 19, 33, 49
 scalar-valued parametric 18, 33
 Outperformed cuckoo search algorithms 96

P

Parabolic trough 36
 Particle swarm optimization (PSO) 92, 94
 Poisson process 3
 Policy 2, 3, 8
 recovery 8
 threshold-recovery 3
 Power 36, 37
 generation 37
 producing 36
 thermal 37
 Pressure 38, 41, 43
 transducer 38
 turbine inlet 41, 43
 Printed circuit board (PCB) 2
 Private key 115, 122, 123, 124
 generator 115
 Probability, complementary 4
 Problem 18, 19, 23, 30, 31, 32, 33, 37, 49, 54,
 55, 56, 75, 76, 77, 85, 88, 93, 104
 biological 93
 complementarity 33
 conventional 75
 environmental 37
 numerical 77, 85
 Process 36, 49, 54, 92, 107, 132, 133, 134,
 142
 analytical hierarchy 133
 computational 133, 142
 computing 132, 134
 radiation 36
 Prostate tumor 100
 PSO 94, 95
 coupled 94
 -dICA technique 95
 Public-key cryptography (PKC) 114, 115,
 120, 128
 Python cryptographic library 124

Q

Quasi-newton method 1, 3, 9, 14
Queueing principle 1
Queueing system 2, 3, 5, 6, 10
 finite-capacity Markovian feedback 2

R

Radial basis function (RBF) 99
Resource-constrained devices 119

S

Security 114, 115, 116, 128
 communications 114, 115
 services 115
Server 1, 2, 3, 5, 7, 8, 10
 breakdown 1, 2, 3, 8
 broken 3
 idle 2
Service 2, 3, 4, 8, 10, 115
 banking 2
 mobile 2
 rate 8, 10
Silicon irradiance sensor 38
Simulated annealing 96
SIS sensor 38
Slack 62, 66, 67, 68, 69, 70, 72, 99
 variable 99
 systems 62, 66, 67, 68, 69, 70, 72
SMO 96, 97, 98
 algorithms 96, 97, 98
 method 98
Solar power 36, 37
 generation 37
 tower plants 36
Spider monkey optimization (SMO) 92, 98
Supply chain 75
 management 75
 problems 75
Support vector machine (SVM) 92, 94, 95, 99,
 100, 101, 102, 103, 104, 108
Surplus variables 53
SVF extension 18
SVM algorithm 99
Synchronization 62, 63
 chaos 62
 problems 62
System 36, 41, 62

satellite 62
vapor absorption refrigeration 36, 41

T

Techniques 3, 6, 63, 93, 94, 118, 133
 adaptive sliding mode control techniques
 feedback control 63
 compact certificateless signature 118
 data generation 93
 gene expression analysis 94
 inclusion-based TOPSIS 133
 nature-inspired metaheuristic 94
 numerical 3, 6
Technologies 36, 62, 63
 -electrical signals 62
 thermal 36
Technology, conventional power generation
 37
Tools, bioinformatics 93
TOPSIS methods 133, 139
 expanded intuitionistic fuzzy 133
Transcription factor binding sites 95
Transport problems 2
Transportation cost 56, 75, 77, 79, 84, 85
Transportation problem 48, 56, 57, 75, 76, 77,
 79, 87, 88
 nonlinear solid 76
 solid 75, 76, 77, 87, 88
Triangular fuzzy number properties 53
Trigeneration system 36, 37, 40, 41, 42, 43,
 44, 46
 solar-operated 37, 40
 solar-powered 40
Triple layer ensemble method 95

V

Variation of energy efficiency 42

W

Water loss management 133



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