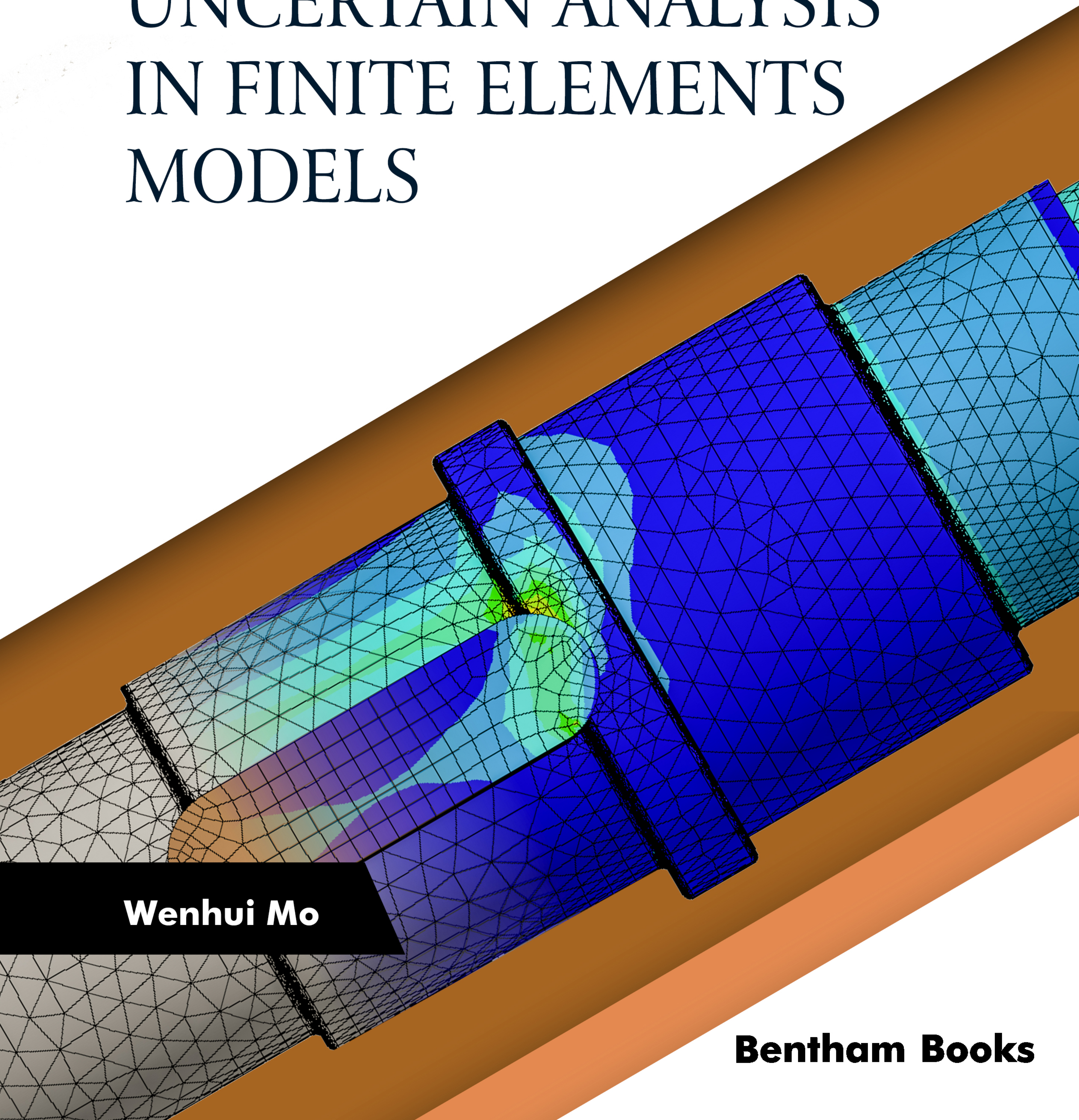
UNCERTAIN ANALYSIS IN FINITE ELEMENTS MODELS



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Uncertain Analysis in Finite Elements Models

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CONTENTS

PREFACE

There are three kinds of uncertainties in engineering problems. One is randomness, the second is fuzziness, and the third is non probability. Sometimes, the impact of uncertainty on engineering problems can not be ignored. Uncertainty has a great impact on buildings, dams, nuclear power plants, bridges, aircraft, machinery, vehicles, warship, *etc*. The material properties, geometry parameters and loads of the structure are assumed to be random, fuzzy and non probabilistic.

In the first chapter, nonlinear stochastic finite elements for general nonlinear problems and elastoplastic problems are discussed, and three methods are proposed. In Chapter 2, the calculation formula of stochastic finite element is given by using the third-order Taylor expansion and a simple calculation method is addressed. The stress-strength interference model, Monte Carlo simulation, a new iterative method (NIM) of reliability calculation for the linear static problem and linear vibration are proposed. Reliability calculation methods using the homotopy perturbation method (MIHPD) and second order reliability method for the nonlinear static problem and nonlinear vibration are proposed. In Chapter 3, the structural fuzzy reliability calculation of static problem, linear vibration, nonlinear problem and nonlinear vibration is studied by using the stochastic finite element method. The normal membership function is selected as the membership function, and the calculation formula of fuzzy reliability is presented. In Chapter 4, Taylor expansion, Neumann expansion, Sherman Morrison Woodbury expansion and a new iterative method (NIM) for interval finite element calculation of static problems are proposed. In Chapter 5, Perturbation technology, Taylor expansion, Neumann expansion, Sherman Morrison Woodbury expansion and a new iterative method (NIM) for interval finite element calculation of structural linear vibration are addressed. Chapter 6 proposes five calculation methods of nonlinear interval finite element for general nonlinear problems and elastoplastic problems. In the seventh chapter, five methods of interval finite element calculation methods for nonlinear structures are presented. In the eighth chapter, two improved methods of random field are proposed. The midpoint method, local average method, interpolation method and improved interpolation method of interval field and fuzzy field are proposed. The calculation method of mixed field is introduced. In the last chapter, calculation methods of random interval finite element, random fuzzy finite element and random fuzzy and interval finite element are proposed by using Taylor expansion and Neumann expansion.

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CHAPTER 1

Nonlinear Stochastic Finite Element Method

Abstract: Considering the influence of random factors on the structure, three stochastic finite element methods for general nonlinear problems are proposed. They are Taylor expansion method, perturbation method and Neumann expansion method. The mean value of displacement is obtained by the tangent stiffness method or the initial stress method of nonlinear finite elements. Nonlinear stochastic finite element is transformed into linear stochastic finite element. The mean values of displacement and stress are obtained by the incremental tangent stiffness method and the initial stress method of the finite element of elastic-plastic problems. The stochastic finite element of elastic- plastic problems can be calculated by the linear stochastic finite element method.

Keywords: Nonlinear stochastic finite element method, Taylor expansion, Perturbation technology, Neumann expansion, Elastic-plastic problem.

INTRODUCTION

In the fields of dams, buildings, earthquakes and so on, random factors have a great impact on the structure. Under random load and working environment, advanced numerical technology and famous finite element method are used to analyze structures. Most applications are limited to certain loads and working environments, although random and uncertain factors reach a considerable degree. Due to the spatial variability of material properties and the randomness of load, the research of stochastic finite elements has attracted more and more attention by many authors. Nonlinear structures are affected by random factors. In order to improve the calculation accuracy, the research of nonlinear stochastic finite elements is very necessary.

The second order perturbation and stochastic second central moment technique solve homogenization of two-phase elastic composites [1]. Both Monte Carlo Simulation and perturbation Methods are examined [2]. The second-order perturbation and the second probabilistic moment method are applied to the stress-

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based finite element method [3]. The variability of displacements and eigenvalues of structures are studied using variability response functions [4]. The weighted integral and local average methods of the triangular composite facet shell element are presented [5]. The nonlinear behaviour of strand-based wood composites is simulated [6]. A stochastic formulation of shell structures with multiple uncertain materials and geometric properties is proposed [7]. Efficient approaches to finite element analysis reinforced concrete structures are dealt with [8]. Stochastic finite element analysis of shells is presented for the case of combined uncertain material and geometric properties [9]. A novel response surface method employing ad hoc ratios of polynomials as a performance function is presented [10]. Efficient iterative algorithms for the stochastic finite element method with application to acoustic scattering for solving problems are described [11]. This work studies elliptic boundary value problems with uncertain coefficients by the stochastic finite element method [12]. A projection scheme based on the preconditioned stochastic Krylov subspace is compared with [13]. Primal mixed finite-element approximation of the second-order elliptic problem is proposed [14]. A biodegradation using the perturbation based stochastic finite element method is analyzed [15]. A perturbation-based stochastic finite element method can be applied to solve some boundary values [16]. Multiscale finite element methods are applied to uncertainty quantification [17]. An alternative unsteady adaptive stochastic finite element is proposed to further improve the accuracy [18]. A Galerkin-based multi –point reduced –order model (ROM) is developed for design optimization [19]. The generalized spectral decomposition method for nonlinear stochastic problems is extended [20]. The stochastic perturbation technique and Monte Carlo simulation (MCS) method are used to analyse a cable-stayed bridge system with varying material properties [21]. This paper presents a generic high dimensional model for stochastic finite element analysis [22]. Nonlinear bending response of laminated composite spherical shell panel with random system properties is investigated [23]. The investigation reported an approach for nonlinear multi-degree-of -freedom systems with uncertain properties to non-Gaussian random excitations [24]. This paper presents a stochastic nonlinear failure analysis of laminated composite plates with random material properties [25]. We consider the convergence properties of return algorithms in computational elasto-plasticity [26]. A plastic–damage model of seismic cracking of concrete gravity dams utilizing two different damping mechanisms is examined [27]. This work examines the effect of random geometric imperfections in the buckling analysis of portal frames with stochastic imperfections [28]. This paper investigates the effect of measurement noise and excitation for nonlinear finite element model updating [29]. This paper investigates stochastic analysis of structures with softening materials [30]. Nonlinear finite element modeling of reinforced concrete haunched beams are presented and discussed [31]. This work explores reliability sensitivity analysis of nonlinear structural systems under stochastic excitation [32]. This paper introduces adaptive condensed SFEs for nonlinear mechanical problems [33]. The Markov diffusion theory is applied in studying stochastic nonlinear ships rolling in random beam seas [34]. In this work, a computational framework for nonlinear finite element models is presented [35]. Subset simulation, a Markov chain Monte Carlo technique, can be used to estimate physical models and nonlinear finite element analysis [36]. In this paper, the nonlinear finite element approach is used to solve laminated composite thin hypar shells [37]. This paper investigates randomness in constituent material properties by presenting a reliable model for solving stochastic nonlinear equations [38]. The paper proposes a spectral stochastic formulation with a nonlinear analysis of framed structures [39]. This paper presents a nonlinear finite element model of adobe masonry structures [40]. A nonlinear proxy finite element analysis (PFEA) technique was developed to predict capacity assessment of older t-beam bridges [41]. Stochastic constitutive modeling of elastic-plastic is developed that is efficient for use in which the material properties are considered random variables [42]. To analyze the influence of uncertainty, the methodology of stochastic cohesive interface analysis of layer debonding is proposed [43].

The finite element has become an important method for analyzing structures. Nonlinear structures are affected by random factors and

sometimes they can not be ignored. Three stochastic finite element methods for general nonlinear problems are proposed. Three stochastic finite element methods for elastic-plastic problems are formulated.

GENERAL NONLINEAR PROBLEMS

When the material stress-strain is nonlinear, the stiffness matrix is not constant, which is related to strain and displacement. The global equilibrium equation of the structure is the following nonlinear equations

$$\{\emptyset\} = [\boldsymbol{A}(\boldsymbol{U})]\{\boldsymbol{U}\} - \{\boldsymbol{F}\}=0 \quad \textbf{(1)}$$

CHAPTER 2

Reliability Calculation of Stochastic Finite Element

Abstract: The stochastic finite element third-order perturbation method for linear static problems is formulated. The stress-strength interference model, Monte Carlo simulation and a new iterative method (NIM) of reliability calculation for the linear static problem and linear vibration are proposed. Reliability calculation methods using modified iteration formulas by the homotopy perturbation method (MIHPD) and second- order reliability method for a nonlinear static problem and nonlinear vibration are proposed.

Keywords: The third-order Taylor expansion, Perturbation method, Linear static problem, Linear vibration, Nonlinear static problem, Nonlinear vibration, Stress-strength interference model, Monte Carlo simulation, A new iterative method, Modified iteration formulas, Homotopy perturbation method.

INTRODUCTION

Finite element is a world recognized tool for analyzing structures. The influence of randomness on some structures cannot be ignored. Material properties, geometric parameters and applied loads of the structure have a great impact on dams, buildings, bridges, mechanical parts, *etc*. Considering the influence of random factors, the stochastic finite element is introduced. Stochastic finite element and structural reliability calculation methods are combined to calculate the reliability of the structure.

Two methods are studied for a combination of finite element and reliability methods: the direct method and the quadratic response surface method [1]. An element-free Galerkin method was developed for the reliability analysis of linear-elastic structures [2]. In this paper, finite element reliability methods of the first-order reliability method (FORM) and importance sampling are considered [3]. Analytical methods, combined analytical and simulation-based methods, direct Monte Carlo simulations and the importance sampling strategies are used to analyze dynamic reliability [4]. Five reliability methods calculating the reliability of a

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composite structure are given [5]. This paper presents a reliability analysis in geotechnics engineering based on mathematical theories [6]. Reliability of linear structures with parameter uncertainty under non-stationary earthquakes, the perturbation stochastic finite element methodis utilized in deriving reliability of linear structures [7]. Finite element reliability analyses of nonlinear frame structures are employed with sophisticated structural models [8]. In this paper, the reliability of a rotating beam with random properties is studied and a second- order perturbation method is used [9]. Probabilistic risk assessment using finite element analysis for bridge construction is proposed [10]. Advanced Monte Carlo methods for reliability analysis is proposed and uncertainty is regarded as random variables [11]. The context for this paper is FORM in finite element reliability analysis in conjunction with advanced finite element models [12]. The reliability assessment of uncertain linear struc- tures using stochastic finite elements is presented [13]. This paper deals with the reliability analysis using the stochastic finite element method [14]. The objective of this paper is to illustrate an approach for the lifetime reliability assessment of bridges [15]. This paper presents a method for a reliability assessment in structural dynamics [16]. This contribution presents a model reduction technique for reliability sensitivity analysis of nonlinear finite elements [17]. The main aim is to present the stochastic perturbation-based finite element method analysis of the reliability of the underground steel tanks [18]. Improving the reliability of the frequency response function (FRF) by semi-direct model updating is reported [19]. This paper presents a reliability analysis of steady-state seepage by the stochastic scaled boundary finite element method [20]. This paper performs a probabilistic stability analysis for an existing earthfill dam using a stochastic finite element method based on field data [21]. The paper presents a method for reliability analysis of slopes by conditional random finite element method [22]. The objective of the present work is to develop a probabilistic analysis of a Carbon-Nanotube-Reinforced-Polymer (CNRP) material by using the stress-strength model and multiscale finite element model to determine the reliability [23]. The main aim of this paper is to present a reliability estimation procedure for a steel lattice tower based on the stochastic finite element method [24]. This work presents a reliability analysis of structures equipped with friction-based devices [25]. The reliability approach and finite element method are used to estimate failure probability [26]. Dynamic reliability of structure is computed using Successive over Relaxation method or Neumann expansion [30].

In order to improve the calculation accuracy, the calculation formula of third-order perturbation stochastic finite element is presented. The reliability calculation methods of linear static problem, linear vibration, nonlinear static problem and nonlinear vibration are studied using a stochastic finite element method. The stress strength interference model, Monte Carlo simulation, a new iterative method and a modified iteration method by homotopy perturbation method for stochastic finite element reliability calculation are developed. Second order reliability methods for nonlinear static problems and nonlinear vibration are proposed.

Reliability Calculation of Static Problems

The equilibrium equation is written as

$$KU = F \quad \textbf{(1)}$$

where U = the displacement vector, F = the external force, K = the global stiffness matrix.

By applying Taylor series at the mean point $\bar{a} = \left(\bar{a}_1, \bar{a}_2, \cdots, \bar{a}_n\right)^T$ of the random variables and the perturbation technology, the following equations are given by

$$U^0 = \left(K^0\right)^{-1} F^0 \quad \textbf{(2)}$$

$$\frac{\partial U}{\partial a_i} = \left(K^0\right)^{-1}\left(\frac{\partial F}{\partial a_i} - \frac{\partial K}{\partial a_i}U^0\right) \quad \textbf{(3)}$$

$$\frac{\partial^2 U}{\partial a_i \partial a_j} = \left(K^0\right)^{-1}\left(\frac{\partial^2 F}{\partial a_i \partial a_j} - \frac{\partial^2 K}{\partial a_i \partial a_j}U^0 - \frac{\partial K}{\partial a_i}\frac{\partial U}{\partial a_j} - \frac{\partial K}{\partial a_j}\frac{\partial U}{\partial a_i}\right) \quad \textbf{(4)}$$

$$\frac{\partial^3 U}{\partial a_i^2 \partial a_j} = (K^0)^{-1}(\frac{\partial^3 F}{\partial a_i^2 \partial a_j} - \frac{\partial^3 K}{\partial a_i^2 \partial a_j}U^0 - 3\frac{\partial^2 K}{\partial a_i \partial a_j}\frac{\partial U}{\partial a_i}$$

$$-3\frac{\partial K}{a_i}\frac{\partial^2 U}{\partial a_i \partial a_j}) \quad \textbf{(5)}$$

The Taylor expansion formula of U is

CHAPTER 3

Fuzzy Reliability Calculation Based on Stochastic Finite Element

Abstract: Based on the stochastic finite element, the fuzzy reliability calculation of structure with static problems, linear vibration, nonlinear problems and nonlinear vibration is studied. The mean and variance of stress are obtained by the stochastic finite element method. The normal membership function is generally selected as the membership function of engineering problems. The fuzzy reliability of structure can be obtained by using the calculation formula of fuzzy reliability.

Keywords: Fuzzy reliability, Stochastic finite element, Static problem, Linear vibration, Nonlinear structure, Nonlinear vibration, Membership function.

INTRODUCTION

Stress is related to load, structure and other factors. As long as one of the factors is fuzzy, the stress is fuzzy. The strength value can be found in the design manual, but this information has a certain degree of fuzziness. Finite element analysis of complex structures is feasible. Based on the stochastic finite element method, a new calculation method of fuzzy reliability of structures is proposed. This method can calculate the fuzzy reliability of complex structures.

A methodology is developed that uses Petri nets and fuzzy Lambda–Tau methodology and solves for reliability [1]. This paper presents an approach to assessing the reliability of concrete structures [2]. This paper discusses the optimisation of the forming load path using a fuzzy logic control algorithm and finite element analysis [3]. Modelling of plane strain compression (PSC) test incorporating both the hybrid and the fuzzy finite element models have been undertaken [4].This work introduces a fuzzy finite element procedure to calculate frequency response functions of damped finite element models [5]. A fuzzy logic method for improving the convergence in nonlinear magnetostatic problems using finite elements is presented [6]. This paper presents an efficient method for the static design of imprecise structures including fuzzy data [7]. In this paper, the spectral element method and a fuzzy set is used to estimate frequency response function envelopes [8]. This paper uses the interval and fuzzy finite element method for

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dynamic analysis of finite elements with uncertain parameters [9]. A methodology of the fuzzy finite element method in the model updating welded joints has been highlighted [10]. Fuzzy logic, neural networks and three-dimensional finite element calculations are employed in order to develop a computerized model in a coalface longwall mining simulation [11]. A fuzzy model of the generator is developed using finite element and fuzzy methods for carrying out its leakage field analysis [12]. Fuzzy finite element methods are becoming increasingly popular for the analysis of structure [13]. A fuzzy finite element analysis based on the α-cuts method analyzed heat conduction problems with uncertain parameters [14]. Dynamical relaxed directional method for fuzzy reliability analysis is proposed for engineering problems under epistemic uncertainty [15]. This paper describes a fuzzy reliability method for the calculation of power systems [16]. A new algorithm has been introduced to construct the membership function of fuzzy system reliability using different types of intuitionistic fuzzy failure rates [17]. A new approach of the weakest t-norm based intuitionistic fuzzy fault-tree is proposed to evaluate system reliability [18]. The objective of this study is to develop a fuzzy reliability algorithm of basic events of fault trees through qualitative data processing [19]. This study is to analyze the fuzzy reliability of a repairable system using soft-computing based hybridized techniques [20]. The stress–strength reliability of fuzziness is investigated [21]. An approach to analyze the fuzzy reliability of a dual-fuel steam turbine mechanical propulsion system is presented [22]. The purpose of the present study is to analyse the fuzzy reliability analysis of DFSMC systems with different membership functions by applying-the fuzzy lambda-tau technique [23]. A novel approach is proposed to evaluate system failure probability using intuitionistic fuzzy fault tree analysis [24]. Fuzzy improved distribution function is introduced and the order statistics based on fuzzy improved distribution function is proposed [25]. A fuzzy maximum entropy approach is proposed to determine fuzzy reliability centered maintenance considering the uncertainty [26].

Based on the stochastic finite element method, a new method for calculating the fuzzy reliability of structures with linear problems, linear vibration, nonlinear problems and nonlinear vibration is proposed. The normal membership function is selected as the membership function. The calculation formulas are given respectively.

Fuzzy Reliability Calculation of Static Problems Based on Stochastic Finite Element

The governing equation of the finite element under static load can be written as

$$[K]\{\delta\}=\{F\} \quad (1)$$

The mean and variance of structural displacement are obtained by Taylor stochastic finite element.

$$E\{\delta\}\approx\{\delta\}|_{a=\bar{a}}+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\{\delta\}}{\partial a_i\partial a_j}|_{a=\bar{a}}Cov(a_i,a_j) \quad (2)$$

where $E\{\delta\}$ is mean value $\{\delta\}$ and $Cov(a_i,a_j)$ is the covariance between a_i and a_j.

The variance of $\{\delta\}$ is given by

$$Var\{\delta\}\approx\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\{\delta\}}{\partial a_i}|_{a=\bar{a}}\cdot\frac{\partial\{\delta\}}{\partial a_j}|_{a=\bar{a}}\cdot Cov(a_i,a_j) \quad (3)$$

where $Var\{\delta\}$ is the variance of $\{\delta\}$.

The mean and variance of the stress on the structure are obtained by Taylor stochastic finite element method

$$E\{\sigma\}\approx\{\sigma\}|_{a=\bar{a}}+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\{\sigma\}}{\partial a_i\partial a_j}|_{a=\bar{a}}Cov(a_i,a_j) \quad (4)$$

CHAPTER 4

Static Analysis of Interval Finite Element

Abstract: Four methods of interval finite element for static analysis are proposed. Using the second-order and third-order Taylor expansion , interval finite element for static analysis is addressed. Neumann expansion of interval finite element for static analysis is formulated. Interval finite element using Sherman -Morrison-Woodbury expansion is presented. A new iterative method (NIM) is used for interval finite element calculation. Four methods can calculate the upper and lower bounds of node displacement and element stress.

Keywords: A new iterative method (NIM), Interval finite element, Neumann expansion, Taylor expansion, Sherman-Morrison-Woodbury expansion, Static analysis.

INTRODUCTION

Final element method deals with deterministic engineering problems has become a world recognized numerical analysis method. Stochastic finite element has been developed to analyze structure with stochastic parameters. Fuzzy finite element has been developed to analyze structure with fuzzy parameters. In the design of engineering problems, material properties, geometry parameters and loads are assumed to be interval variables. Instead of conventional finite elements, the interval finite element has been studied by many authors.

This work analyzes structural systems using interval analysis [1]. A formulation is proposed for the interval estimation of displacement input with uncertainty [2].Interval calculation is used to analyze mechanical systems modeled with interval finite elements [3]. The uncertain parameters are assumed to be interval variables, and the bounds of the displacement are obtained by interval finite element methods [4]. To account for uncertainties in linear static problems, a interval linear equations is proposed [5]. The interval and fuzzy finite element method is used to analyze the eigenvalue and frequency response function analysis of structures [6]. This paper presents a method for computing linear systems with large uncertainties

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[7].Affine arithmetic is an improving interval analysis in finite element calculations [8].This paper presents chosen computational algorithms for interval finite element analysis [9]. The article focuses on a new interval finite element formulation to reduce overestimation problems [10]. This study is deemed a contribution to novel parameterized intervals for solving problems with uncertainty [11]. Improved interval analysis of the second-order statistics of the response is proposed [12]. Optimization and anti-optimization solution of combined parameterized and improved interval analyses for structures with uncertainties is presented [13].The objective is to validate a methodology for multivariate uncertainty in interval finite elements [14]. Interval Finite element analysis of linear-elastic structures with uncertain properties is addressed [15]. This paper presents a flexible approach for intervalfinite element analysis [16].

Four static analysis methods of interval finite element are proposed. They are Taylor expansion method, Neumann expansion method, Sherman-Morrison-Woodbury expansion method and a new iterative method (NIM). The calculation process and formulas of the four methods are given in detail

Taylor Expansion for Interval Finite Element

Interval variable $\left[\underline{a},\overline{a}\right]$ is generated by the following formula

$$a_i = \underline{a} + \frac{\overline{a}-\underline{a}}{n} i = \frac{i\overline{a}+(n-i)\underline{a}}{n} \quad \textbf{(1)}$$

$i = 1, 2, \cdots, n$

Material properties, geometry parameters and applied loads of structures are assumed to be interval variables. They are $[\underline{a}_1,\overline{a}_1],[\underline{a}_2,\overline{a}_2],\cdots,[\underline{a}_j,\overline{a}_j],\cdots,[\underline{a}_n,\overline{a}_n]$.

The equilibrium equation is written as

$$KU = F \quad \textbf{(2)}$$

where U = the displacement vector, F = the external force, K = the global stiffness matrix.

By applying Taylor series at the midpoint of the interval variables, the following equations are given by

$$U^0 = \left(K^0\right)^{-1} F^0 \tag{3}$$

$$\frac{\partial U}{\partial a_i} = \left(K^0\right)^{-1} \left(\frac{\partial F}{\partial a_i} - \frac{\partial K}{\partial a_i} U^0 \right) \tag{4}$$

$$\frac{\partial^2 U}{\partial a_i \partial a_j} = \left(K^0\right)^{-1} \left(\frac{\partial^2 F}{\partial a_i \partial a_j} - \frac{\partial^2 K}{\partial a_i \partial a_j} U^0 - \frac{\partial K}{\partial a_i} \frac{\partial U}{\partial a_j} - \frac{\partial K}{\partial a_j} \frac{\partial U}{\partial a_i} \right) \tag{5}$$

$$\frac{\partial^3 U}{\partial a_i^2 \partial a_j} = (K^0)^{-1} (\frac{\partial^3 F}{\partial a_i^2 \partial a_j} - \frac{\partial^3 K}{\partial a_i^2 \partial a_j} U^0 - 3\,\frac{\partial^2 K}{\partial a_i \partial a_j} \frac{\partial U}{\partial a_i} - 3\,\frac{\partial K}{a_i} \frac{\partial^2 U}{\partial a_i \partial a_j}) \tag{6}$$

The Taylor expansion formula of U is

$$U = U^0 + \sum_{k=1}^{m} \frac{1}{k!} \sum_{i_1, i_2, \cdots, i_k = 1}^{n} \frac{\partial^k U}{\partial a_{i_1} \partial a_{i_2} \cdots \partial a_{i_k}} \left(a^0\right) \left(a_{i_1} - a^0{}_{i_1}\right)$$

$$\left(a_{i_2} - a^0{}_{i_2}\right) \cdots \left(a_{i_k} - a^0{}_{i_k}\right) +$$

$$\frac{1}{(m+1)!} \sum_{i_1, i_2, \cdots, i_{m+1} = 1}^{n} \frac{\partial^{m+1} U}{\partial a_{i_1} \partial a_{i_2} \cdots \partial a_{i_k}} \left(a^0\right) \left(a_{i_1} - a^0{}_{i_1}\right) \tag{7}$$

$$\left(a_{i_2} - a^0{}_{i_2}\right) \cdots \left(a_{i_k} - a^0{}_{i_k}\right)$$

CHAPTER 5

Interval Finite Element for Linear Vibration

Abstract: Interval variables have an effect on linear vibration. The linear vibration is transformed into a static problem by Newmark method. The perturbation method, Neumann expansion method, Taylor expansion method, Sherman Morrison Woodbury expansion method and a new iterative method of interval finite element for linear vibration are proposed. The detailed derivation processes are explored.

Keywords: Linear vibration, Perturbation, Neumann expansion, Taylor expansion, Sherman Morrison Woodbury expansion, A new iterative method.

INTRODUCTION

Stochastic finite element has been studied for more than 50 years. Stochastic finite element requires statistical data. Obtaining statistics data is troublesome. Interval finite element does not need probability density function and statistical data. It is difficult to determine the probability density function in engineering applications. Linear vibration is sometimes greatly affected by interval variables.

A combinatorial approach and an inequality-based method are used to solve interval equations [1]. The scatter of external loads identified by displacement input with uncertainty is estimated by the Lagrange multiplier method [2]. An iterative algorithm is a conservative solution for linear interval finite element analysis [3]. The newmethod is based on an element-by-element technique to solveuncertainty in mechanics problems [4]. Anti-optimisation of interval finite element is proposed for uncertain structures analysis [5]. Interval boundary element methods have been explored in finite element analysis with parametric uncertainties [6]. The objective is to give a general overview of non-probabilistic finite element analysis with parametric uncertainty [7]. A method to calculate the static structures with uncertain-but-bounded axial stiffness is proposed [8].The elastic modulus of one-dimensional heterogeneous solids is considered both a probabilistic and a non-probabilistic approach [9]. The present paper is to determine bounds for the stationary stochastic response of truss structures *via* interval analysis [10] .A novel expression of the frequency response function (FRF) matrix of discretized

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structures with uncertain-but-bounded parameters are formulated [11]. Reliability analysis of randomly excited structures with interval uncertainties is addressed and the improved interval analysis handles uncertain-but-bounded parameters [12]. A novel Interval finite element method is formulated to improve interval analysis [13]. Finite element analysis of structures is addressed for interval and stochastic finite element analysis [14]. The paper presents the formulation of a stochastic B-spline wavelet on the interval finite element in elasto-statics analysis [15]. The paper presents an interval finite element method based on stochastic B-spline wavelet for beams [16].

Considering the influence of interval variables on linear vibration, five calculation methods of interval finite element for linear vibration are proposed. They are the perturbation method, Neumannexpansion method, Taylor expansion method, Sherman Morrison Woodbury expansion method and a new iterative method. The calculation formulas are given respectively.

Interval Perturbation Finite Element for Linear Vibration

For a linear system, the dynamic equilibrium equation is given by

$$[M]\{\ddot{\delta}\}+[C]\{\dot{\delta}\}+[K]\{\delta\}=\{F\} \quad \textbf{(1)}$$

Where $\{\ddot{\delta}\},\{\dot{\delta}\},\{\delta\}$ are the acceleration, velocity and displacement vectors. $[M][K]$ and $[C]$ are the global mass, stiffness and damping matrices obtained by assembling the element variables in the global coordinate system.

For ease of programming, the comprehensive calculation steps of the Newmark method are as follows

The matrices $[K]$, $[M]$ and $[C]$ are formed.

The initial values $\{\delta_t\},\{\dot{\delta}_t\},\{\ddot{\delta}_t\}$ are given.

After selecting step Δt and parameters γ,β, the following relevant parameters are calculated:

$$\gamma \geq 0.50 \quad \textbf{(2)}$$

$$\beta \geq 0.25(0.5+\gamma)^2 \quad \textbf{(3)}$$

$$b_0 = \frac{1}{\beta(\Delta t)^2} \quad \textbf{(4)}$$

$$b_1 = \frac{\gamma}{\beta \Delta t} \quad \textbf{(5)}$$

$$b_2 = \frac{1}{\beta \Delta t} \quad \textbf{(6)}$$

$$b_3 = \frac{1}{2\beta} - 1 \quad \textbf{(7)}$$

$$b_4 = \frac{\gamma}{\beta} - 1 \quad \textbf{(8)}$$

$$b_5 = \frac{\Delta t}{2}\left(\frac{\gamma}{\beta} - 2\right) \quad \textbf{(9)}$$

$$b_6 = \Delta t(1-\gamma) \quad \textbf{(10)}$$

$$b_7 = \gamma \Delta t \quad \textbf{(11)}$$

The stiffness matrix is defined as

$$\left[\tilde{K}\right] = [K] + b_0[M] + b_1[C] \quad \textbf{(12)}$$

The stiffness matrix inversion $\left[\tilde{K}\right]^{-1}$ is solved.

CHAPTER 6

Nonlinear Interval Finite Element

Abstract: Nonlinear structures in engineering are affected by uncertain parameters. Firstly, the displacement when the interval variable takes the midpoint value is obtained, and the nonlinear problem is transformed into a linear problem. Five calculation methods of nonlinear interval finite element for general nonlinear problems and elastoplastic problems are proposed. According to the perturbation technique, a perturbation method is proposed. According to Taylor expansion, Taylor expansion method is proposed. Neumann expansion, Sherman Morrison Woodbury expansion and a new iterative method are proposed.

Keywords: Nonlinear structures, Elastoplastic problems, Perturbation method, Taylor expansion method, Neumann expansion, Sherman Morrison Woodbury expansion , A new iterative method.

INTRODUCTION

The influence of uncertain parameters on nonlinear structures can not be ignored sometimes. In order to improve the calculation accuracy of finite element, it is necessary to study the interval finite element of nonlinear structure. The influence of interval variables on engineering problems should be paid attention to. A static structural analysis problem with uncertain parameters can beexpressed a system of linear interval equations [1]. The sensitivity analysis of interval finite element is evaluated [2]. A new formulation has been given for the analysis of mechanical systems using interval finite elements methods [3]. A modified interval perturbation analysis of uncertain structures is presented [4]. Under loading, material and geometric uncertainty, a very sharp enclosure for the solution is obtained [5]. In order to reduce the calculation time, Interval and fuzzy dynamic analysis of finite elements are proposed [6]. The merits of the new approach are demonstrated by computing linear systems with large uncertainties, with applications to truss structures, leading to over 5000 variables and over 10000 interval parameters [7]. This paper presents a novel method to solve interval finite element analysis [8]. A new interval finite element states sharp displacement bounds are produced by the Lagrange multiplier method [9]. Intervals describing variation are parameterized by trigonometric functions [10]. The method is adopted to improve the ordinary interval analysis, based on the so-called affine arithmetic [11]. Chosen numerical algorithms for interval finite element analysis are compared

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with the Monte Carlo method [12]. The improved interval analysis *via* extra unitary interval is proposed [13]. The principal idea is to solve an inverse problem for analyzing multivariate interval uncertainty [14]. A response surface approach is adopted for uncertainty propagation analysis which provides a method of interval finite element [15]. One model parameter over the domain is usually modelled using a series expansion for interval finite element analysis via convex hull pair constructions [16].

Five methods for interval finite element calculation of nonlinear structures are proposed. They are perturbation method, Taylor expansion method, Neumann expansion, Sherman MorrisonWoodbury expansion and a new iterative method. The detailed derivation of five calculation methods is given respectively.

General Nonlinear Problems

When the material stress-strain is nonlinear, the stiffness matrix is not constant, which is related to strain and displacement. The global equilibrium equation of the structure is the following nonlinear equations.

$$\{\emptyset\} = \mathrm{K}\{\delta\} - \{\mathrm{F}\}=0 \quad \textbf{(1)}$$

where $[\mathrm{K}(\delta)]$ is global stiffness matrix, $\{\delta\}$ is displacement matrix and

$\{\mathrm{F}\}$ is a load matrix.

Midpoint values of the elastic modulus, Poisson's ratio and load are substituted into the above formula. The displacement is calculated by the tangent stiffness method or initial stress method (See Chapter 1 for details). $\{\delta\}_{n+1}$ is the solution of Eq.1 .After $\{\delta\}_{n+1}$ represents Eq.1, it becomes a linear equation containing interval variables of material properties, geometry parameters and applied loads of structures.

Perturbation technology for nonlinear interval finite element

Material properties, geometry parameters and applied loads are assumed to be interval variables. They are expressed as

$$\bar{\mathrm{a}} = \mathrm{a} + \Delta\mathrm{a} \quad \textbf{(2)}$$

$$\underline{a} = a - \Delta a \quad (3)$$

Where a is midpoints of interval variables,Δa is a small perturbation.

Eq.1 is rewritten

$$[K]\{\delta\}=\{F\} \quad (4)$$

Using perturbation technology and representing$\{\delta\}_{n+1}$, we obtain

$$([K]+\Delta[K])(\{\delta\}+\Delta\{\delta\})=(\{F\}+\Delta\{F\}) \quad (5)$$

where Δ[K] is a small perturbation of [K],Δ{F} is a small perturbationof {F}. Δ{δ} is a small perturbation of {δ}in the following equations.

Eq.5 is rewritten

$$(\{\delta\}+\Delta\{\delta\})=([K]+\Delta[K])^{-1}(\{F\}+\Delta\{F\}) \quad (6)$$

The Neumann expansion of $[\tilde{K}]+\Delta[\tilde{K}]^{-1}$ takes the following form:

$$([K]+\Delta[K])^{-1}=\sum_{i=0}^{\infty}\left(-[K]^{-1}\Delta[K]\right)^{i}[K]^{-1} \quad (7)$$

Neglecting second-order terms, we get

$$\Delta\{\delta\}=[K]\Delta\{F\}-[K]^{-1}\Delta[K]\{\delta\} \quad (8)$$

δ^{l} can be written as

$$\delta^{l}=[\{\underline{\delta}\},\{\bar{\delta}\}]= \delta^{c}+\Delta \delta^{l}= \delta^{c}+[-\Delta \delta ,\Delta \delta] \quad (9)$$

We obtain

CHAPTER 7

Nonlinear Vibration Analysis of Interval Finite Element

Abstract: For the influence of non-probabilistic parameters on nonlinear vibration, the nonlinear vibration analysis of interval finite element is proposed. Using the Newmark method, nonlinear vibration is transformed into nonlinear equations. The midpoint values of interval variables are substituted into the nonlinear equations to calculate the displacement. The displacement value is substituted into the nonlinear equations, and the nonlinear equations become linear equations. Five calculation methods of interval finite element for linear vibration are extended to nonlinear vibration.

Keywords: Nonlinear vibration analysis, Newmark method, nonlinear equations, linear equations, five calculation methods, interval variable, interval finite element.

INTRODUCTION

The influence of non-probabilistic parameters on nonlinear vibration can not be ignored sometimes. The influence of non-probabilistic parameters on nonlinear vibration must be considered in some engineering problems. The membership function of fuzzy finite element is difficult to determine in engineering. Interval variables in interval finite element are relatively simple. The engineering application of interval finite element is also more convenient. An interval truncation method is proposed to obtain solutions of large amounts of uncertainty [1]. The validity of the proposed method is investigated by finite element interval analysis in a flat plate [2]. This novel interval formulation is based on a adapting Rump's algorithm for solving interval linear equations [3]. Anti-optimisation of uncertain structures is that the displacement surface produced by the uncertain parameters is monotonic [4]. The Lagrange multiplier method is applied in interval finite element [5]. The interval boundary element method is developed for considering uncertain boundary conditions [6]. The paper gives an overview of non-probabilistic finite element analysis in applied mechanics [7]. An interval-valued Sherman–Morrison–Woodbury formula is used to inverse the interval stiffness matrix [8]. Interval versus stochastic analysis of structure are derived for the bounds of the interval

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field [9]. Interval frequency response function matrix of truss structures with uncertain-but-bounded parameters is evaluated [10]. Rational Series Expansion (RSE) provides an approximate explicit expression of the frequency response functions with uncertain parameters [11]. The bounds of the interval reliability of structures under stationary stochastic excitations are evaluated [12]. Interval rational series expansion applying interval finite element analysis is proposed for the lower bound and upper bound of interval displacements and stresses [13]. A unified response surface method is thus developed for interval and stochastic finite element analysis under different uncertainty models [14]. Perturbation approach of interval finite element is used to calculate elasto-statics problems [15].The response statistics are obtained using the perturbation method of interval finite element based on a stochasticB-spline wavelet [16].

Five calculation methods of nonlinear vibration of interval finite element are presented. They are the perturbation method, Taylor expansion method, Neumann expansion, Sherman Morrison Woodbury expansion and a new iterative method. The detailed derivation of five calculation methods is given respectively.

For a nonlinear system, the dynamic equilibrium equation is given by

$$[M]\{\ddot{\delta}\}+[C]\{\dot{\delta}\}+[K]\{\delta\}=\{F\} \quad \textbf{(1)}$$

where $\{\ddot{\delta}\},\{\dot{\delta}\},\{\delta\}$ are the acceleration, velocity and displacement vectors. $[M],[K]$ and $[C]$ are the global mass, stiffness and damping matrices.

By using the Newmark method, Eq.1 becomes

$$\{\delta_{t+\Delta t}\}=\left[\tilde{K}\left(\delta_{t+\Delta t}\right)\right]^{-1}\{\tilde{F}_{t+\Delta t}\} \quad \textbf{(2)}$$

where, $\{\delta_{t+\Delta t}\}$, $[\tilde{K}]$ and $\{\tilde{F}_{t+\Delta t}\}$ indicate the displacement vector, stiffness matrix and load vector at a time $t+\Delta t$.

Eq.2 is rewritten as

$$\left[\tilde{K}\left(\delta_{t+\Delta t}\right)\right]\left\{\delta_{t+\Delta t}\right\}=\left\{\tilde{F}_{t+\Delta t}\right\} \tag{3}$$

Material properties, geometry parameters and applied loads of structures are assumed to be interval variables. Midpoint values of the elastic modulus, Poisson's ratio ,geometry parameters and load are substituted into Eq.3.The incremental tangent stiffness method or initial stress method are used to solve displacement and stress(See Chapter 1 for details). $\{\delta\}_{n+1}$ is the solution of Eq.40 . $\{\sigma\}_{n+1}$ is the solution of stress. After $\{\delta\}_{n+1}$ represents Eq.3, it becomes a linear equation containing interval variables of Young's modulus , Poisson's ratio ,geometry parameters and loads. After $\{\sigma\}_{n+1}$ represents stress formula (Eq.10,Eq.20 ,Eq.27 ,Eq.34,Eq.39), it contains interval variables of Young's modulus , Poisson's ratio and geometry parameters . Nonlinear stochastic finite element is transformed into linear stochastic finite element.

Interval Perturbation Finite Element for Nonlinear Vibration

Material properties, geometry parameters and applied loads are assumed to be interval variables. They are expressed as

$$\overline{a} = a + \Delta a$$

$$\underline{a} = a - \Delta a \tag{4}$$

where a is midpoints of interval variables,Δa is a small perturbation.

Eq.3 is rewritten

$$\left[\tilde{K}\right]\left\{\delta_{t+\Delta t}\right\}=\left\{\tilde{F}_{t+\Delta t}\right\} \tag{5}$$

Using perturbation technology and representing $\{\delta\}_{n+1}$, we obtain

$$\left[\tilde{K}\right]+\Delta\left[\tilde{K}\right] \quad \delta_{t+\Delta t} \;+\Delta\; \delta_{t+\Delta t} \quad = \quad \tilde{F}_{t+\Delta t} \;+\Delta\; \tilde{F}_{t+\Delta t} \tag{6}$$

CHAPTER 8

Random Field, Interval Field, Fuzzy Field and Mixed Field

Abstract: Material properties are assumed to be random parameters, interval parameters and fuzzy parameters. If the variation range is large, they are not assumed to be constants. Two improved methods of the random field are developed. The midpoint method, local average method, interpolation method and improved interpolation method of interval field are addressed. The midpoint method, local average method, interpolation method and improved interpolation method of the fuzzy field are presented. The calculation method of mixed field is discussed and the calculation formula is proposed.

Keywords: Stochastic field, interval field, Fuzzy field, mixed field, The midpoint method, Local average method, Interpolation method, Improved interpolation method.

INTRODUCTION

Concrete and other composite materials have spatial variability, so the material properties should not be regarded as constants, but as random processes. The material properties are assumed to be interval parameters or fuzzy parameters. If the variation range is large, it should be treated as an interval field or fuzzy field. The material properties are random, non-probabilistic and fuzzy, and should be treated as mixed field .Midpoint method for the discretization of random fields is proposed and the influence of material properties and applied loads of structures is investigated [1]. Local spatial averages efficiently evaluate the matrix of covariances [2]. Uncertain structural parameter is regarded as Gaussian stochastic process and the two-dimensional local averaging technique is extended for 3D random field [3]. The local averages method of inhomogeneous random field and non-rectangular elements is proposed using Gaussian quadrature [4]. The method that the random field is discretized is analogous to the discretization of the displacement in finite element methods [5]. A method to evaluate the stochastic fields using Karhunen- Loeve expansion is developed [6]. A weighted integral method is proposed to compute the stochastic field of material parameters [7，8].

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An improvement of the midpoint method of stochastic field is presented [9]. The Karhunen–Loève (KL) expansion represents a random field that relies on the solution of an eigenvalue problem [10]. In order to cope with limitations on the applicability of the interval finite element analysis, the concept of interval fields is introduced [11]. The present paper determines the region of material properties and applied loads with uncertain-but-bounded parameters via interval analysis [12]. The uncertain flexibility is represented by both a random and an interval field to analyze the response variability of Euler–Bernoulli beams [13]. A method consisting of a combination of interval and random fields is proposed [14]. The improved interval analysis via extra unitary interval is proposed to solve the generalized interval eigenvalue problem [15]. A new discretization of the coupled interval field is performed to analyze the effects of Young's modulus uncertainty via interval finite element [16]. Anapproach for imprecise random field analysis using parametrized kernel functions is presented to handle imprecision [17].

Two improved methods of the random field are introduced. Four calculation methods of interval field are developed. Four calculation methods of fuzzy fields are presented. A mixed field calculation method is proposed.

Stochastic Field

The elastic modulus of the material, Poisson's ratio and the load on the structure are assumed to be random processes. If the random vector field is non-homogeneous or homogeneous, the quadrant is asymmetric, and the local average region of the random field is not rectangular. Gaussian integral is used to calculate the mean vector and covariance matrix. The elastic modulus is discussed as an example. Poisson's ratio, load, and so on.

Improved local average method

The elastic modulus of the element *l* is expressed as

$$b_l(x) = \frac{\iiint_{v_l} b(x)}{v_l} \qquad \textbf{(1)}$$

where v_l is the volume of element l, b(x) is a random process representing the elastic modulus and $b_l(x)$ is the elastic modulus of element l

After coordinate transformation, the above formula is written in Jacobi form. The above formula is converted to Gaussian numerical integration.

The elastic modulus of element *l* is expressed as

$$b_l(x) = \frac{\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n} W_i W_j W_k b_l(\zeta_i, \eta_j, \xi_k)}{V_l} \quad \textbf{(2)}$$

where w is the weight coefficient of the one-dimensional Gaussian integral. ζ_i, η_j, ξ_k are the integration points. n is the number of integral points in each coordinate direction.

The mean value of elastic modulus of the element is

$$E(b_l(x)) = \frac{\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n} W_i W_j W_k \overline{b_l}(\zeta_i, \eta_j, \xi_k)}{V_l} \quad \textbf{(3)}$$

where $\overline{b_l}(\zeta_i, \eta_j, \xi_k)$ is the mean of the elastic modulus of the Gaussian integral points , $E(\ )$ is the mean..

The covariance of elastic modulus of element and element m is

$$\operatorname{cov}(b_l(x), b_m(x)) = \frac{\sum_{k'=1}^{n}\sum_{j'=1}^{n}\sum_{i'=1}^{n}\ \sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n} W_{i'} W_{j'} W_{k'} W_i W_j W_k \operatorname{cov}(b_l(\zeta_i, \eta_j, \xi_k), b_m(\zeta_i, \eta_j, \xi_k))}{V_l V_m} \quad \textbf{(4)}$$

where cov( ) is the covariance, V_m is the volume of element m.

Improved Interpolation Method

The stochastic process c (x) is approximately expressed as a shape function

CHAPTER 9

Mixed Finite Element

Abstract: The parameters of the structure contain random variables and interval variables. The Taylor expansion method and Neumann expansion method of random interval finite element are proposed. The parameters of the structure are random and fuzzy. Taylor expansion method and Neumann expansion method of the random fuzzy finite element are illustrated. The parameters of the structure are random, fuzzy and non-probabilistic. The mixed finite element calculation should be carried out using Taylor expansion and Neumann expansion.

Keywords: Mixed finite element, Neumann expansion, Random interval finite element, Random fuzzy finite element, Random fuzzy and interval finite element, Taylor expansion.

INTRODUCTION

Finite element method deals with deterministic engineering problems. The influence of uncertain factors must be considered. Uncertain factors influence the strength and life of a structure. Many engineering problems should consider uncertain properties of material, geometry and loads. The structure is affected by two or three uncertain factors, and the structure should be calculated by the mixed finite element method.

Several types of stochastic finite element methods exist in the literature :the Monte Carlo Simulation (MCS) [1-5] , the perturbation method [2, 6-9] and the spectral stochastic finite element method [10-13]. According to first-order or second-order perturbationmethods, calculation formulas of perturbation stochastic finite element method (PSFEM) are derived [2, 6-8]. Finite element solutions for material variability can be obtained by means of perturbation stochastic finite element [2]. A major advantage of perturbation stochastic finite element is that the multivariate distribution function need not be known [6]. The vibration equation of a system is transformed into a static problem by using the Newmark method and the Taylor expansion [7]. Considering the influence of random factors, sensitivity computation for a linear vibration is illustrated [8, 9]. The PSFEM is an adequate tool for nonlinear structural dynamics [14]. This paper presents a framework for probability

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sensitivity estimation of a class of problems involving linear stochastic finite element models [15]. Fuzzy finite element analysis based on the theory of fuzzy sets is presented to take account of the uncertainty of the elastic modulus and Poisson's ratio [16]. An elastoplastic finite element analysis with fuzzy parameters is proposed using fuzzy mathematics [17]. A fuzzy finite-element approach for vibration analysis involving vagueness is developed [18]. Finite element analysis of flexible multibody systems with fuzzy parameters is presented to predict the dynamic response and evaluate the sensitivity [19]. Neumann expansion for fuzzy finite element analysis can resolve the uncertain eigenvalue problem with fuzzy parameters [20].A unified respons esurface framework for interval and stochastic finite element analysis of structures with both probabilistic and non-probabilistic parameters is developed [21]. Modifications for the fuzzy and fuzzy–stochastic FEM is proposed [22]. The static analysis of structures subjected to uncertain loads using the fuzzy and intervalfinite element method is investigated [23].

The calculation formula of random interval finite element is investigated using Taylor expansion and Neumann expansion. The stochastic fuzzy finite element method is developed. Two calculation methods of mixed finite elements are presented.

Stochastic and Interval Finite Element

The elastic modulus , Poisson's ratio of the material and the load on the structure are assumed to be random process and interval variables. The global equilibrium equation of the structure is the following linear equations

$$[K]U = \{F\} \qquad (\mathbf{1})$$

The elastic modulus , Poisson's ratio and loads of the structure are regarded as n random variables $a_1, a_2, \cdots, a_i, \cdots, a_n$ and n interval variables $[\underline{b}_1, \bar{b}_1], [\underline{b}_2, \bar{b}_2], \ldots, [\underline{b}_j, \bar{b}_j], \cdots, [\underline{b}_n, \bar{b}_n]$

Taylor expansion method

The partial derivative of Eq.1 with respect to a_i is given by

$$\frac{\partial U}{\partial a_i}=\left[K\right]^{-1}\left(\frac{\partial\{F\}}{\partial a_i}-\frac{\partial[K]}{\partial a_i}U\right) \quad \textbf{(2)}$$

where $\frac{\partial U}{\partial a_i}$ is the partial derivative of U with respect to a_i.

The partial derivative of Eq.2 with respect to a_j is given by

$$\frac{\partial^2 U}{\partial a_i \partial a_j}=\left[K\right]^{-1}\left(\frac{\partial^2\{F\}}{\partial a_i \partial a_j}-\frac{\partial\left[K\right]}{\partial a_i}\frac{\partial U}{\partial a_j}-\frac{\partial\left[K\right]}{\partial a_j}\frac{\partial U}{\partial a_i}-\frac{\partial^2\left[K\right]}{\partial a_i \partial a_j}U\right) \quad \textbf{(3)}$$

where $\frac{\partial^2 U}{\partial a_i \partial a_j}$ is the partial derivative of $\frac{\partial U}{\partial a_i}$ with respect to a_j. The displacement is expanded at the mean points of the random variables, and the mean value is taken on both sides, we get

$$E\{U\}\approx\{U\}\big|_{a=\bar{a}}+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\{U\}}{\partial a_i \partial a_j}\Big|_{a=\bar{a}}Cov\left(a_i,a_j\right) \quad \textbf{(4)}$$

The variance of $\{U\}$ can be calculated by the following formula:

$$Var\{U\}\approx\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\{U\}}{\partial a_i}\Big|_{a=\bar{a}}\cdot\frac{\partial\{U\}}{\partial a_j}\Big|_{a=\bar{a}}\cdot Cov\left(a_i,a_j\right) \quad \textbf{(5)}$$

where $Var\{U\}$ is the variance of U.

Chebyshev inequality can be rewritten as

$$P\left\{\left|x-\mu'\right|<\varepsilon\right\}\geq 1-\frac{\sigma^2}{\varepsilon^2} \quad \textbf{(6)}$$

where, μ' is the mean, σ is the standard deviation, ε is any positive number.

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