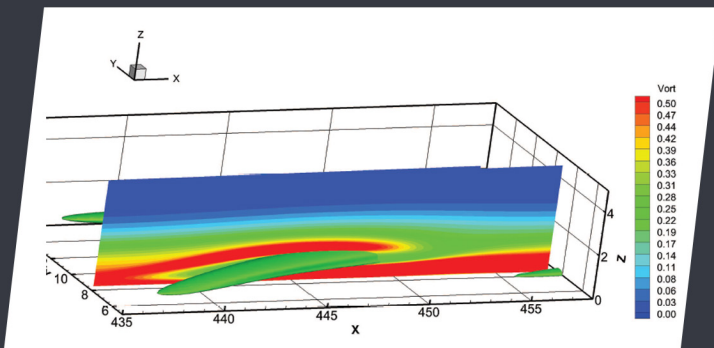
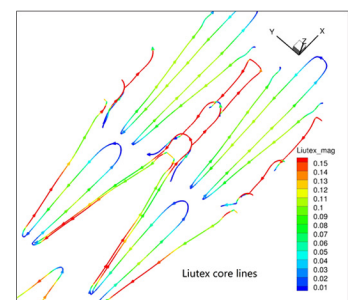
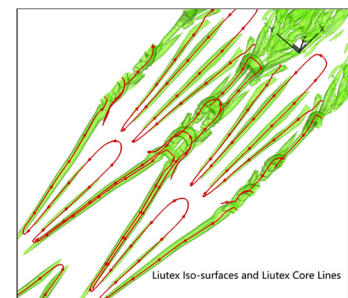
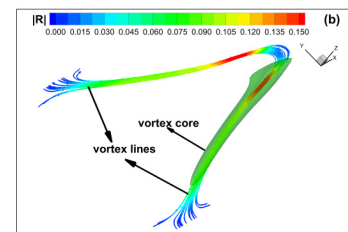
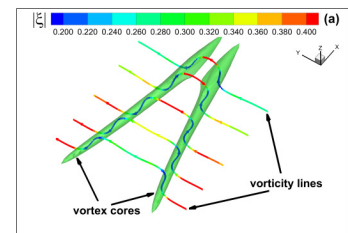


LIUTEX-BASED AND OTHER MATHEMATICAL, COMPUTATIONAL AND EXPERIMENTAL METHODS FOR TURBULENCE STRUCTURE



Editors:
Chaoqun Liu
Yisheng Gao



Bentham Books

**Current Developments in Mathematical
Sciences**
(Volume 2)

*Liutex-based and Other Mathematical,
Computational and Experimental Methods for
Turbulence Structure*

Edited by

Chaoqun Liu

Department of Mathematics,
University of Texas at Arlington,
Arlington, Texas 76019,
USA

&

Yisheng Gao

College of Aerospace Engineering,
Nanjing University of Aeronautics and Astronautics ,
Nanjing 210016,
China

Current Developments in Mathematical Sciences

Volume # 2

Liutex-based and Other Mathematical, Computational and Experimental Methods for Turbulence Structure

Editor: Chaoqun Liu & Yisheng Gao

ISSN (Online): 2589-272X

ISSN (Print): 2589-2711

ISBN (Online): 978-981-14-3760-1

ISBN (Print): 978-981-14-3758-8

ISBN (Paperback): 978-981-14-3759-5

© 2020, Bentham Books imprint.

Published by Bentham Science Publishers Pte. Ltd. Singapore. All Rights Reserved.

BENTHAM SCIENCE PUBLISHERS LTD.

End User License Agreement (for non-institutional, personal use)

This is an agreement between you and Bentham Science Publishers Ltd. Please read this License Agreement carefully before using the book/echapter/ejournal (“**Work**”). Your use of the Work constitutes your agreement to the terms and conditions set forth in this License Agreement. If you do not agree to these terms and conditions then you should not use the Work.

Bentham Science Publishers agrees to grant you a non-exclusive, non-transferable limited license to use the Work subject to and in accordance with the following terms and conditions. This License Agreement is for non-library, personal use only. For a library / institutional / multi user license in respect of the Work, please contact: permission@benthamscience.net.

Usage Rules:

1. All rights reserved: The Work is the subject of copyright and Bentham Science Publishers either owns the Work (and the copyright in it) or is licensed to distribute the Work. You shall not copy, reproduce, modify, remove, delete, augment, add to, publish, transmit, sell, resell, create derivative works from, or in any way exploit the Work or make the Work available for others to do any of the same, in any form or by any means, in whole or in part, in each case without the prior written permission of Bentham Science Publishers, unless stated otherwise in this License Agreement.
2. You may download a copy of the Work on one occasion to one personal computer (including tablet, laptop, desktop, or other such devices). You may make one back-up copy of the Work to avoid losing it.
3. The unauthorised use or distribution of copyrighted or other proprietary content is illegal and could subject you to liability for substantial money damages. You will be liable for any damage resulting from your misuse of the Work or any violation of this License Agreement, including any infringement by you of copyrights or proprietary rights.

Disclaimer:

Bentham Science Publishers does not guarantee that the information in the Work is error-free, or warrant that it will meet your requirements or that access to the Work will be uninterrupted or error-free. The Work is provided "as is" without warranty of any kind, either express or implied or statutory, including, without limitation, implied warranties of merchantability and fitness for a particular purpose. The entire risk as to the results and performance of the Work is assumed by you. No responsibility is assumed by Bentham Science Publishers, its staff, editors and/or authors for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products instruction, advertisements or ideas contained in the Work.

Limitation of Liability:

In no event will Bentham Science Publishers, its staff, editors and/or authors, be liable for any damages, including, without limitation, special, incidental and/or consequential damages and/or damages for lost data and/or profits arising out of (whether directly or indirectly) the use or inability to use the Work. The entire liability of Bentham Science Publishers shall be limited to the amount actually paid by you for the Work.

General:

1. Any dispute or claim arising out of or in connection with this License Agreement or the Work (including non-contractual disputes or claims) will be governed by and construed in accordance with the laws of Singapore. Each party agrees that the courts of the state of Singapore shall have exclusive jurisdiction to settle any dispute or claim arising out of or in connection with this License Agreement or the Work (including non-contractual disputes or claims).
2. Your rights under this License Agreement will automatically terminate without notice and without the

need for a court order if at any point you breach any terms of this License Agreement. In no event will any delay or failure by Bentham Science Publishers in enforcing your compliance with this License Agreement constitute a waiver of any of its rights.

3. You acknowledge that you have read this License Agreement, and agree to be bound by its terms and conditions. To the extent that any other terms and conditions presented on any website of Bentham Science Publishers conflict with, or are inconsistent with, the terms and conditions set out in this License Agreement, you acknowledge that the terms and conditions set out in this License Agreement shall prevail.

Bentham Science Publishers Pte. Ltd.

80 Robinson Road #02-00

Singapore 068898

Singapore

Email: subscriptions@benthamscience.net



CONTENTS

PREFACE	i
LIST OF CONTRIBUTORS	vii
CHAPTER 1 LIUTEX – A NEW MATHEMATICAL DEFINITION OF VORTEX AND VORTICITY DECOMPOSITION FOR TURBULENCE RESEARCH	1
<i>Chaoqun Liu, Yisheng Gao and Yifei Yu</i>	
INTRODUCTION	2
LIUTEX – A NEW EIGENVECTOR BASED MATHEMATICAL DEFINITION FOR FLUID ROTATION	5
1) Four Principles	5
2) Definition of Liutex	5
3) Calculation Procedure for Liutex	9
Velocity Decomposition	9
Vortex Gradient Tensor Decomposition Based on Liutex	11
Vortex Identification	12
LIUTEX FOR COMPRESSIBLE FLOW	17
CONCLUSION	17
CONSENT FOR PUBLICATION	18
CONFLICT OF INTEREST	18
ACKNOWLEDGEMENTS	18
REFERENCES	18
CHAPTER 2 LIUTEX AND ITS CALCULATION AND GALILEAN INVARIANCE	21
<i>Yiqian Wang, Yisheng Gao and Chaoqun Liu</i>	
INTRODUCTION	21
REVISIT OF THE LIUTEX VECTOR DEFINITION	22
THE PHYSICAL MEANING OF LIUTEX, VORTICITY AND λ_{CI}	24
DERIVATION OF THE EXPLICIT FORMULA TO CALCULATE LIUTEX	30
Derivation - Approach 1	30
Derivation – Approach 2	39
GALILEAN INVARIANCE OF LIUTEX VECTOR	41
CONCLUSIONS	42
LIST OF SYMBOLS	42
CONSENT FOR PUBLICATION	43
CONFLICT OF INTEREST	43
ACKNOWLEDGEMENTS	43
REFERENCES	43
CHAPTER 3 NEW OMEGA VORTEX IDENTIFICATION METHOD BASED ON DETERMINED EPSILON	45
<i>Xiangrui Dong, Yisheng Gao and Chaoqun Liu</i>	
INTRODUCTION	45
DEFINITION OF A VORTEX BY NEW OMEGA METHOD	46
APPLICATION OF OMEGA METHOD WITH DETERMINED EPSILON	48
Case 1: Boundary Layer Transition	48
Case 2: SWBLIs Controlled by MVG	54
Case 3: Channel Flow with $Re\delta=950$	56
CONCLUSIONS	57
CONSENT FOR PUBLICATION	58
CONFLICT OF INTEREST	58

ACKNOWLEDGEMENTS	58
REFERENCES	58
CHAPTER 4 STABILITY ANALYSIS ON SHEAR FLOW AND VORTICES IN LATE BOUNDARY LAYER TRANSITION	60
<i>Jie Tang</i>	
INTRODUCTION	60
A Short History Review of Research on Flow Transition and Turbulence Generation	60
Liu's New Theory on Boundary Layer Transition	63
The Purpose of the Current Chapter	64
Organization of this Chapter	64
CHEBYSHEV SPECTRAL METHODS	64
Introduction to Spectral Methods	64
Chebyshev Polynomials of First Kind	65
Chebyshev Collocation Approach	69
Convergence of Chebyshev Spectral Method	70
Advantages of Chebyshev Nodes	71
DIRECT NUMERICAL SIMULATION (DNS) CASE SET UP AND CODE VALIDATION	72
Case Set-up	72
Governing Equation in Generalized Curvilinear Coordinates	73
Numerical Methods	76
Code Validation	77
DNS Visualization Method	77
INSTABILITY OF TWO-DIMENSIONAL SHEAR FLOW	78
Linear Stability Equation	78
Chebyshev Discretization of the Orr-Sommerfeld Equation	80
Numerical Results for Typical Shear Flows	81
LINEAR STABILITY EQUATION FOR QUASI-ROTATION FLOW IN CYLINDRICAL COORDINATES	84
Derivation of Linear Perturbation System	84
Eigenvalue Function	87
Shifted Chebyshev Polynomials and Discretization	91
<i>Shifted Chebyshev Polynomials with Linear Argument</i>	91
<i>Shifted Chebyshev Polynomials with Quadratic Argument</i>	93
<i>Equation Discretization</i>	95
DNS OBSERVATIONS AND NUMERICAL RESULTS	98
Comparison of Two Shifted Chebyshev Polynomials in a Hyperbolic Case	98
DNS Leg-Like Vortices Cases	101
<i>DNS Observations</i>	101
<i>Numerical Results</i>	105
DNS Ring-Like Vortices Cases	107
<i>DNS Observations</i>	107
<i>Numerical Results</i>	107
CONCLUSION	112
CONSENT FOR PUBLICATION	113
CONFLICT OF INTEREST	113
ACKNOWLEDGEMENTS	113
REFERENCES	113
CHAPTER 5 POD AND DMD ANALYSIS IN LATE FLOW TRANSITION WITH OMEGA METHOD	116
<i>Sita Charkrit and Chaoqun Liu</i>	

INTRODUCTION	116
CASE SETUP AND CODE VALIDATION	118
THE VORTEX IDENTIFICATION AND THE OMEGA METHOD	119
Vortex Visualization	121
MODAL DECOMPOSITION	121
The Proper Orthogonal Decomposition (POD)	122
<i>POD Algorithm</i>	122
<i>Dimension Reduction and the Matrix Reconstruction</i>	124
<i>Linear Combination of POD Mode</i>	125
<i>POD Analysis for Late Transition Flow</i>	126
Dynamic Mode Decomposition (DMD)	139
<i>DMD Algorithm</i>	141
<i>Linear Combination of DMD Mode</i>	142
<i>The Matrix Reconstruction</i>	143
<i>Diagnostic from Eigenvalues and Eigenvectors of A</i>	144
<i>DMD Analysis for Late Transition Flow</i>	145
DISCUSSION AND CONCLUSION	152
CONSENT FOR PUBLICATION	153
CONFLICT OF INTEREST	153
ACKNOWLEDGEMENT	153
REFERENCES	153
CHAPTER 6 COMPARISON OF LIUTEX AND EIGENVALUE-BASED VORTEX IDENTIFICATION CRITERIA FOR COMPRESSIBLE FLOWS	156
<i>Yisheng Gao and Chaoqun Liu</i>	
INTRODUCTION	156
REVIEW OF THE DEFINITION OF LIUTEX	160
EIGENVALUE-BASED VORTEX IDENTIFICATION CRITERIA AND COMPRESSIBLE EXTENSION	163
THE APPLICABILITY OF LIUTEX TO COMPRESSIBLE FLOWS	167
ANALYTICAL RELATIONS BETWEEN LIUTEX, Q_D AND λ_{ci}	168
TEST CASE	169
CONCLUDING REMARKS	174
CONSENT FOR PUBLICATION	174
CONFLICT OF INTEREST	175
ACKNOWLEDGEMENT	175
REFERENCES	175
CHAPTER 7 OBSERVATION OF COHERENT STRUCTURES OF LOW REYNOLDS NUMBER TURBULENT BOUNDARY LAYER BY DNS AND EXPERIMENT	178
<i>Panpan Yan, Chaoqun Liu, Yanang Guo and Xiaoshu Cai</i>	
INTRODUCTION	179
CASE SET UP	180
DNS Case Setup and Code Validation	180
<i>Numerical Methods</i>	182
<i>Code Validation</i>	184
Experiment Setup	188
<i>Introduction to MSFLE Method</i>	188
<i>Experimental Facility</i>	190
<i>Experimental Validation</i>	192
RESULT AND DISCUSSION	193
Comparison Between DNS and Experiment Results	193

Discussion on Multilevel Vortex Structures	198
Multilevel Vortices Ejections and Sweeps	200
CONCLUSIONS	207
NOMENCLATURE	208
CONSENT FOR PUBLICATION	209
CONFLICT OF INTEREST	209
ACKNOWLEDGEMENTS	209
REFERENCES	209
CHAPTER 8 DIRECT NUMERICAL SIMULATION OF INCOMPRESSIBLE FLOW IN A CHANNEL WITH RIB STRUCTURES	214
<i>Ting Yu, Duo Wang, Heng Li and Hongyi Xu</i>	
INTRODUCTION	214
MATHEMATICAL-PHYSICAL MODELS AND METHODS	216
Governing Equations	216
Computation Scheme	217
Vortex Identification Method	217
Surface Roughness Geometry	218
RESULTS OF CASES WITH SINGLE RIB	219
Computed Cases	219
Results of Validation	220
Flow Structures and Heat Transfer	225
RESULTS OF CASE WITH ROUGHENED SURFACE	230
Structure of the Computed Case with Surface Roughness	230
Results of Thermal Fields	231
Results of Rortex	232
CONCLUSIONS	233
CONSENT FOR PUBLICATION	233
CONFLICT OF INTEREST	234
ACKNOWLEDGEMENTS	234
REFERENCE	234
CHAPTER 9 VORTEX AND FLOW STRUCTURE INSIDE HYDROTURBINES	236
<i>Yuning Zhang and Yuning Zhang</i>	
A SUMMARY OF TYPES OF VORTEX IN HYDROTURBINES	236
THE EFFECTS OF VORTEX ON PRESSURE FLUCTUATION	238
THE EFFECTS OF VORTEX ON VIBRATIONS OF HYDROTURBINES	239
VORTEX AROUND THE GUIDE PLATE IN THE FRANCIS TURBINE	241
SWIRLING VORTEX ROPE IN FRANCIS TURBINE	242
Swirl Number Analysis	242
VORTEX IN VANELESS SPACE OF REVERSIBLE PUMP TURBINE	243
CONCLUSIONS	244
CONSENT FOR PUBLICATION	245
CONFLICT OF INTEREST	245
ACKNOWLEDGEMENT	245
REFERENCES	245
CHAPTER 10 A COMPARATIVE STUDY OF COMPRESSIBLE TURBULENT FLOWS BETWEEN THERMALLY AND CALORICALLY PERFECT GASES	247
<i>Xiaoping Chen</i>	
INTRODUCTION	247
GOVERNING EQUATIONS	249
Thermally Perfect Gas (TPG)	250

Calorically Perfect Gas (CPG)	251
DESCRIPTION OF DNS	251
DNS RESULTS AND DISCUSSION	253
Turbulent Statistics	253
Strong Reynolds Analogy	257
Flow Structures	261
CONCLUSION AND OUTLOOK	263
CONSENT FOR PUBLICATION	264
CONFLICT OF INTEREST	264
ACKNOWLEDGEMENT	264
REFERENCES	265
CHAPTER 11 THE EXPERIMENTAL STUDY ON VORTEX STRUCTURES IN TURBULENT BOUNDARY LAYER AT LOW REYNOLDS NUMBER	269
<i>Yanang Guo, Xiaoshu Cai, Wu Zhou, Lei Zhou and Xiangrui Dong</i>	
INTRODUCTION	269
EXPERIMENTAL METHODS	271
Motion Single Frame and Long Exposure (MSFLE)	271
<i>Experiment Apparatus</i>	272
Experimental Validation	274
EXPERIMENTAL RESULTS AND ANALYSIS	275
Measurements in the Streamwise-Normal (x-y)-Plane	275
Measurements in the Streamwise-Spanwise (x-z)-Plane	279
CONCLUSIONS	282
CONSENT FOR PUBLICATION	283
CONFLICT OF INTEREST	283
ACKNOWLEDGEMENTS	283
REFERENCES	283
CHAPTER 12 EXPERIMENTAL STUDIES ON COHERENT STRUCTURES IN JET FLOWS USING SINGLE-FRAME-LONG-EXPOSURE (SFLE) METHOD	284
<i>Lei Zhou, Xiaoshu Cai, Wu Zhou and Yiqian Wang</i>	
INTRODUCTION	284
EXPERIMENTAL METHODS	285
Single-Frame-Long-Exposure (SFLE) and Moving SFLE (MSFLE)	285
Experiment Apparatus	287
EXPERIMENTAL RESULTS AND ANALYSIS	287
The Measurements of Coherent Structures in Jet Entrainment Boundary Layer	288
Special Pathline Structures	290
Investigation on the Coherent Structures of Jet Entrainment Boundary Layer	290
Results from MSFLE	293
CONCLUSIONS	295
CONSENT FOR PUBLICATION	296
CONFLICT OF INTEREST	296
ACKNOWLEDGEMENTS	296
REFERENCES	296
CHAPTER 13 HYBRID COMPACT-WENO SCHEME FOR THE INTERACTION OF SHOCK WAVE AND BOUNDARY LAYER	297
<i>Jianming Liu and Chaoqun Liu</i>	
A SHORT REVIEW ON STUDY OF HIGH ORDER FINITE DIFFERENCE SCHEME FOR COMPRESSIBLE FLOWS	297

Governing Equations	298
High-order Central Finite Difference Scheme	299
Compact Finite Difference Scheme	301
WENO Schemes	302
HYBRID WENO SCHEMES	303
TIME DISCRETIZATION	305
NUMERICAL EXAMPLES	306
CONCLUSION	313
CONSENT FOR PUBLICATION	314
CONFLICT OF INTEREST	314
ACKNOWLEDGEMENT	314
REFERENCES	314
SUBJECT INDEX	312

PREFACE

Turbulence is a centuries-long world puzzle. Turbulence coherent structure really means vortex structure. However, there was no mathematical definition of vortex ever before and there was no mathematical definition for turbulence either. Therefore, there was no vortex science or vortex dynamics since vortex had no definition. Since turbulence is built up and driven by vortices, there was no serious scientific research on turbulence theory and turbulence structure because there was no definition of vortex. This book collected a lot of scientific efforts to give an accurate and mathematical definition of vortex, that is Liutex developed by Liu *et al* [C. Liu *et al.*, Phys. Fluids **30**, 035104 (2018)].

The core of this book is a collection of papers presented in the 13th World Congress of Computational Mechanics (WCCM2018), Symposium 704, **Mathematics and Computations for Multiscale Structures of Turbulent and Other Complex Flows**, New York, United States on July 27, 2018. This book also collects quite a number of other research papers working on the vortex definition, vortex identification and turbulence structure from different insight angles including mathematics, computations and experiments. Of course, the priority is dedicated to an accurate and mathematical definition for vortex, which was first named “RORTEX” in 2018 and was changed to “LIUTEX” approved unanimously by alliance of six universities and Alliance of Vortex Research in 2019. Besides Liutex, this book also publishes a lot of efforts to do analysis on turbulence structure by unobjectionable mathematics, incredible DNS computations, and marvelous experiments.

This book contains thirteen chapters which are briefly introduced in this preface.

The first chapter, Liutex – A New Mathematical Definition of Vortex and Vorticity Decomposition for Turbulence Research, is written by Chaoqun Liu, Yisheng Gao and Yifei Yu at University of Texas at Arlington. For long time, people recognize vortex as vorticity tube and measure the vortex rotation strength by vorticity magnitude. These misunderstandings have been carried out by thousands of research papers and almost all textbooks. Robinson (1989) has found the association between regions of strong vorticity and actual vortices can be rather weak. Many vortex identification criteria have been proposed. However, vortex still has no rigorous mathematical definition with direction and magnitude and the relationship between vortex and vorticity was unknown. Because we do not have definition for vortex, there was really no vortex science. Since vortex is the building block and the muscle of turbulence, lack of mathematical definition for vortex becomes a bottle neck for turbulence research. Really, there was no serious turbulence research without definition of vortex. In our recent work, a mathematical definition called Liutex (Previously called Rortex) is given to identify the rigid rotation of fluid motion. Since vortex core is near the rigid rotation, Liutex naturally represents the vortex cores. Liutex is a local mathematical vector definition with direction and magnitude of pure rotation without shear contamination, which is unique and Galilean invariant. The Liutex direction is defined as the local rotation axis and the Liutex magnitude is the local rotation. More important, we derive the accurate mathematical relation between vorticity and vortex, which is $\text{Vorticity} = \text{Liutex} + \text{Antisymmetric Shear}$ (RS decomposition). This new discovery is an important breakthrough in modern fluid dynamics and is extremely important for turbulence research. In addition, the velocity gradient tensor has been decomposed to two parts, R (rigid rotation) and NR (non-rotation part) as a counterpart of the traditional Cauchy-Stokes decomposition (Helmholtz decomposition) which is improper since

vorticity cannot represent flow rotation. Liutex is a new physical quantity like velocity, vorticity, temperature, pressure, which has been ignored by our founding fathers of fluid dynamics for centuries but is particularly important for vortex dynamics and turbulence research. Introduction of Liutex, RS decomposition of vorticity, and R-NR decomposition of velocity gradient would open a new era for vortex dynamics and new turbulence research, likely new fluid dynamics.

The second chapter, Liutex – a New Vortex definition, and its Calculation and Galilean Invariance, is written by Yiqian Wang, Yisheng Gao and Chaoqun Liu from University of Shanghai for Science and Technology in China and University of Texas at Arlington in USA. The Liutex (previously known Rortex) method introduces a vortex vector field to mathematically and systematically describe vortices in flow fields. In the present study, the calculation procedure of Liutex is revisited which includes two-step reference coordinate rotation. Then, for the first time, an explicit formula to calculate Liutex is derived and the physical intuition and efficiency improvement brought by this formula are discussed. In addition, the Galilean invariance, which has been widely accepted as a preliminary check for a successful vortex identification method is discussed for Liutex vector.

The third chapter, New Omega Vortex Identification Method Based on Determined Epsilon, is authored by Xiangrui Dong, Yisheng Gao, Chaoqun Liu from University of Shanghai for Science and Technology and University of Texas at Arlington. A new Omega method with ε determination is introduced to represent the ratio of vorticity square over the sum of vorticity square and deformation square, for the vortex identification. the advantages of the new Ω method can be summarized as follows: (1) Omega, as a ratio of the vorticity squared over the sum of the vorticity squared and deformation squared, is a normalized and case-independent function which satisfies $\Omega \in [0,1]$; (2) Compared with the other vortex visualization methods, which require a wide threshold change to capture the vortex structures, Ω can always be set as 0.52 to capture vortex for different cases and time steps; (3) ε is defined as a function without any adjustment on its coefficient in all cases; (4) The Ω method can capture both strong and weak vortices simultaneously. In addition, Ω method is quite robust with no obvious change in vortex visualization.

The fourth chapter, Stability Analysis on Shear Flow and Vortices in Late Boundary Layer Transition, is solely authored by Jie Tang from University of Texas at Arlington. Turbulence is still an unsolved scientific problem, which has been regarded as “the most important unsolved problem of classical physics”. Liu proposed a new mechanism about turbulence generation and sustenance after decades of research on turbulence and transition. One of them is the transitional flow instability. Liu believes that inside the flow field, shear (dominant in laminar) is unstable while rotation (dominant in turbulence) is relative stable. This inherent property of flow creates the trend that non-rotational vorticity must transfer to rotational vorticity and causes the flow transition. To verify this new idea, this chapter analyzed the linear stability on two-dimensional shear flow and quasi-rotational flow. Chebyshev collocation spectral method is applied to solve Orr–Sommerfeld equation. Several typical parallel shear flows are tested as the basic-state flows in the equation. The instability of shear flow is demonstrated by the existence of positive eigenvalues associated with disturbance modes (eigenfunctions), *i.e.* the growth of these linear modes. Quasi-rotation flow is considered under cylindrical coordinates. An eigenvalue perturbation equation is derived to study the stability problem with symmetric flows. Shifted Chebyshev polynomial with Gauss collocation points is used to solve the equation. To investigate the stability of vortices in flow transition, a ring-like vortex and a leg-like vortex over time from our Direct Numerical Simulation (DNS) data are

tracked. The result shows that, with the development over time, both ring-like vortex and leg-like vortex become more stable as Omega becomes close to 1.

The fifth chapter, POD and DMD Analysis in Late Flow Transition with Omega Method, is authored by Sita Charkrit and Chaoqun Liu at Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA. In this chapter, the proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD) are applied to analyze the 3D late transitional flow on the flat plate obtained from Direct numerical simulation (DNS). POD is used to find the most persistent spatial structures while DMD is used to find single frequency modes. The omega method is applied as a vortex identification to visualize vortices with iso-surfaces $\Omega = 0.52$. The results in POD and DMD are discussed and compared to show the same and different features such as shapes, amplitudes and time evolutions.

The sixth chapter, Comparison of Liutex and Eigenvalue-based Vortex Identification Criteria for Compressible Flows, is written by Yisheng Gao and Chaoqun Liu at Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA. Most of the currently popular vortex identification methods, including the \mathbf{Q} criterion, the Δ criterion and the λ_{ci} criterion, are exclusively determined by the eigenvalues or invariants of the velocity gradient tensor and thereby can be classified as eigenvalue-based criteria. However, these criteria will suffer from several shortcomings, such as inadequacy of identifying the rotational axis and contamination by shearing. Recently, a new eigenvector-based Liutex method (previously named Rortex) was proposed to overcome the issues associated with the eigenvalue-based criteria. In this paper, the comparison of Liutex and two eigenvalue-based criteria, namely the λ_{ci} criterion and a modification of the original \mathbf{Q} criterion, are performed to assess these methods for compressible flows. According to the analysis of the deviatoric part of the local velocity gradient tensor, all the scalar, vector and tensor forms of Liutex are valid for compressible flows without any modification, while two eigenvalue-based criteria, though applicable to compressible flows, are prone to severe contamination by shearing as in incompressible flows. Vortex structures in the problem of shock-vortex interaction are examined to confirm the validity and superiority of Liutex in compressible flows.

The seventh chapter, Observation of Coherent Structures of Low Reynolds Number Turbulent Boundary Layer by DNS and Experiment, is written by Panpan Yan from Beijing Jiaotong University, Beijing, 100044, China, Chaoqun Liu from University of Texas at Arlington, Yanang Guo and Xiaoshu Cai from University of Shanghai for Science and Technology, Shanghai, 200093, China. In order to study the characteristics of coherent structures of the turbulent boundary layer, the motion single frame, and long exposure imaging (MSFLE) method is proposed and an elaborate direct numerical simulation experiment was also conducted. MSFLE method is a Lagrangian measurement method, the speed of the camera is kept the same as the speed of the coherent structure, and the particle trajectory was captured by long exposure. By calculating the trace of the points on a chosen plane of the DNS result, we can obtain the particle trajectory like MSFLE method. Multilayer of vortex structures was observed and the evolution of the vortex packets with time was recorded. The result of the DNS simulation agrees well with the experiment. The size of the vortex of the different layer is almost the same, and no vortex breakdown was observed. The formation of the small-scale vortex is caused by sweeps and ejections of the larger coherent structures rather than the breakdown process.

The eighth chapter, Direct Numerical Simulation of Incompressible Flow in a Channel with Rib Structures, is authored by Ting Yu, Duo Wang, Heng Li and Hongyi Xu from Aeronautics and

Astronautics Department, Fudan University Shanghai PR China. The paper applied the state-of-the-art flow simulation method, i.e. the Direct Numerical Simulation (DNS), and strongly coupled the DNS with the heat-transfer governing equation to solve the thermal-turbulence problem in both 2-dimensional(2D) and 3-dimensional(3D) channel with rib tabulator structures. An innovative approach was applied to the simulations in one case. The surface roughness effects of the cooling vane were considered by including the roughness geometry in the DNS and the immersed-boundary method were invented to handle the geometry complexities due to the roughness. Two inlet conditions, namely the uniform flow and full-developed turbulence, were applied at the inflow surface of the channel. Half height of the channel was used as the scale length. The Prandtl number was set at $Pr = 0.7$. Five Reynolds number of 1000, 2500, 5000, 7500 and 1000 were calculated in the 2D cases and the Reynolds numbers of 2500 and 5000 were applied in 3D cases where a periodical condition was applied in the span-wise direction. Additionally, Reynolds number of 10000 was set in the case with roughened surface. The stream-wise velocity, turbulence intensity, the Nusselt (Nu) number were analyzed. Results in 2D cases and 3D cases presented a great difference on flow structure. At the same time, with increasing Reynolds number, the length of recirculation zone and the enhancement of heat transfer showed a decreasing trend. A vortex identification method, the newly-defined Rortex, was applied.

The ninth chapter, Vortex and Turbulent Structure Inside Hydroturbines, is written by Yuning Zhang from Key Laboratory of Condition Monitoring and Control for Power Plant Equipment (Ministry of Education), School of Energy, Power and Mechanical Engineering, North China Electric Power University, Beijing, China and Yuning Zhang from College of Mechanical and Transportation Engineering, China University of Petroleum-Beijing, Beijing China and Beijing Key Laboratory of Process Fluid Filtration and Separation, China University of Petroleum-Beijing, Beijing China. In this chapter, various kinds of vortex in the hydroturbines are briefly introduced with a focus on the swirling vortex rope in Francis turbine and the vortex in the vaneless space of the reversible pump turbine. The vortex induced pressure fluctuation and vibrations are initially demonstrated based on the on-site measurement in the power stations. Then, detailed characteristics of the vortex in the hydroturbines are demonstrated based on the plenty of examples together with the aid of the quantitative analysis.

The tenth chapter, Comparative Study of Supersonic Turbulent Channel flows between Thermally and Calorically Perfect Gases, is written by Xiaoping Chen from National-Provincial Joint Engineering Laboratory for Fluid Transmission System Technology, Zhejiang Sci-Tech University, Hangzhou, Zhejiang, China. In this chapter, to study the effects of gas model on the turbulent statistics and flow structures, direct numerical simulations (DNSs) of supersonic turbulent channel flow for thermally perfect gas and calorically perfect gas are conducted at Mach number 3.0 and Reynolds number 4800 combined with two wall temperature of 298.15K (low temperature condition) and 596.30 K (high temperature condition). The results show that, for high temperature condition, the effects of thermally perfect gas are important because the vibrational energy excited degree exceeds 0.1. Many of turbulent statistics used to express low temperature condition for calorically perfect gas still can be generalized for high temperature condition. The gas model does not have a significant influence on the strong Reynolds analogy. Omega could capture both strong and weak vortices simultaneously for supersonic flows, even under thermally perfect gas, which is difficult to

obtain by Q . Compared to the results of calorically perfect gas, the vortex structure becomes smaller, sharper and more chaotic by considering thermally perfect gas.

The eleventh chapter The Experimental Study on Vortex Structures in Low Reynolds Number Turbulent Boundary Layer, was authored by Yanang Guo, Xiaoshu Cai, Wu Zhou, Lei Zhou, Xiangrui Dong from Institute of Particle and Two-phase Flow Measurement, University of Shanghai for Science and Technology, Shanghai, China. A motion single frame and long exposure (MSFLE) imaging method, which is a Lagrangian-type measurement, is experimentally carried out to study the vortex structures in a fully developed turbulent boundary layer with a low Reynolds number on a flat plate. In order to give the process of the vortex generation and evolution, on the one hand, the measurement system moves at the substantially same velocity as the vortex structure; on the other hand, a long exposure time is selected for recording the paths of the particles. In the experiment, the vortex structure characteristics as well as the temporal-spatial development can be shown by the streamwise-normal and streamwise-spanwise images which are extracted from a fully developed turbulent boundary layer. The result shows that the interaction between high and low-speed streaks induces the generation, deformation and ‘breakdown’ of the vortex structures, and badly influences the vortex evolution.

The twelfth chapter, Experimental Studies on Coherent Structures in Jet Flows using Single-Frame-Long-Exposure (SFLE) Method is authored by Lei Zhou, Xiaoshu Cai, Wu Zhou and Yiqian Wang from Institute of Particle and Two-phase Flow Measurement, University of Shanghai for Science, China. An experimental investigation on the flow structures in jet entrainment boundary layer flows based on the Single-Frame-Long Exposure (SFLE) method is carefully performed. It is found that two entrainment mode of ‘engulfing’ and ‘nibbling’ alternatively appear in the region of $2d$ to $3.5d$ in the axial direction and $1d$ to $1.25d$ in the radial direction with d being the diameter of jet nozzle. The appearance probability of such a pattern and the proportion of the ‘engulfing’ mode increases with Reynolds number Re when $Re \geq 1981$ (the Reynolds number is based on the nozzle diameter and jet velocity). However, the influence of Reynolds number on this flow pattern becomes weaker when $Re > 2245$. The main frequency of this structure is found to be between 10-19Hz with Fourier analysis. The vortical structures are further explored with the moving SFLE (MSFLE) method, and it is found that vortices always exist near the turbulent and non-turbulent interface (TNTI).

The last chapter (thirteenth), Hybrid Compact-WENO Scheme for the Interaction of Shock Wave and Boundary Layer, is co-authored by Jianming Liu from Jiangsu Normal University of China and Chaoqun Liu from Department of Mathematics, University of Texas at Arlington, Arlington, USA. In this chapter, an introduction on hybrid Weighted Essentially non-oscillatory (WENO) method is given. The hybrid techniques including both central and compact finite difference schemes are introduced. The paper reviews the driven mechanism of the high order finite scheme required for compressible flow with shock. The detailed constructing processes of the compact and WENO schemes are given and the hybrid detector.

I hope this book will be useful to scientists and engineers who are interested in fundamental fluid dynamics, vortex science and turbulence research.

In conclusion, I want to thank the numerous authors for their incredible contributions and having patience in assisting us. Furthermore, I want to acknowledge and thank the referees for their tiresome work on making this book come to fruition. Last but not the least, I would also like to thank my family including Weilan Jin (my wife), Haiyan Liu (my daughter) and Haifeng Liu (my

vi

son) for their unconditional support. The co-editor, Yisheng Gao is also grateful to his family for the strong support.

Chaoqun Liu
Department of Mathematics,
University of Texas at Arlington,
Arlington, Texas 76019,
USA

List of Contributors

- Chaoqun Liu** Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA
- Duo Wang** Department of Aeronautics and Astronautics, Fudan University, Shanghai, PR China
- Heng Li** Department of Aeronautics and Astronautics, Fudan University, Shanghai, PR China
- Hongyi Xu** Department of Aeronautics and Astronautics, Fudan University, Shanghai, PR China
- Jianming Liu** School of Mathematics and Statistics, Jiangsu Normal University, Xuzhou 221116, China
- Jie Tang** Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA
- Lei Zhou** Institute of Particle and Two-phase Flow Measurement, University of Shanghai for Science and Technology, Shanghai 200093, China
- Panpan Yan** Shenyang Aircraft Design and Research Institute, Aviation Industry of China, Shenyang, 110035, China
- Sita Charkrit** Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA
- Ting Yu** Department of Aeronautics and Astronautics, Fudan University, Shanghai, PR China
- Xiaoshu Cai** Institute of Particle and Two-phase Flow Measurement, College of Energy and Power Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China
- Xiaoping Chen** National-Provincial Joint Engineering Laboratory for Fluid Transmission System Technology, Zhejiang Sci-Tech University, Hangzhou, Zhejiang 310018, China
- Xiangrui Dong** Institute of Energy and Power Engineering, University of Shanghai for Science and Technology, Shanghai, China
- Yisheng Gao** College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

viii

- Yanang Guo** Institute of Particle and Two-phase Flow Measurement,
University of Shanghai for Science and Technology, Shanghai,
China
- Yiqian Wang** School of Mathematical Science, Soochow University, Suzhou
215006, China
- Yifei Yu** Department of Mathematics, University of Texas at Arlington,
Arlington, Texas 76019, USA
- Yuning Zhang** Key Laboratory of Condition Monitoring and Control for Power
Plant Equipment (Ministry of Education), School of Energy,
Power and Mechanical Engineering, North China Electric
Power University, Beijing 102249, China
- Yuning Zhang** College of Mechanical and Transportation Engineering, China
University of Petroleum-Beijing, Beijing 102249, China
- Wu Zhou** Institute of Particle and Two-phase Flow Measurement,
University of Shanghai for Science and Technology, Shanghai,
China and School of Aerospace Engineering, Tsinghua
University, Beijing, China

Liutex – A New Mathematical Definition of Vortex and Vorticity Decomposition for Turbulence Research

Chaoqun Liu^{1,*}, Yisheng Gao² and Yifei Yu¹

¹Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA

²College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

Abstract: For a long time, people recognize a vortex as a vorticity tube and measure the vortex rotation strength by vorticity magnitude. These misunderstandings have been carried out by thousands of research papers and almost all textbooks. It has been found that the association between regions of strong vorticity and actual vortices can be rather weak. Accordingly, many vortex identification criteria have been proposed. However, the vortex still has no rigorous mathematical definition and the relationship between the vortex and the vorticity is still not clear. Because we do not have a definition for the vortex, there exists no vortex science. Since the vortex is the building block and the muscle of turbulence, the lack of the mathematical definition for the vortex becomes a bottleneck for turbulence research. Actually, there is no serious turbulence research without the definition of the vortex. In our recent work, a mathematical definition called Liutex (previously called Rortex) is introduced to identify the rigid rotation of fluid motion. Liutex is a local mathematical vector definition with the direction and magnitude of pure rotation without shear contamination, which is unique and Galilean invariant. The Liutex direction is defined as the local rotation axis and the Liutex magnitude is the local rotation strength. More importantly, we derive the accurate mathematical relation between the vorticity and the vortex, which is $\text{Vorticity} = \text{Liutex} + \text{Antisymmetric Shear}$ (RS decomposition). This new discovery is an important breakthrough in modern fluid dynamics and is extremely important for turbulence research. In addition, the velocity gradient tensor has been decomposed to two parts, R (rigid rotation) and NR (non-rotation part) as a counterpart of the traditional Cauchy-Stokes decomposition which is improper since the vorticity cannot represent flow rotation. Liutex is a new physical quantity like velocity, vorticity, temperature, pressure, which has been ignored by our founding fathers of fluid dynamics for centuries but is particularly important for vortex dynamics and turbulence research. The introduction of Liutex, RS decomposition of vorticity, and R-NR decomposition of the velocity gradient tensor would open a new era for vortex dynamics and new turbulence research, likely new fluid dynamics.

*Corresponding author **Chaoqun Liu**: Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA; Tel: +1-8172725151; Fax: +1-8172725802; E-mail: cliu@uta.edu

Keywords: Angular velocity, Coherent structures, Liutex, Turbulence, Velocity gradient tensor, Vortex identification, Vortex.

INTRODUCTION

Vortex is intuitively recognized as the rotational/swirling motion of the fluids. However, a universally accepted definition for vortex is yet to be achieved, which is probably one of the major obstacles causing considerable confusions and misunderstandings in turbulence research. Vorticity is mathematically defined as the curl of velocity. Vortices are ubiquitous in nature. As addressed by Küchemann “vortices are the sinews and muscles of turbulence” [1], some vortical structures, such as hairpin vortices, referred to as coherent turbulent structures [2], are recognized as one of the most important characteristics of turbulent flow and have been studied for more than 60 years [3]. It is generally acknowledged that intuitively, vortices represent the rotational/swirling motion of the fluids. However, a precise and rational definition of vortex is deceptively complicated and remains an open issue [4-5]. The lack of a consensus on the vortex definition has caused considerable confusions in visualizing and understanding the vortical structures, their evolution, and the interaction in complex vortical flows, especially in turbulence [4].

In classical vortex dynamics [4, 6-7], the vortex is usually associated with the vorticity which has a rigorous mathematical definition (the curl of velocity). Wu *et al.*, [4] define a vortex as “a connected region with high concentration of vorticity compared with its surrounding.” Lamb [8] uses vorticity tubes to define vortices. Nitsche [9] asserts, “A vortex is commonly associated with the rotational motion of fluid around a common centerline. It is defined by the vorticity in the fluid, which measures the rate of local fluid rotation.” An immediate contradiction to these definitions is that the Blasius boundary layer where the vorticity is large near the wall, but no rotational/swirling motion (considered as a vortex) is observed, as the vorticity cannot distinguish a vortical region from a shear layer region. In addition, the maximum vorticity does not necessarily represent the center of the vortex. Robinson [10] pointed out that the association between regions of strong vorticity and actual vortices can be rather weak in the turbulent boundary layer, especially in the near wall region. Wang *et al.*, [11] also found that vorticity magnitude will be reduced when vorticity lines enter the vortex region and vorticity magnitude inside the vortex region is much smaller than the surrounding area, especially near the solid wall in a flat plate boundary layer, for most three-dimensional vortices like Λ -shaped vortices.

Another possible candidate for vortex definition is the one based on closed or spiraling streamlines [12]. Robinson *et al.*, [13] claim, “A vortex exists when

instantaneous streamlines mapped onto a plane normal to the vortex core exhibit a roughly circular or spiral pattern, when viewed from a reference frame moving with the center of the vortex core". Although it seems intuitive, Lugt [12] pointed out that "the definition and identification of a vortex in unsteady motions is difficult since streamlines and pathlines are not invariant with respect to Galilean and rotational transformations. Recirculated streamline patterns at a certain instant in time do not necessarily represent vortex motions in which fluid particles are moving around a common axis. Thus, instantaneous streamline patterns do not provide enough information to be used for the definition of a vortex."

Due to the essential requirement for visualizing vortical structures and their evolution in turbulence, several vortex identification criteria have been developed, including λ_2 -criterion [14], Q-criterion [15], λ_{ci} -criterion [16], and $\lambda_{cr}/\lambda_{ci}$ -criterion [17], *etc.* Nevertheless, these methods still fail to provide a rigorous definition of vortices. Moreover, these methods require proper thresholds. It is difficult to determine which threshold is proper, since different thresholds will indicate different vortical structures. For example, even if the same DNS data on the late boundary layer transition is employed, "vortex breakdown" will be exposed for some large threshold in Q-criterion while no "vortex breakdown" can be found for some smaller threshold. This will directly influence one's understanding and explanation on the mechanism of turbulence generation, *i.e.* turbulence is caused by "vortex breakdown" or not caused by "vortex breakdown". Recently, a new vortex identification method called Ω -method is given by the proposer, based on the idea that a vortex is a connected area where the vorticity overtakes the deformation [18]. The vorticity represents the fluid's intention to rotate but deformation would resist rotation. Ω -method possesses several advantages, such as normalized from 0 to 1, no need for a case-related threshold, clear physical meaning and capability to capture both strong and weak vortices simultaneously. However, it is still not the ideal answer to the question of the mathematical definition of vortex, owing to some limitations such as the introduction of an artificial parameter ε and the incapability to identify the swirl axis and its orientation. Kolář [19] formulated a triple decomposition from which the residual vorticity can be obtained after the extraction of an effective pure shearing motion and represents a direct measure of the pure rigid-body rotation of a fluid element. However, the triple decomposition is not unique, so a so-called basic reference frame must be first determined. Searching for the basic reference frame results in an expensive optimization problem for every point in the flow field, which limits the applicability of the method. Kolář *et al.*, [20-21] also introduced the concepts of the maximum corotation and the average corotation of line segments near a point and apply these methods for vortex identification. However, the so-called maximum-corotation method suffers from the unstable

Liutex and Its Calculation and Galilean Invariance

Yiqian Wang^{1,*}, Yisheng Gao² and Chaoqun Liu³

¹*School of Mathematical Science, Soochow University, Suzhou 215006, China*

²*College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*

³*Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA*

Abstract: The Liutex (previously known Rortex) method introduces a vortex vector field to mathematically and systematically describe vortices in flow fields. In this study, the previous calculation procedures of Liutex which includes a two-step reference coordinate rotation is revisited first. An explicit formula to calculate Liutex is then derived and the physical intuition and efficiency improvement brought by this formula are discussed. It is estimated that the computation time of Liutex vector from velocity gradient field can be reduced by 36.6% compared with that of the previous method. Besides, the Galilean invariance widely accepted as a preliminary check for a successful vortex identification method is discussed for Liutex vector.

Keywords: Angular velocity, Coherent structures, Galilean invariance, Liutex, Vortex identification, Velocity gradient tensor.

INTRODUCTION

Despite that turbulence is often considered random, instantaneous organized structures, or coherent structures exist in turbulence and play a significant role in turbulent momentum transport [1-2]. Two typical coherent structures found in near-wall turbulence are the hairpin vortex and low-speed streaks. Theodorsen [3], back in 1952, proposed a conceptual model of “horseshoe” or “hairpin” vortex to describe the regeneration cycle of turbulence. Actually, these hairpin shaped vortices are ubiquitously found in wall turbulence both from numerical simulations and experiments. Adrian [4] has stipulated that hairpins may autogenerate to form hairpin vortex packets which are presumed to be the prevalent coherent structures in wall bounded turbulence. In a transitional boundary layer flow, the development from the Λ -vortex to the hairpin vortex was carefully studied by Wang *et al.* [5] and its preponderance and statistical importance in turbulence and transition was further investigated by Eitel-Amor [6]. The second type of coherent structure considered

*Corresponding author Yiqian Wang: School of Mathematical Science, Soochow University, Suzhou 215006, China; Tel: +1-15950504139; E-mail: yiqianw@sina.com

here is the near-wall streaks of alternating high and low streamwise momentum fluids which were first reported by Kline *et al.* [7] based on flow visualization in the viscous sublayer using hydrogen bubbles. Thereafter, substantial attention had been drawn to the streaks with the surrounding staggered quasi-streamwise vortices and the dynamics of the nonlinear self-sustain cycle concerning the streak instability that leads to the formation of quasi-longitudinal vortices was proposed [8]. Clearly, both the hairpins and the streaks are related to the notion of vortex, which has a clear physical intuition but hardly a mathematical definition. Δ , λ_{ci} , \mathbf{Q} and $\mathbf{\Omega}$ methods [9-12] which are classified into velocity-gradient-based Eulerian scalar vortex identification methods have been proved to be able to capture the vortical rotational strength to some extent. To give a more precise and unambiguous definition of a vortex, a vector field which includes information of both the direction and the magnitude named Liutex (previously named Rortex) [13-14] has been introduced recently. Then Gao and Liu [15] have improved the calculation method of Liutex by pointing out that the local rotational axis is actually the real eigenvector of the velocity gradient tensor provided that the other two corresponding eigenvalues are complex conjugates which serves as the sufficient and the necessary condition of local fluid rotation.

However, an explicit formula for Liutex vector has not been reached, making the derivation of governing equation more difficult and the appreciation of the physical meaning ambiguous. In the present study, a careful derivation of such an explicit formula is given after a revisit of the Liutex vector calculation procedure. Thereafter, the physical meaning and efficiency improvement of calculating the Liutex vector from this formula are discussed and also the Galilean invariance of Liutex is reassured according to the explicit formula.

REVISIT OF THE LIUTEX VECTOR DEFINITION

The computation of Liutex vector includes the following three steps.

- 1) Obtain the directional information of Liutex.

Calculate the eigenvalues of the 3×3 matrix $\nabla \vec{v}$ (velocity gradient tensor) in the original xyz -frame. If all the three eigenvalues are real, there is no fluid rotation. Thus, the Liutex vector equals to zero. If $\nabla \vec{v}$ has two complex conjugated eigenvalues and one real eigenvalue, the corresponding real unit eigenvector \vec{t}_r is the local rotational axis and thus Liutex direction. Then, make a coordinate rotation (Q rotation) that rotates the original z -axis to the local rotational axis \vec{t}_r and obtain

the new velocity gradient tensor $\nabla\vec{V}_Q$ in the resulting XYZ_Q frame by $\nabla\vec{V}_Q = Q\nabla\vec{v}Q^T$.

2) Obtain the magnitude of Liutex.

After the Q rotation, $\nabla\vec{V}_Q$ in the XYZ_Q reference frame has the form of:

$$\nabla\vec{V}_Q = \begin{bmatrix} \frac{\partial U_Q}{\partial X_Q} & \frac{\partial U_Q}{\partial Y_Q} & 0 \\ \frac{\partial V_Q}{\partial X_Q} & \frac{\partial V_Q}{\partial Y_Q} & 0 \\ \frac{\partial W_Q}{\partial X_Q} & \frac{\partial W_Q}{\partial Y_Q} & \frac{\partial W_Q}{\partial Z_Q} \end{bmatrix} \quad (1)$$

Another rotation (P rotation) is then applied to rotate the reference frame around the Z_Q -axis and the velocity gradient tensor $\nabla\vec{V}_P$ in the resulting XYZ_P coordinate can be expressed as:

$$\nabla\vec{V}_P = P\nabla\vec{V}_QP^{-1} \quad (2)$$

where

$$P = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The resulting $\partial U_P/\partial Y_P$ under rotation angle θ is the angular velocity at this azimuth angle θ , and could be obtained as:

$$\frac{\partial U_P}{\partial Y_P} |_{\theta} = \alpha \sin(2\theta + \varphi) - \beta \quad (4)$$

with α and β determined by the elements of $\nabla\vec{V}_Q$:

$$\alpha = \frac{1}{2} \sqrt{\left(\frac{\partial V_Q}{\partial Y_Q} - \frac{\partial U_Q}{\partial X_Q}\right)^2 + \left(\frac{\partial V_Q}{\partial X_Q} + \frac{\partial U_Q}{\partial Y_Q}\right)^2} \quad (5)$$

$$\beta = \frac{1}{2} \left(\frac{\partial V_Q}{\partial X_Q} - \frac{\partial U_Q}{\partial Y_Q}\right) \quad (6)$$

New Omega Vortex Identification Method Based on Determined Epsilon

Xiangrui Dong^{1,2}, Yisheng Gao³ and Chaoqun Liu^{2,*}

¹Institute of Energy and Power Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China

²Department of Mathematics, University of Texas at Arlington, Arlington, Vgzcw'9823;, USA

³College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

Abstract: A new Omega (Ω) method with ε determination is introduced to represent the ratio of vorticity square over the sum of vorticity squared and deformation squared, for vortex identification. the advantages of the new Ω method can be summarized as follows: (1) Ω , as a ratio of the vorticity squared over the sum of the vorticity squared and deformation squared, is a normalized and case-independent function which satisfies $\Omega \in [0,1]$; (2) Compared with the other vortex visualization methods, which require a wide threshold to capture the vortex structures, Ω can always be set as 0.52 to capture vortex for different cases and time steps; (3) ε is defined as a function without any adjustment on its coefficient for all cases; (4) The Ω method can capture both strong and weak vortices simultaneously. In addition, Ω is quite robust with no obvious change in vortex visualization.

Keywords: Case-independent, Deformation, Omega method, Vortex identification, Vorticity.

INTRODUCTION

The definition and identification of vortex has been a longstanding issue in fluid dynamics. Robinson *et al.* [1] proposed a rather accurate definition: a vortex exists when instantaneous streamlines mapped onto a plane normal to the vortex core exhibit a roughly circular or spiral pattern, when viewed from a reference frame moving with the center of the vortex core. Several vortex identification methods based on the velocity gradient tensor ∇V have been widely used to investigate the

*Corresponding author Chaoqun Liu: Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA; Tel: +1-8172725151; Fax: +1-8172725802; E-mail: cliu@uta.edu

vortex structures in turbulent flows. Perry and Chong [2] suggested a $\tilde{\Delta}$ -method with an idea that the vortices exist where eigenvalues of velocity gradient tensor $\nabla\mathbf{V}$ are complex, which implies the streamline pattern is spiral or closed viewed from a reference frame moving with the point. This method was further developed by Zhou *et al.* [3] and called λ_{ci} . They suggested employing iso-surfaces of imaginary part of the complex eigenvalue to capture vortices. A famous Q -criterion was introduced by Hunt *et al.* [4], in which an eddy is defined as the region with positive second invariant Q of the velocity gradient tensor. Another well-known scheme is the λ_2 method, introduced by Jeong and Hussain [5]. They suggested the usage of second eigenvalue of the symmetric tensor $\mathbf{S}^2 + \mathbf{\Omega}^2$ trying to capture the pressure minimum in a plane normal to the vortex axis. All these methods have achieved some success. However, a threshold is required case by case and time by time, which means different thresholds will lead to different vortex structures. Zhang *et al.* [6] discussed various vortex identification methods in their study, and pointed out that those identification methods are too sensitive to the chosen threshold, making them inadequate for the quantitative analysis of the vortex. The improper thresholds may be able to only capture strong vortices but lose weak ones.

DEFINITION OF A VORTEX BY NEW OMEGA METHOD

A new vortex identification method, called Omega (Ω), first proposed by Liu *et al.* [7], appears to overcome the above-mentioned weaknesses. Recently, Zhang *et al.* [6] applied this new Ω method into the analysis of the reversible pump turbine and indicated that this new omega method is quite suitable for the analysis of complex flow of hydro-turbines, especially for the unsteady flow cases. Tao *et al.* [8] also utilized Ω to investigate the wake flow from moving bodies in their study, and they pointed out that comparing to other vortex identification methods, the new Ω method has a clear physical meaning and vortex is formed when the vorticity is strong but deformation is weak. Ω method was also used by other researchers [9-11] to compare with the existing vortex identification methods. Thus, the definition of Ω is introduced in the following section.

A parameter Ω is introduced to represent the ratio of vorticity over the whole velocity gradient inside a vortex core. According to Helmholtz velocity decomposition, the velocity gradient tensor $\nabla\mathbf{V}$ can be decomposed into a symmetric tensor and an anti-symmetric tensor,

$$\nabla\mathbf{V} = \frac{1}{2}(\nabla\mathbf{V} + \nabla\mathbf{V}^T) + \frac{1}{2}(\nabla\mathbf{V} - \nabla\mathbf{V}^T) = \mathbf{A} + \mathbf{B} \quad (1)$$

where \mathbf{A} is symmetric part which represents deformation and \mathbf{B} is anti-symmetric part which is related to the whole vorticity. Now the ratio Ω is defined as a ratio of vorticity squared over the sum of vorticity squared and deformation squared, which shows vortex is formed when the vorticity is strong but deformation is weak,

$$\Omega = \frac{\|\mathbf{B}\|_F^2}{\|\mathbf{A}\|_F^2 + \|\mathbf{B}\|_F^2} = \frac{b}{a+b} \quad (2)$$

where $\|\cdot\|_F$ is the Frobenius norm. a and b is given below,

$$a = \text{trace}(\mathbf{A}^T \mathbf{A}) = \sum_{i=1}^3 \sum_{j=1}^3 (\mathbf{A}_{ij}^2) \quad (3)$$

$$b = \text{trace}(\mathbf{B}^T \mathbf{B}) = \sum_{i=1}^3 \sum_{j=1}^3 (\mathbf{B}_{ij}^2) \quad (4)$$

There is no doubt that $\Omega \in [0, 1]$, since both a and b are not negative. In fact vortex is a measurement of fluid stiffness. If $\Omega = 1$, fluid will behave as a solid rotation. If $\Omega = 0.5$, fluid has strong shear without rotation. Fluid is different from solid and is a mixture of vorticity and deformation. $\Omega > 0.5$ represents the region where vorticity overtakes deformation ($b > a$), which is defined as vortex. Although Ω is non-dimensional and satisfies $\Omega \in [0, 1]$, some serious noises (clouds) may appear inside the flow domain if both term a and b in equation (2) are in close proximity to zero due to the systematic computational errors. These noises can be reduced or even removed by introducing a proper positive number, ε , in the denominator of Ω . Therefore, in application, we pick,

$$\Omega = \frac{b}{a+b+\varepsilon} \quad (5)$$

ε is introduced in fact to remove noises caused by the consequence of division by zero. Apparently, as a positive number, ε is dependent upon the dimension of the physical variables and needs to be adjusted to a proper number case by case and time by time. A linear correlation is found between ε and the maximum of $b - a$. The Epsilon ε is defined as a function of $(b - a)_{\max}$, which is a fixed parameter at each time step in each case. In this study, ε is proposed as follows,

$$\varepsilon = 0.001 * (b - a)_{\max} = 0.002 * Q_{\max} \quad (6)$$

It should be noted that equation (6) is an empirical formula based on a large number of test results from different cases. The term $(b - a)_{\max}$ represents the maximum of the difference of vorticity squared and deformation squared, and is easy to obtain as a fixed number at each time step in a certain case. The adjustment of ε in any

Stability Analysis on Shear Flow and Vortices in Late Boundary Layer Transition

Jie Tang*

Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA

Abstract: Turbulence is still an unsolved scientific problem, which has been regarded as “the most important unsolved problem of classical physics”. Liu proposed a new mechanism about turbulence generation and sustenance after decades of research on turbulence and transition. One of them is the transitional flow instability. Liu believes that inside the flow field, shear (dominant in laminar) is unstable while rotation (dominant in turbulence) is relatively stable. This inherent property of flow creates the trend that non-rotational vorticity must transfer to rotational vorticity and causes the flow transition. To verify this new idea, this chapter analyzed the linear stability on two-dimensional shear flow and quasi-rotational flow. Chebyshev collocation spectral method is applied to solve Orr–Sommerfeld equation. Several typical parallel shear flows are tested as the basic-state flows in the equation. The instability of shear flow is demonstrated by the existence of positive eigenvalues associated with disturbance modes (eigenfunctions), *i.e.* the growth of these linear modes. Quasi-rotation flow is considered under cylindrical coordinates. An eigenvalue perturbation equation is derived to study the stability problem with symmetric flows. Shifted Chebyshev polynomial with Gauss collocation points is used to solve the equation. To investigate the stability of vortices in flow transition, a ring-like vortex and a leg-like vortex over time from our Direct Numerical Simulation (DNS) data are tracked. The result shows that, with the development over time, both ring-like vortex and leg-like vortex become more stable as Ω becomes close to 1.

Keywords: Shear flow, Stability analysis, Transition, Turbulence, Vortices.

INTRODUCTION

A Short History Review of Research on Flow Transition and Turbulence Generation

In fluid flow, the process of a laminar flow becoming turbulent is a fundamental scientific phenomenon, known as laminar-turbulent transition. Laminar flow describes the fluid flows in parallel layers, with no disruption between the layers [1]. Turbulent flow is characterized by eddies or small packets of fluid particles

*Corresponding author Jie Tang: Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA; Tel: 8179081296; E-mail: jietanguta@gmail.com

which result in lateral mixing [2]. Laminar-turbulent transition is an extraordinarily complicated process which at present is still far from fully understood. Nevertheless, as the result of many decades of intensive research, classical comprehensive theories of physical mechanisms of the transition phenomenon have been proposed [3-5].

Boundary layer is a very important concept in transition theory. It is a thin layer of viscous fluid close to the solid surface of a wall in contact with a moving stream [6]. The flow velocity varies from zero at the wall up to approximate free stream velocity at the boundary. The fundamental concept of the boundary layer was suggested by L. Prandtl [7] in 1904. Modern research on fluid transition is most often studied in the context of boundary layers due to their ubiquity in real flows and their importance in many fluid-dynamic processes [8].

In a thin boundary layer, the velocity gradient is significant, and consequently the viscous shear stresses defined by is large, where μ is the dynamic viscosity, $u = u(y)$ describes the profile of the boundary layer longitudinal velocity component, y is the normal-to-wall direction. In other words, in a thin boundary layer, laminar flow is dominant with shear layers.

$$\tau = \mu \frac{du}{dy} \quad (1.1)$$

Computation of the boundary layer parameters is based on the solution of equations obtained from the Navier–Stokes equations for viscous fluid motion. Navier-Stokes equations describe the conservation of mass, momentum, and energy.

For boundary-layer flows, two main classes of transition are known [9-11] depending on the character of environmental disturbances. The first of them is usually observed when environmental disturbances are rather small. It is regarded as natural transition and has fundamental and practical importance in problems involving moving vehicles in air and water. The second class of transition, usually called bypass, is observed when high enough levels of environmental perturbations are present.

Classical theory on natural transition can be described by four stages: receptivity, linear instability, non-linear growth and vortex breakdown as shown in Fig. (1-1) [5].

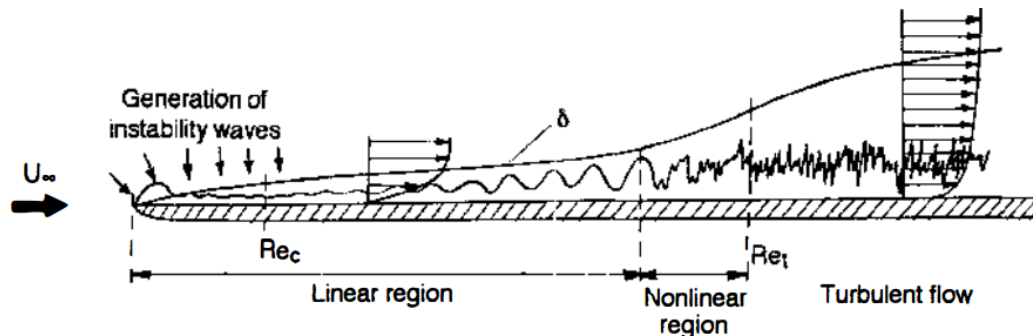


Fig. (1-1). Qualitative sketch of the process of turbulence onset in a boundary layer. δ is the thickness of the boundary layer, Re represents the Reynolds number and U_∞ is the income free stream [5].

The initial stage of the natural transition process is known as the receptivity phase and consists of the transformation of environmental disturbances into small perturbations (*i.e.* instability waves, usually called Tollmien-Schlichting waves) within the boundary layer. This aspect of the transition process was clearly formulated for the first time by Morkovin [10] in 1968. Many experimental and theoretical work of this process appeared in the 1970s [11-16]. Details of the subsequent rapid development of investigations on receptivity can be found in a number of books and review papers [17-21].

The second stage of transition corresponds to the linearly propagation of small-amplitude instability waves in the boundary. This stage is described by linear hydrodynamic stability theory, also called linear stability theory. Tollmien [22] started the research on linear stability theory in 1929. In the following century, it becomes the most developed branch of the transition problem with a lot of research achievements for two-dimensional and three-dimensional flows. For example, Schlichting [23], Lin [24], Herbert [25] and many others.

When the growth of linear instability waves reaches considerable values, the flow enters a phase of three-dimensional nonlinear growth, then the turbulent flow formed (so-called vortex breakdown). They are the last two stages. Although the region of nonlinear growth has been studied for more than half century, there are still many questions unanswered [26-31]. For example, the mechanism of vortices generation and deformation, the formation of turbulence and turbulence coherent structure.

POD and DMD Analysis in Late Flow Transition with Omega Method

Sita Charkrit and Chaoqun Liu*

Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA

Abstract: In this paper, the proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD) are applied to analyze the 3D late transitional flow on the flat plate obtained from direct numerical simulation (DNS). POD is used to find the most persistent spatial structures while DMD is used to find single frequency modes. The Omega method is applied as a vortex identification to visualize vortices with isosurfaces $\Omega = 0.52$. The results in POD and DMD are discussed and compared to show the same and different features such as shapes, amplitudes and time evolutions.

Keywords: Dynamic mode decomposition, Identification method, Modal decomposition, Late flow transition, Omega method, Proper orthogonal decomposition.

INTRODUCTION

In the study of fluid flow structure in computation fluid dynamics (CFD), the modal decomposition is a useful tool to extract the whole structure into coherent structures in different features such as energy content, mode shape, amplitude and frequency since a complex flow structure often consists of a combination of coherent structure. The two popular modal decomposition methods, *i.e.*, the proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD), are presented in this paper. The POD and DMD have been widely applied to explore the complex flow fields since they can be used to decouple the spatial and temporal applications. These two methods help in further understanding of fundamental fluid processes since they can examine the dominant and coherent structures in fluid flows.

The POD is one of the most widely used techniques to analyze fluid flows. There are two versions of POD technique. The original POD, proposed by Lumley [1] in

*Corresponding author Chaoqun Liu: Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA; Tel: +1-8172725151; Fax: +1-8172725802; E-mail: cliu@uta.edu;

1967, is used to investigate the turbulent flow. The other version is called snapshot POD, which was introduced by Sirovich [2] in 1987. The snapshot POD is applied in order to optimize the computation. In POD, the flow structure is decomposed into orthogonal mode ranking by their kinetic energy content. In recent years, there have been many applications about POD in many fields of fluid dynamics. For examples, POD was used to study the turbulent pipe flow [3, 4]. In some studies [5, 6], POD was applied to identify turbulent discontinuous and nonlinear flows. A mixing layer downstream on a thick splitter plate obtain form DNS was analyzed by POD in a study [7]. The flow structure in transition stage has been analyzed by POD. For example, POD was applied to analyze coherent structures in pipe flow [8, 9] and a transitional boundary layer with and without control [10]. POD was also used to investigate asymmetric structures of flow on the flat plate [11]. The vortex structure in MVG wake was examined [12]. The entropy generation in a laminar separation boundary layer was analyzed by POD [13].

The other popular decomposition method used in this paper is DMD, which was first introduced by Schmid [14] in 2010 to extract the dynamic features by finding the relationship between each time step. Each coherent structure has a single feature in temporal mode. This method relates to Koopman operator explained in the paper of Rowley [15]. Many researchers applied DMD and compared both POD and DMD to analyze the flows in CFD. For instance, Premaratne and Hu [16] studied turbine wake characteristics by DMD. Alina and Navon [17] used DMD in shallow water and a swirling flow problem. Mohan and Gaitondey [18] worked on analysis of deep dynamic stall on a plunging airfoil by DMD. Both methods were compared in terms of identification of multi-dominant coherent structures and high-order harmonics buried in fluid flow in a study [19]. The POD and DMD on LES of subsonic jets were presented in a study [20].

The purpose of this work is to apply POD and DMD to investigate the complex flow on the flat plate in late transition stage obtained from direct numerical simulation (DNS) and to better understand its three-dimensional structure. Moreover, the vortex identification method called the omega method is introduced to visualize the vortex structures of spatial modes. This paper is organized as follows. In section 2, the detail of DNS data used in this paper is introduced. In section 3, the concept of Omega method is described. In section 4, descriptions of POD and DMD are explained. In addition, the analyses in both POD and DMD are discussed and compared in section 4.

CASE SETUP AND CODE VALIDATION

The case and code validation are presented in studies [21, 22]. The computational domain is demonstrated in Figs. (1 and 2). The grid level is $1920 \times 128 \times 241$, representing the number of grids in streamwise (x), spanwise (y), and wall normal (z) directions. The grid is stretched in the normal direction and uniform in the streamwise and spanwise directions. The length of the first grid interval in the normal direction at the entrance is found to be 0.43 in wall units ($Z^+ = 0.43$). The flow parameters, including Mach number, Reynolds number, etc. are listed in Table 1. The DNS code, DNSUTA, has been validated by NASA Langley and UTA researchers carefully to make sure the DNS results are correct. For more detail about case setup and code validation, see [21, 22].

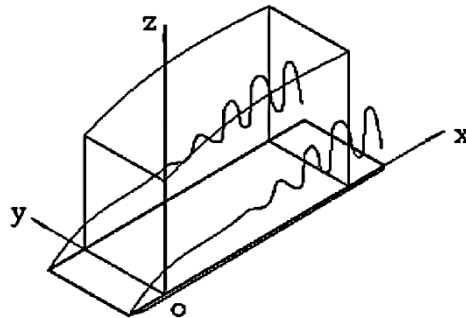


Fig. (1). Computation domain.

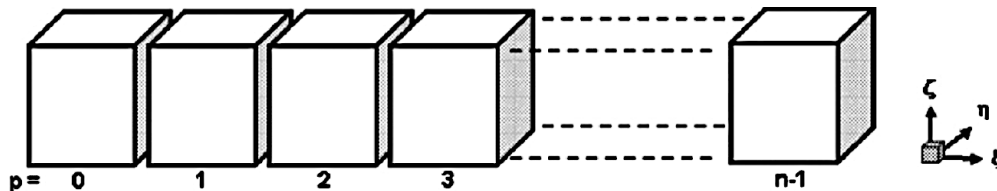


Fig. (2). Domain decomposition along the streamwise direction in the computational space.

Table 1. DNS parameters.

M_∞	Re	x_{in}	Lx	Ly	Lz_{in}
0.5	1000	$300.79\delta_{in}$	$798.03\delta_{in}$	$22\delta_{in}$	$40\delta_{in}$

Comparison of Liutex and Eigenvalue-based Vortex Identification Criteria for Compressible Flows

Yisheng Gao¹ and Chaoqun Liu^{2,*}

¹College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

²Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA

Abstract: Currently, the Q criterion, the Δ criterion and the λ_{ci} criterion are representative among the most widely used vortex identification criteria. These criteria can be categorized as eigenvalue-based criteria since they are exclusively determined by the eigenvalues or invariants of the velocity gradient tensor. However, these criteria are not always satisfactory and suffer from several defects, such as inadequacy of identifying the rotational axis and contamination by shearing. Recently, a novel concept of Liutex (previously named Rortex), including the scalar, vector and tensor form, was proposed to overcome the issues associated with the eigenvalue-based criteria. In the present paper, the comparison of Liutex and two eigenvalue-based criteria, namely the λ_{ci} criterion and the Q_D criterion, a modification of the Q criterion, is performed to assess these methods for compressible flows. According to the analysis of the deviatoric part of the velocity gradient tensor, all the scalar, vector and tensor forms of Liutex are valid for compressible flows without any modification, while two eigenvalue-based criteria, though applicable to compressible flows, will tend to be severely contaminated by shearing as for incompressible flows. Vortical structures induced by supersonic microramp vortex generator (MVG) at Mach 2.5 are examined to confirm the validity and superiority of Liutex for compressible flows.

Keywords: Compressible flows, Liutex/Rortex, Vortex identification, Vortex structures, Turbulence.

INTRODUCTION

Vortical structures, or formally referred to as coherent structures [1-3], are widely regarded as one of the most principal features of transitional and turbulent flows and serve a crucial role in turbulence generation and sustenance. During the last

*Corresponding author Chaoqun Liu: Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA; Tel: +1-8172725151; Fax: +1-8172725802; E-mail: cliu@uta.edu

several decades, some typical elementary structures, such as hairpin vortices [4-7], vortex braids [8-9] and quasi-streamwise vortices [1, 10-11] *etc.*, have been identified and intensively studied. Unfortunately, despite the ubiquity and significance of such spatially and temporally coherent vortical motions in transitional and turbulent flows, an unambiguous and rigorous definition of the vortex is yet to be achieved (without doubt, the concept of coherent structures is also somewhat ambiguous). The lack of a well-accepted definition has been perceived as one of the chief obstacles hindering the thorough understanding of vortical structures and the mechanism of turbulence generation and sustenance [12-13].

The classic vortex dynamics generally associate the vortex with the vorticity since the vorticity is mathematically mathewell-defined (the curl of the velocity vector). For example, Saffman [14] regards a vortex as a “finite volume of vorticity immersed in irrotational fluid.” Nitsche [15] suggests that “a vortex is commonly associated with the rotational motion of fluid around a common centerline. It is defined by the vorticity in the fluid, which measures the rate of local fluid rotation.” Wu *et al.* [16] declares that “a vortex is a connected region with high concentration of vorticity compared with its surrounding”. However, the usage of vorticity-based methods is not always satisfactory, especially for turbulence flows. The prominent issue is that the vorticity cannot distinguish between a region with real vortical motion and a shear layer region. A typical example is the Blasius boundary layer where the magnitude of the vorticity is relatively large in the near-wall regions, but no real rotational motion will be observed. In addition, the vorticity can be somewhat misaligned with the direction of vortical structures [17]. It is not uncommon that the vorticity vector angle will be significantly larger than the local inclination of the vortex structure over almost the entire length of the quasi-streamwise vortex in the channel flow [18]. And the association between regions of strong vorticity and actual vortices can be rather weak in the turbulent boundary layer, especially in the near wall region [19]. In fact, it has been found by Wang *et al.* [20] that in the near-wall regions, the magnitude of the vorticity will be dramatically reduced along vorticity lines entering the vortex core.

To overcome the issues associated with vorticity-based methods for the identification and visualization of vortex structures, numerous vortex identification methods, including Eulerian non-local methods, Eulerian local region-type methods, Eulerian local line-type methods and Lagrangian methods have been proposed during the last three decades [21]. Most of the currently popular vortex identification criteria belong to Eulerian local region-type methods. Most of these criteria are exclusively determined by the eigenvalues or invariants of the velocity

gradient tensor and thereby can be categorized as eigenvalue-based criteria. For example, the Q criterion [22] defines vortices as the regions where the vorticity magnitude prevails over the strain-rate magnitude. The Δ criterion [23-25] identifies the region where the velocity gradient tensor has complex eigenvalues by the discriminant of the characteristic equation. The λ_{ci} criterion [18] is an extension of the Δ criterion, which uses the (positive) imaginary part of the complex eigenvalue to determine the swirling strength. The λ_2 criterion [26] is a dynamical definition and based on the second-largest eigenvalue of $\mathbf{S}^2 + \mathbf{\Omega}^2$ (\mathbf{S} and $\mathbf{\Omega}$ represent the symmetric and the antisymmetric parts of the velocity gradient tensor, respectively). Also, it should be noted that λ_2 can be exclusively determined by the eigenvalues only if the eigenvectors of the velocity gradient tensor are orthonormal). These methods can discriminate against shear layers, offering more detectable vortex structures than vorticity-based methods. Nevertheless, there exist several shortcomings of the eigenvalue-based criteria. One is the user-specified threshold. On one hand, the threshold is case-dependent and cannot be determined beforehand. On the other hand, no one can confirm if a threshold is appropriate or not, because different thresholds will present different vortical structures [27]. Therefore, the educed structures obtained from these criteria should be interpreted with care. As a remedy, relative values can be employed to avoid the usage of case-related thresholds, and one such example is the Omega method [28-30]. The Omega method (Ω) is originated from an idea that the vortex is a region where the vorticity overtakes the deformation and Ω is defined as a ratio of vorticity tensor norm squared over the sum of vorticity tensor norm squared and deformation tensor norm squared. Thus, the Omega method is robust to moderate threshold change and capable to capture both strong and weak vortices simultaneously. Another obvious drawback is the inadequacy of identifying the swirl axis or orientation. The existing eigenvalue-based criteria are scalar-valued criteria, which means that only iso-surfaces will be obtained, and no rotational axis can be identified by these criteria. In addition, eigenvalue-based criteria are prone to contamination by shearing [31-33]. The problem of contamination by shear motivates Kolář [34] to propose a triple decomposition from which the residual vorticity can be obtained after the extraction of an effective pure shearing motion and represents a direct and accurate measure of the pure rigid-body rotation of a fluid element. However, a basic reference frame (BRF) should be first determined and searching for BRF in 3D cases is very challenging, which severely limits the applicability of the triple decomposition. The extensive overview of the currently available vortex identification methods has been provided by several review papers [21, 35-36].

Observation of Coherent Structures of Low Reynolds Number Turbulent Boundary Layer by DNS and Experiment

Panpan Yan^{1,2}, Chaoqun Liu^{2,*}, Yanang Guo³ and Xiaoshu Cai³

¹Shenyang Aircraft Design and Research Institute, Aviation Industry of China, Shenyang 110035, China

²Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA

³Institute of Particle and Two-phase Flow Measurement, University of Shanghai for Science and Technology, Shanghai 200093, China

Abstract: An elaborate direct numerical simulation (DNS) for late boundary layer transition has been conducted. The DNS results are qualitatively compared with a new Lagrangian property experimental technique named the moving single-frame and long-exposure (MSFLE) imaging method to obtain a deeper understanding on the coherent structures of a transitional and turbulent boundary layer at low Reynolds number. Multilevel vortex structures are clearly observed by both experiment and DNS. This study found that there are multilevel co-rotating vortices, showing how energy is transported from the main flow to the bottom of the boundary layer and how the streaks or white-black strips are formed. The results also show that the lower level vortices cannot simply be produced by the upper-level vortex induction. There are multilevel hairpin vortex ejections and sweeps inside the boundary layer of the transitional and low Reynolds number turbulent flows. The ejections and sweeps are much stronger around the hairpin legs and necks than those in the ring area. This clearly shows that the ring-like vortices are the production of the strong vortex neck rotation. In conjunction with ejections and sweeps, a lot of strong shear layers are produced. Because fluid cannot tolerate the strong shear, the shear layer must turn to rotation and form many vortices. This would help reveal the mechanism of multilevel vortices and turbulence generation. Although the upper-level vortices could be larger than the lower ones, the lower level vortices sometimes have the same size as the neighboring upper-level vortices. The vortex cascade and large vortex breakdown are not observed by either DNS or experiment.

Keywords: DNS, Ejection and sweep, Hairpin vortices, Multilevel vortices, experiment, Turbulent boundary layer.

*Corresponding author Chaoqun Liu: Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA; Tel: +1-8172725151; Fax: +1-8172725802; E-mail: cliu@uta.edu

Chaoqun Liu and Yisheng Gao (Eds.)
All rights reserved-©2020 Bentham Science Publishers

INTRODUCTION

Wall-bounded turbulent flow is a fundamental scientific topic and has received a lot of attention for over a century due to its importance to both scientific research and many industrial applications such as transition control and drag reduction in aerospace engineering [1, 2]. Many efforts have been made for turbulent boundary layer flow [3-5]. However, the mechanism of wall-bounded turbulent flow still remains a puzzle. Nearly four decades ago, Falco [6] gave a well-known visualization of a low Reynolds number turbulent boundary layer that illustrates several known types of coherent structures. Theodorsen [7] had identified the horseshoe vortex in wall-bounded turbulent flow by experimental observation. Robinson [8] believed that the coherent structure is responsible for the production and dissipation of turbulence in a boundary layer, and the study of turbulence structures is of fundamental importance to understand and control the turbulent boundary layer. Kline *et al.* [9] found that the long streamwise streaks of hydrogen bubbles in the near-wall region by experiment and that the spanwise spacing of these streaks were about 100 wall units. They believed that the instability of these streaks plays a vital role in turbulence generation. Working with Kline, Robinson [10] gave a summary of the structures. He observed that quasi-streamwise vortices are located close to the wall, arches or horseshoe vortices in the wake region, and there is a mixture of quasi-streamwise vortices and arches in the logarithmic layer. Liu *et al.* [11-13], Rist *et al.* [14], and Wu and Moin [15] obtained transitional and low Reynolds number turbulent flow with a forest of hairpin vortices through DNS while Eitel-Amor found that the hairpin vortex structures are not a feature in fully developed turbulence [38]. Scaling theories coupled with the notion of coherently organized motions were first proposed by Townsend [16, 17] and advanced by Perry and Chong [18]. They gave a model of individual hairpin vortices scattered randomly in the flow and found that the vortices are statistically independent of each other. They imagined the wall layer as a forest of single layer hairpin vortices which can be modeled with simplified shapes in a hierarchy of scales above the wall. Yan *et al.* [19] found that the hairpin vortex is a combination of the Λ -vortex roots and vortex rings. Λ -vortex roots and vortex rings are formed separately and independently, and the mechanism of the Λ -vortex self-deformation to hairpin vortex does not exist. Liu *et al.* [20, 21] believed that turbulence is an inherent property of fluid flow, although the external disturbance is needed. The nature of turbulence generation is that fluids, away from the wall, cannot tolerate the shear, and shear must transfer to rotation. Therefore, we believe shear layer instability is the mother of turbulence. However, physics of turbulence is very complex and a number of articles have given a variety of theories [22-27]. Adrian [28] believed

hairpins could be auto-generated to form packets that populate a significant fraction of the boundary layer, and he addressed the important role of hairpin vortex ejections and sweeps. Marusic's review paper [3] mentioned that there is still a dichotomy on whether the hairpin vortex exists or not. Schoppa and Hussain [29] thought complete hairpin vortices do not exist in wall bounded turbulence. Therefore, more work must be conducted to get a deep understanding about the vortex structure of the low Reynolds number turbulent boundary layer.

A new Lagrangian property experiment technique, which is named as the moving single-frame and long-exposure (MSFLE) imaging method, is proposed for measuring the coherent structure in a turbulent boundary layer. Meanwhile, a high order direct numerical simulation with nearly 60 million grid points with 400,000 time steps is carried out in order to get a deep understanding of the coherent structure for the low Reynolds number turbulent boundary layer flow.

This chapter is organized in the following way. In Section 2, the DNS case setup and validation and the experiment setup are described. Section 3 provides the comparison of the DNS results with the experiment and describes our new DNS observations. Finally, some conclusions are presented in Section 4.

CASE SET UP

DNS Case Setup and Code Validation

Fig. (1) shows the computational domain where x , y , z represent the streamwise, spanwise and wall-normal directions respectively. There are $1920 \times 128 \times 241$ grid points in the computation domain. The points are distributed uniformly in the streamwise and spanwise directions and stretched in the wall normal direction to ensure the grid has enough resolution to capture all small length scales. The first grid interval is set to 0.43 in wall units ($z^+ = 0.43$) in the normal direction. As shown in Fig. (1b), the whole domain is decomposed in the x -direction to implement parallel computation by using the Message Passing Interface (MPI) technique. Table I gives the details of the flow conditions including Mach number, Reynolds number, *etc.* Here δ_{in} is the inflow displacement thickness, and other parameters are non-dimensionalized by δ_{in} as reference length. L_x and L_y are the length of computational domain in the x and y directions, and $L_{z_{in}}$ is the height of the inlet. x_{in} is the distance between the leading edge and inlet. T_w and T_∞ represent the wall temperature and freestream temperature. The Reynolds number of the inlet is defined as $Re = \rho_\infty U_\infty \delta_{in} / \mu_\infty$.

Direct Numerical Simulation of Incompressible Flow in a Channel with Rib Structures

Ting Yu, Duo Wang, Heng Li, Hongyi Xu*

Aeronautics and Astronautics Department, Fudan University, Shanghai, PR China

Abstract: This chapter applied the state-of-the-art flow simulation method, *i.e.* the Direct Numerical Simulation (DNS), and strongly coupled the DNS with the heat-transfer governing equation to solve the thermal turbulence in both 2-dimensional (2D) and 3-dimensional (3D) channels with the rib tabulator structures. An innovative approach was applied to the simulations. The surface roughness effects of the cooling vane were directly tackled by including the roughness geometry in the DNS and applying the immersed-boundary method to handle the geometry complexities due to the roughness. Two inlet conditions, namely the uniform flow and full-developed turbulence, were applied at the inflow surface of the channel. Half height of the channel was used as the scale length. The Prandtl (Pr) number was set at $Pr = 0.7$. Five Reynolds (Re) number of 1000, 2500, 5000, 7500 and 10000 were calculated in the 2D cases and the Reynolds numbers of 2500 and 5000 were applied in 3D cases where a periodical condition was applied in the span-wise direction. Additionally, Reynolds number of 10000 was set in the case with roughened surface. The stream-wise velocity, turbulence intensity, and the Nusselt (Nu) number were analyzed. Results in the 2D and 3D cases presented a significant difference on flow structure. At the same time, with increasing Reynolds number, the length of recirculation zone and the enhancement of heat transfer showed a decreasing trend.

Keywords: Direct Numerical Simulation, Heat Transfer, Rib Tabulator, Roughness, Rortex.

INTRODUCTION

Surface structures, such as pin-fins, ribs and dimples, are effective cooling methods in modern aero-engine design. These structures are commonly used to enhance heat exchange by increasing the heat-exchange surfaces and the turbulence level in flow.

*Corresponding author Hongyi Xu: Aeronautics and Astronautics Department, Fudan University, Shanghai, PR China; Tel: 86-021-55665062; Email: hongyi_xu@fudan.edu.cn

The rib tabulator is one of the important structures applied in aero-engine components, for example, the combustion ducts and the internal cooling channel of turbine blade. Therefore, it is important to study the flow around a rib and to analyze its effects on the flow field and closely-related heat transfer process. As well known, the flow separation occurs in front of a rib and then reattachment can be found at the bottom wall after rib. Many researches have been conducted to investigate the interactions of these structures with the strongly-coupled flow and heat transfer effects.

The experiments from Eaton *et al.* [1], Chun *et al.* [2], and Liu *et al.* [3] showed the strong unsteadiness of these flow structures. Bergeles and Athanassiadis [4] found that the structures included the combination of complex phenomena, such as flow separation and reattachment. Liu *et al.* [5] provided their experiment results about the flow over a rib, which tried to elucidate the unsteady behaviors of the separation and reattachment over a 2D square rib.

As the computational capabilities grew, more numerical results of flow over the rib were given. Leonardi *et al.* [6] presented the periodic flow in channel with square ribs, the relationship between the flow structure and the ratio of rib height to gap of two ribs was studied. LES method was applied by Cui *et al.* [7], which focused on two types of flows between the ribs. Matsubara *et al.* [8] and Miura *et al.* [9] simulated the flow structure and heat transfer of 3D rib with different Reynolds numbers and aspect ratios.

The current chapter numerically analyzed the flow and heat transfer processes in both 2D and 3D single-rib geometries. Ten cases were investigated including seven 2D cases and three 3D cases. The Reynold numbers (Re) of 2500, 5000 and 7500 were applied in the 2D cases, and the Re of 2500 and 5000 were applied in the 3D cases. Fully-developed turbulent inlet conditions were used in the case with Re=2500. Additionally, most previous work focused on the smooth boundary wall. Since these studies were not able to reflect the effects of the roughness, a 2D case with roughened boundary surfaces was simulated in this study. This case had six dis-symmetric ribs. The Reynolds number of 10000 was applied. Fully-developed turbulent inlet condition was used. X-ray scanning technologies were applied to obtain the geometric contours of the roughness on the cooling-vane surface. An immersed boundary (IB) method coupled with the adaptive mesh refinement technology was implemented to tackle the extremely complex geometries of the surface roughness [10]. In all cases, Prandtl number (Pr) was fixed at 0.7 in these studies to represent air flow. The state-of-the-art simulation method in fluid mechanics, namely Direct Numerical Simulation (DNS), was applied to obtain the

flow solution, and multi-grid method was applied to increase the solution efficiency. In order to capture the turbulence and the associated vortical structure, a new vortex identification method was applied, namely, newly defined Rortex proposed by Liu *et al.* [10].

The data are useful and helpful in pushing the nowadays frontiers in each of these relevant science fields, such as the fluid mechanics, heat transfer as well as computations. Also, the database will help transforming the design and manufacture of the turbine blade from the current one based on engineering experiments and empirical data to the future one intelligently-guided by reliable simulation databank.

MATHEMATICAL-PHYSICAL MODELS AND METHODS

Governing Equations

The mathematical models included the conservations of mass, momentum and energy, which are written in the non-dimensional form under Cartesian coordinators:

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0 \quad (1)$$

$$\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial \bar{u}_j \bar{u}_i}{\partial x_j} = -\frac{\partial p}{\partial x_j} + \frac{1}{Re} \frac{\partial}{\partial x_j} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad (2)$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial \bar{u}_j \theta}{\partial x_j} = \frac{1}{Re Pr} \frac{\partial^2 \theta}{\partial x_j \partial x_j} + \frac{Ec}{Re} \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad (3)$$

where the subscripts $i, j, k = 1, 2, 3$ represented the three spatial directions x, y, z , with x being the stream-wise and y, z being the cross-streamwise directions; the repeated subscripts followed the Einstein summation; \bar{u}_j and θ are the non-dimension forms of velocities and the temperature, respectively, and p is the static pressure. The criterion numbers of Re , Pr and Ec are defined as $Re = \rho U h / \mu$, $Pr = \mu C_p / k$ and $Ec = U^2 / C_p (Th - Tc)$ with ρ, μ, C_p being the fluid density, molecular viscosity and specific heat capacity, U being the fluid incoming velocity, Th and Tc being the hot(wall) and cool(incoming) temperatures, respectively, and h being the height of rib as seen in Fig. (2).

Vortex and Flow Structure inside Hydroturbines

Yuning Zhang^{1*}, Yuning Zhang^{2,3,**}

¹Key Laboratory of Power Station Energy Transfer Conversion and System (Ministry of Education), School of Energy, Power and Mechanical Engineering, North China Electric Power University, Beijing, China

²College of Mechanical and Transportation Engineering, China University of Petroleum-Beijing, Beijing 102249, China

³Beijing Key Laboratory of Process Fluid Filtration and Separation, China University of Petroleum-Beijing, Beijing 102249, China

Abstract: In this chapter, various kinds of vortex in the hydroturbines are briefly introduced with a focus on the swirling vortex rope in Francis turbine and the vortex in the vaneless space of the reversible pump turbine. The vortex induced pressure fluctuation and vibrations are initially demonstrated based on the on-site measurement in the prototype power stations. Then, influences of the vortex in the upstream on the flow status in the downstream are discussed. Finally, detailed characteristics of the swirling vortex in the draft tube section of the hydroturbines are demonstrated based on the plenty of examples together with the aid of a quantitative swirl number analysis.

Keywords: Hydroturbines, Pressure fluctuations, Vibrations, Vortex, Vortex rope.

A SUMMARY OF TYPES OF VORTEX IN HYDROTURBINES

There are various kinds of hydroturbines including Francis turbine, reversible pump turbine, Kaplan turbine, Pelton turbine etc. For a complete review of the flow-induced vortex in hydroturbines together with the vortex identification methods and their applications in hydroturbines, readers are referred to Zhang *et al.* [1]. For reference books relating with the associated pressure fluctuations and vibrations in hydroturbines, readers are referred to Wu *et al.* [2] and Dörfler *et al.* [3]. Fig. (2) of Chen *et al.* [4] shows the Francis turbine of Three Gorges hydro power station, which was the largest hydroturbines (in terms of the electricity generation capacity)

*Corresponding author Yuning Zhang: Key Laboratory of Power Station Energy Transfer Conversion and System (Ministry of Education), School of Energy, Power and Mechanical Engineering, North China Electric Power University, Beijing, China; Tel: 86 (0)1061773958; E-mail: y.zhang@ncepu.edu.cn

Chaoqun Liu and Yisheng Gao (Eds.)
All rights reserved-©2020 Bentham Science Publishers

of the given type in the world when it was commissioned. In the above figure, the detailed components of the aforementioned hydroturbines are also shown. Basically speaking, the fluid passing components of a typical Francis turbine include the spiral casing, the stationary guide vane, the wicket gate (also named as the adjustable guide vane, with the function of the water flux control), runner (also named as impeller), draft tube (including cone and elbow sections respectively). For other types of the hydroturbines, the basic structures and principles are quite similar. In the present chapter, two paramount types of the hydroturbines are discussed: the aforementioned Francis turbine and the reversible (Francis type) pump turbines of a pumped hydro energy storage power plant.

For different kinds of hydroturbines, the dominant types of vortex are quite different. For example, for the Francis turbine, the dominant vortex is usually the swirling vortex rope in the draft tube (referring to the figure 13 of Chen *et al.* [4]). When the turbine is operated in the partial loads (off-design conditions), this kind of vortex is quite strong with significant rotating momentum. Meanwhile, prominent pressure fluctuation will be also generated by the swirling vortex rope with the propagation to the upstream or the downstream also possibly leading to the vibrations of the whole unit. Generally speaking, the rotational speed of this kind of vortex is rather slow and is far less than the rotational speed of the runner. Other prominent vortex also exists in the channel of runner (termed as the channel vortex).

For the reversible pump turbine, as shown in figure 16 of Zhang *et al.* [5], the dominant vortex is the vortex shown in the vaneless space between the wicket gate and the runner (also named as the impeller). For more details about the reversible pump turbine, readers are referred to Zhang *et al.* [5]. In the vaneless space, as shown in figure 12 of Hasmatuchi *et al.* [6], there are prominent backflows during the low discharge mode. Comparing with the best design point (BEP), the flow status is much distorted in the low discharge working conditions, leading to the strong channel blockage of the flow. For the reversible pump turbine, the vortex rope in the draft tube also exists but is no longer the primary source of the vortex as those shown in the Francis turbine.

For the Kaplan turbine, the tip vortex between the runner and the hub is very significant, leading to serious damage on the fluid components. Generally speaking, because the tip is quite small according to the design (for the enhancement of the efficiency), cavitation usually occurs near the tip. Hence, as shown in figure 14 of Motycak *et al.* [7], the tip leakage vortex is often accompanied by the cavitation bubbles. Because the force and micro-jet generated by the bubble final collapse are quite prominent, serious damage could be observed on the runner edges.

Other types of vortex also include the inter-blade vortex in the runner, the Kármán vortex in the wake flow of the vanes and the cavitating vortex.

The relationships between the vortex and the turbulence could be illustrated as follows. On the one hand, the vortex phenomenon could induce the generations of the turbulence together with associated structures. For example, during the rotating stall status in the reversible pump turbine, the vortex generated in the impeller channels could block the fluid passing through the component. With the increment of the fluid distortion, a strong turbulence flow will be finally demonstrated with prominent pressure fluctuations. On the other hand, the existing turbulence will lead to the intensive generations of the vortex. For example, in the low load condition, the turbulence will be induced inside the guide vane channels due to the large incidence angle. Then, many small vortex will appear especially near the pressure side of the vanes.

THE EFFECTS OF VORTEX ON PRESSURE FLUCTUATION

In this section, various kinds of negative effects of the vortex on the hydroturbine performances will be introduced including the pressure fluctuation together with its characteristic frequency.

Fig. (2) by Zhang *et al.* [8] shows the non-dimensional peak-to-peak values of pressure fluctuation versus load variations (from 25.41% to 96.82% of the full load) at four monitoring points (referring to the ref. [8] for the positions). The identified three zones could be summarized as follows:

Zone 1: This zone corresponds to the conditions of the low partial load. The pressure fluctuation is mainly generated by the vortex flow in the vaneless space. Fig. (4) of Zhang *et al.* [8] further shows the cascade plot of frequency spectrums measured at the vaneless space. The dominant frequency is $9f_n$ (also termed as blade passing frequency, with f_n representing the impeller rotational frequency), which is generated by vortex induced by the rotating impeller.

Zone 2: This zone corresponds to the conditions of the medium partial load. The pressure fluctuation is mainly generated by the swirling vortex rope in the draft tube cone section. Figure 5 of Zhang *et al.* [8] further shows the cascade plot of frequency spectrums measured at the draft tube cone section. The dominant frequency of this kind of vortex is less than f_n , which is generated by the vortex induced by the swirling vortex rope.

A Comparative Study of Compressible Turbulent Flows Between Thermally and Calorically Perfect Gases

Xiaoping Chen*

National-Provincial Joint Engineering Laboratory for Fluid Transmission System Technology, Zhejiang Sci-Tech University, Hangzhou, Zhejiang 310018, China

Abstract: In this chapter, direct numerical simulations (DNSs) of compressible turbulent flows for thermally perfect gas (TPG) and calorically perfect gas (CPG), including two wall temperature of 298.15K (low temperature condition) and 596.30K (high temperature condition), are performed to investigate the influence of a gas model on the turbulent statistics and flow structures. The results show that the influence of TPG is negligible and remarkable for low and high-temperature conditions, respectively. Many of the statistical characteristics used to express low-temperature conditions for CPG still can be applied to high-temperature conditions for TPG. The smaller the influence of the gas model on the mean and fluctuating velocity, the stronger the Reynolds analogy. The static temperature for TPG is smaller than that for CPG, whereas an inverse trend is found for turbulent and root square mean Mach numbers. Omega could capture both strong and weak vortices simultaneously for compressible flow, even TPG, which is difficult from Q. Compared to the results of CPG, the vortex structure becomes smaller, sharper and more chaotic considering TPG.

Keywords: Calorically perfect gas, Compressible flow, Direct numerical simulation, Thermally perfect gas, Vortex structure.

INTRODUCTION

Compressible flow has critical importance in gas dynamics and engineering application. The heat flux and frictional resistance in turbulent flow are obviously higher than that for laminar flow. For compressible turbulent flows, the near wall

*Corresponding author **Xiaoping Chen:** National-Provincial Joint Engineering Laboratory for Fluid Transmission System Technology, Zhejiang Sci-Tech University, Hangzhou, Zhejiang, 310018, China; Tel+86 18626878196; E-mail: chenxp@zstu.edu.cn

temperature may be higher than 500K. Here, it will be changed the thermodynamic environment, which means that the calorically perfect gas (CPG) no longer appropriate and the thermally perfect gas (TPG) [1] should be considered. Therefore, assumed to be TPG, the behavior of turbulent statistical characteristics and flow structures in compressible turbulent flow urgently needs to be investigated.

Direct numerical simulation (DNS) does not involve any modeling errors and solves the Navier-Stokes equations directly [2], which is a powerful tool to simulate the turbulent flows, including channel flows [3-10], boundary layers [11-15], compression ramps [16, 17], and blunt cones [18-20]. A large number of DNS study is reported to investigate the statistical characteristics in the compressible turbulent flows, such as strong Reynolds analogy (SRA) and Morkovin's hypothesis. The Morkovin hypothesis [21] denotes that, when the Mach number isn't very large, the relationship of turbulent statistical characteristics between incompressible and compressible flows can be connected by mean variations of fluid properties. Huang *et al.* [5] investigated compressibility effects based on the DNS data of compressible turbulent channel flow performed by Coleman *et al.* [4]. Lechner *et al.* [6] and Foysi *et al.* [7] studied compressible effects and turbulence scaling in the compressible turbulent channel flow. The differences in turbulence statistics near both the adiabatic and isothermal walls were reported by Mamano *et al.* [8] and Morinishi *et al.* [9]. In addition, to some extent, the energy equation can be explained by SRA—a relationship between velocity and temperature fluctuations. Morkovin [21] firstly proposed SRA in 1963. Then, researches proposed several modified SRA, such as ESRA was introduced by Cebeci *et al.* [22], GSRA was introduced by Gaviglio [23], RSRA was introduced by Rubesin [24], HSRA was introduced by Huang *et al.* [5] and GHSRA was introduced by Duan *et al.* [25]. So far, many studies of supersonic turbulent boundary layer flows have been performed to check the validity of SRA and Morkovin's hypothesis by the DNS data. For example, Duan and Martin [13, 14] assessed the influence of Mach number and wall temperatures on the SRA and Morkovin's hypothesis. Liang and Li [15] investigated many turbulent characteristics, such as Walz equation, mean and fluctuating velocity, compressibility effect and SRA

For the compressible turbulent flow, the behaviour of instantaneous vortex structures is also very important. Many criterions for identify turbulent structures have been introduced in many literatures, such as $\tilde{\lambda}$ -criterion [26-27], q -criterion [28], λ_2 -criterion [29], λ_{ci} -criterion [30], the Ω criterion [31, 32], and Rortex [33]. Based on the DNS results of boundary layer, these criterions were evaluated by Sayadi *et al.* [34] and Pierce *et al.* [35]. They found that these criterion produce the

same images as chosen the threshold of $10^{-3} \left(\frac{\partial u}{\partial y} \right)_w^2$. Coleman *et al.* [4] illustrated that near-wall streak in the stream-wise direction become more coherent as the Mach number increased. They also argued that the weakly compressible hypothesis modifies the near-wall structures little. Morinishi *et al.* [9] showed that the thermal wall boundary condition has very little effect on the near-wall streaks in semi-local units. For the supersonic boundary layers, Lagha *et al.* [36, 37] studied the influence of Mach number on the near-wall structure. Guo and Adams [38] illustrated that near-wall streak structures are larger than that of incompressible flow. Most previous studies have been carried out on calorically perfect gas (CPG), a good understanding is gained due to these works.

So far, many DNS results for TPG were performed based on DNS. Marxen *et al.* [39, 40] studied the effects of gas model on the stability of hypersonic boundary layer. In the hypersonic boundary layer, Jia and Cao [41] investigated the behavior of stability of flat plate under different variable specific heat. Recently, taking temporally evolving compressible turbulent flows as a research object, Chen *et al.* [42-46] not only performed several DNS, but also investigated the similarities and differences between TPG and CPG. However, the turbulent statistics and flow structures in compressible turbulent flows for TPG have not been studied clearly, especially for vortex structures, SRA and Morkovin's hypothesis.

In the present study, based on the DNS database, we focus on the behavior of turbulent statistical characteristics and flow structures in the compressible turbulent channel flow for TPG, and discuss how they depend on the wall temperature.

GOVERNING EQUATIONS

The governing equations are the time-dependent three-dimensional Navier-Stokes equations in non-dimensional form, which can be described as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0 \quad (1)$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial}{\partial x_j} \left(\rho u_i u_j + p \delta_{ij} - \frac{1}{\text{Re}} \sigma_{ij} \right) = \rho f_i \quad (2)$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j} \left[(E + p) u_j - \frac{1}{\text{Re}} (u_i \sigma_{ij} + q_j) \right] = \rho f_i u_i \quad (3)$$

The Experimental Study on Vortex Structures in Turbulent Boundary Layer at Low Reynolds Number

Yanang Guo, Xiaoshu Cai*, Wu Zhou, Lei Zhou and Xiangrui Dong

Institute of Particle and Two-phase Flow Measurement, College of Energy and Power Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China

Abstract: Experiments with a moving single-frame and long-exposure (MSFLE) imaging method, which is a Lagrangian-type measurement, is carried out to study the vortex structures in a fully developed turbulent boundary layer at low Reynolds number on a flat plate. In order to give the process of the vortex generation and evolution, on the one hand, the measurement system moves at the substantially same velocity as the vortex structure; on the other hand, a long exposure time is selected for recording the paths of the particles. In the experiment, the vortex structure characteristics as well as the temporal-spatial development can be shown by the streamwise-normal (x - y)-plane and streamwise-spanwise (x - z)-plane images which are extracted from a fully developed turbulent boundary layer. The result shows that the interaction between high- and low-speed streaks induces the generation, deformation and ‘breakdown’ of the vortex structures, and badly influences the vortex evolution.

Keywords: Boundary layer, High- and low-speed streaks, Low Reynolds number, Moving single-frame and long-exposure time, Vortex structures, Vortex generation, Vortex evolution, Vortex breakdown, Turbulent.

INTRODUCTION

The coherent structures play a significant role in the friction drag, heat and mass transfer, and the turbulence kinetic energy of turbulent boundary layer. However, the vortex structure is dominant and badly affects the generation and evolution of other coherent structures. Thus, the study on vortex structures is the starting point of the turbulence research.

In 1952, Theodorsen [1] proposed the horseshoe vortex as the basic structures in

*Corresponding author Xiaoshu Cai: Institute of Particle and Two-phase Flow Measurement, College of Energy and Power Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China; Tel: +86-21-55275059; E-mail: usst_caixs@163.com

wall-bounded turbulent flow. Kline *et al.* [2] analyzed a turbulent boundary layer flow field by hydrogen bubble. The high- and low-speed spots and the long streamwise streaks of hydrogen bubble were found in the near-wall region. In addition, they used dye to be the tracer for flow visualization and suggested that bursting is an important factor for energy generation of turbulence. A visual study of a turbulent boundary layer flow was conducted by photographing the motions of small tracer particles ($d = 62 - 74\mu\text{m}$) using a stereoscopic medium-speed camera system moving with the flow by Brodkey *et al.* [3]. They found that the ejections could be a consequence of low-speed fluid being trapped between fingers of high-speed fluid. Head *et al.* [4] conducted an experiment by a smoke tunnel with a 45 degree light sheet to visualize the hairpin vortex in turbulent boundary layer flow. Adrian *et al.* [5-9] studied the structure of energy-containing turbulence in the outer region of a zero-pressure gradient boundary layer by using particle image velocimetry (PIV) to measure the instantaneous velocity fields in a streamwise-normal plane. They found that the hairpin vortices in the outer layer occur in streamwise-aligned structures and could form large scale packets. The experimental investigations by the hydrogen bubble technique were performed by Lian [10] to show the coherent structures of turbulent boundary layer. In their experiment, the streamwise- and normal-vortices were observed along the interface regions between high- and low-speed streaks, while, the transverse (spanwise) vortices were observed at the front of the high-speed regions. Lozanoduran *et al.* [11] and Zandonade *et al.* [12] pointed out that the coherent structures in turbulent boundary layer could be illustrated by the high- and low-speed streaks and the vortices. The generation of these high- and low-speed streaks is related to the ejection and sweep, while, the generation of the vortices is related to the shear layer. Gao *et al.* [13] proposed and implemented a moving tomographic particle image velocimetry method to measure temporal evolution of velocity fields in three-dimensional volumes and to track coherent structures within a turbulent boundary layer with $Re_\tau \approx 2410$.

Although the vortex structure in turbulent boundary layer has been widely studied, and people already have had some understandings on its structural characteristics, its mechanism is still not clear. To further study the essence of vortex structure in turbulent boundary layer, the dynamic evolution of vortex structure with time and space must be obtained. In this paper, the moving single-frame and long-exposure (MSFLE) imaging method is utilized to study the vortex structure in a fully developed turbulent boundary layer on the flat plate at low Reynolds number. The evolution process of the vortex structure as well as the interaction between streaks and vortices have been discussed.

EXPERIMENTAL METHODS

Moving Single-Frame and Long-Exposure (MSFLE)

For most imaging measurements, the camera is usually fixed without moving, such as PIV and single-frame and long-exposure (SFLE). Fig. (1) gives the schematic diagram of the SFLE imaging measurement. The principle of SFLE is introduced in the following.

Firstly, the tracer particles with good tracking property are interspersed in the flow field; then, these tracer particles are illuminated by a light sheet from the laser; finally, the scattered light of the particles can be received by the camera. It means that the trajectory of the particles can be recorded in a single frame image by setting a proper exposure time of the camera. The length of the path line represents the movement distance of the tracer particle during the exposure time, thus the velocity of the tracer particle V , can be obtained as:

$$V = \frac{S}{M\Delta t} \quad (1)$$

Where S is the total length of trajectory, M is magnification factor of lens and Δt is exposure time. Moreover, the direction of the particle velocity can be determined by two consecutive frames.

However, it would be failed to capture the process of the fast-moving vortex structure evolution if the camera is fixed. Here, the MSFLE imaging method, which is developed from the SFLE method, is utilized to show both of the temporal and spatial development of the vortex structure in a single frame image, without using vortex identification criteria or Galilean velocity decomposition. Compared to SFLE method, the advantage of MSFLE is that the camera can move in the measuring system so that vortex structures with the same speed as the camera can be captured. MSFLE is a Lagrangian-type measurement and it is easy to observe the evolution process of vortex structure intuitively.

Experimental Studies on Coherent Structures in Jet Flows Using Single-Frame-Long-Exposure (SFLE) Imaging Method

Lei Zhou¹, Xiaoshu Cai^{1,*}, Wu Zhou¹ and Yiqian Wang²

¹*Institute of Particle and Two-phase Flow Measurement, University of Shanghai for Science and Technology, Shanghai, China*

²*School of Mathematical Science, Soochow University, Suzhou, 215006, China*

Abstract: On the axisymmetric water jet experimental apparatus, the flow field structures in entrainment boundary layers are measured using Single Frame Long Exposure image method, in the range of Reynolds number (Re) 1849~2509. It is found that engulfing and nibbling entrainment model occur intermittently with time, in the region of $L=2\sim 3.5d$ streamwise and $H=1\sim 1.25d$ radial direction. It concludes that the occurrence probability of engulfing increases with Reynolds number when $Re>1915$, the influence of Reynolds number on the occurrence probability of this structures decreases when $Re>2311$; the occurrence frequency of this coherent structures obtained by fast Fourier transform is between 10 and 19Hz; special vortex structures were observed in the flow field during the occurrence of engulfing. The jet flow field is measured using Moving Single Frame Long Exposure image method in Lagrangian coordinate system, and it is found that the vortex structures generally exist near the interface of turbulent regions and non-turbulent regions.

Keywords: Entrainment layer, Jet, Move single frame long Exposure, Vortex structures, Single frame single exposure.

INTRODUCTION

The observation of alternating vortical structures on the two sides of a gaseous jet using stroboscopic cinematography by Brown [1] provided one of the earliest experimental evidence for the existence of coherent structures in jet flows. Davis, Fisher and Barratt [2] then reported that a chain of vortex rings exists in round turbulent jets and Beavers and Wilson [3] further confirmed that these vortical

*Corresponding author Xiaoshu Cai: Institute of Particle and Two-phase Flow Measurement, University of Shanghai for Science and Technology, Shanghai, China; Tel: +86-21-55275059; E-mail: usst_caixs@163.com

Chaoqun Liu and Yisheng Gao (Eds.)
All rights reserved-©2020 Bentham Science Publishers

structures are all located inside the shear layer region. Ever since, the investigation of coherent vortical structures in jet flows became a fundamental topic and received attentions from researchers. Westerweel, Hofmann, Fukushima and Hunt [4] studied the characteristics of the TNTI (turbulent/non-turbulent interface) in self-similarity jets by combining the techniques of PIV (Particle Image Velocimetry) and LIF (Laser Induced Fluorescence) and concluded that the profile of TNTI rapidly changes along the axial direction with violent mass, momentum and energy exchange. Gan [5-7] investigated the initial development of gaseous turbulent vortex rings by PIV and found that this coherent pattern forms around $x/d = 2.5$, where x is the axial coordinate and d is the nozzle diameter. Two-dimensional (2D) and three-dimensional (3D) PTV (Particle Tracking Velocimetry) are also applied to study the small-scale coherent structures in the developed region of turbulent jets. Moreover, Silva, Taveira and Borrell [8-12] classified the entrainment at the TNTI into engulfing (big-scale eddy motions) and nibbling small-scale eddy motions).

Undoubtedly, the coherent structures play an important role in the entrainment of turbulent jet flows. However, the generation and the development of these coherent structures, especially their relationship with the TNTI is still unclear. To study the dynamics as well as the physical mechanism of the entrainment in jet flows, a detailed analysis on the evolution of the coherent structures in a water jet flow by utilizing Single Frame Long Exposure (SFLE) and Moving Single Frame Long Exposure (MSFLE) developed in our group is carried out in this chapter.

EXPERIMENTAL METHODS

Single-Frame-Long-Exposure (SFLE) and Moving SFLE (MSFLE)

The trajectories of illuminated tracer particles are obtained by using a relatively long exposure time and a typical trajectory is shown in Fig. (1a). The length of the trajectory S can be regarded as the distance covered by the tracer particle during the exposure time Δt plus the diameter of the tracer particle D as shown in Fig. (1b). Given that Δt is sufficiently small, the velocity magnitude of the particle V can be estimated as

$$V = \frac{L}{K \cdot \Delta t} = \frac{S-D}{K \cdot \Delta t}$$

where K is the magnification factor of the camera lens, D is the diameter of the tracer particle and L is the distance covered by the tracer particle over Δt .

In the frame work of SFLE, the exposure time can be adjusted to accommodate to the targeting velocity, or Reynolds number based on the nozzle velocity V_j and the nozzle diameter d , *i.e.*, $Re = V_j d / \nu$, where ν is kinematic viscosity. In addition, information of the flow field with various resolution can be obtained by adjusting the magnification factor of the lens. Additional information of the moving direction can be acquired by the correlation between two consecutive snapshots or the SFME (Single-Frame-Multiple-Exposure) [13] technique. A schematic diagram of the SFLE is shown in Fig. (2). To study the coherent vortical structures from the instantaneous flow field, moving SFLE (MSFLE) is developed to remove the mean flow velocity by making the camera moving with a constant velocity in the axial direction. SFLE and MSFLE, which have the advantages of providing more intuitively vision of the flow field and more comprehensive information about the moving trajectories during an exposure time, thus are selected in this study to investigate the coherent vortical structures in a water submerged jet flow.

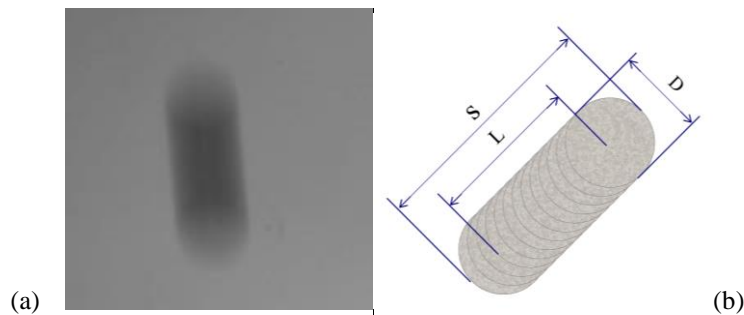


Fig. (1). (a) A typical trajectory by SFLE; (b) Sketch of a particle trajectory.

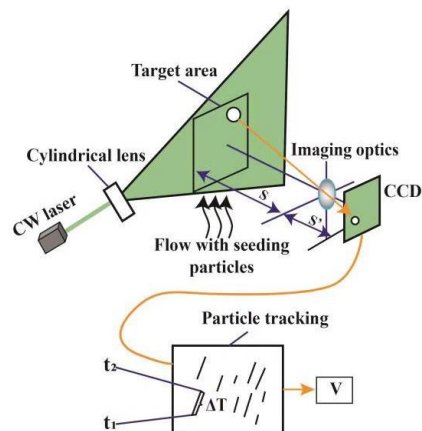


Fig. (2). A schematic diagram of SFLE.

Hybrid Compact-WENO Scheme for the Interaction of Shock Wave and Boundary Layer

Jianming Liu^{1,2} and Chaoqun Liu^{2,*}

¹*School of Mathematics and Statistics, Jiangsu Normal University, Xuzhou 221116, China*

²*Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA*

Abstract: In this chapter, an introduction to hybrid Weighted Essentially non-oscillatory (WENO) method is given. The hybrid techniques including both central and compact finite difference schemes are introduced. The paper review about the driven mechanism of the high order finite scheme required for compressible flow with shock is presented. The detailed constructing processes of the compact and WENO schemes are given and the hybrid detector is introduced. Further, in particular, a series of examples in the field of the compressible flow are designed to illustrate the different methods.

Keywords: Navier-Stokes equation, High order finite difference scheme, Hybrid Compact-WENO, Shock wave and boundary layer interaction turbulence.

A SHORT REVIEW ON STUDY OF HIGH ORDER FINITE DIFFERENCE SCHEME FOR COMPRESSIBLE FLOWS

The compressible flow field is in general governed by the Navier-Stokes equations deduced from physical conservation law. Due to the complexity of the flow problems, if there is no high-order scheme, it is impossible to obtain exact results which can embody complex fluid structures with different scales. To date, there are many effective numerical discretization methods for compressible flow, such as finite difference method (FDM), finite volume method (FVM), discontinuous Galerkin method (DGM) [1], high-order flux reconstruction [2], Spectral volume/difference Method (SVM) [3], *etc.* The finite difference method is one of the main numerical methods of computational fluid dynamics (CFD) because of its simplicity and easy to achieve high precision with the longest history. In addition, it is one of the most mature, widely used, and the most effective method. Hence, in this chapter, we focus on the high-order FDM.

*Corresponding author Chaoqun Liu: Department of Mathematics, University of Texas at Arlington, Arlington, Texas 76019, USA; Tel: +1-8172725151; Fax: +1-8172725802; E-mail: cliu@uta.edu

In the field of CFD, in general, we consider the scheme with at least third-order accuracy is high order. Since the 1990s, the research and application of high-order finite difference methods have begun to make substantial progress. In the present, the multi-scale complex flow such as simulating turbulence requires high-order numerical methods, which has become the consensus of the scientific community. Many scholars have developed a number of high-order numerical methods with advanced algorithms and good computational effects. Emerging advanced numerical methods include the ENO/WENO method [4-7], non-oscillatory containing no free parameter and dissipative (NND) scheme [8], group velocity control (GVC) scheme [9], compact scheme [10-12], *etc.* Among these numerical methods, the compact scheme only requires small number of grid points to get the high-order accuracy. The compact scheme had relatively small dissipation and gained a lot of favor in the field of direct numerical simulation (DNS) of turbulence flow [10-13]. The high-order central finite difference scheme is another kind of more effective method, which is straightforward and easy to implement and doesn't need to calculate the derivative performed by the compact scheme. Although compact scheme and high-order central scheme have obvious advantages in the simulation of multi-scale turbulent flow, it is hard to simulate the compressible flows with shock wave. On the other hand, to date, upwind or bias upwind high-order WENO scheme has achieved great success in capturing the shocks sharply. In the DNS of turbulence flow, the small length scale vortex is very important in the flow transition and turbulence process and thus very sensitive to any artificial numerical dissipation [11]. But the dissipation caused by WENO scheme is still harmful to the simulation of the flow transition of turbulent flow. Hence, in order to capture the shock waves sharply and resolve the small-scale turbulent flow simultaneously, a combination of compact or central and WENO schemes is desirable [11, 13-15].

Governing Equations

We consider the compressible Navier-Stokes equations in Cartesian tensor form as

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} &= 0, \\ \frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} &= -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}, \\ \frac{\partial E}{\partial t} + \frac{\partial u_i E}{\partial x_i} &= -\frac{\partial p u_i}{\partial x_i} + \frac{\partial u_i \tau_{ij}}{\partial x_j} - \frac{\partial q_i}{\partial x_i}, \end{aligned} \quad (1)$$

where

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right),$$

$$q_i = -k \frac{\partial T}{\partial x_i}.$$

With the reference values of characteristic length L , the free stream speed U_∞ , temperature T_∞ , viscous coefficient μ_∞ , density ρ_∞ , and the pressure $\rho_\infty U_\infty^2$, the Navier-Stokes equation (1) can be non-dimensioned and the only changed terms are

$$\tau_{ij} = \frac{\mu}{Re} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right),$$

$$q_i = -\frac{1}{Re(\gamma-1)PrMa_\infty^2} k \frac{\partial T}{\partial x_i}.$$

In order to illustrate the establishment of the finite difference scheme of Equation (1), we use the following simple scalar model equation as sample.

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2}. \quad (2)$$

The first derivative term in Equation (2) is used to simulate the convective term, and the second derivative term is used to simulate the viscous term in Navier-Stokes equation (1).

High-order Central Finite Difference Scheme

The main difficulty in the calculation of compressible fluid problems lies in the discretization of convective terms. To simulate the complex turbulence phenomena, it is significant to discretize the inviscid term by a high order method. In this study, we only consider the fourth/sixth- order approximation. The convect term $\frac{\partial f}{\partial x}$ at the grid point x_i can be approximated by

$$\left. \frac{\partial f}{\partial x} \right|_{x_i} = \frac{\hat{f}_{i+1/2} - \hat{f}_{i-1/2}}{\Delta x}. \quad (3)$$

The fourth order central difference scheme to approximate the face flux can be formulated as

$$\hat{f}_{i+1/2} = \frac{1}{12} (-f_{i-1} + 7f_i + 7f_{i+1} - f_{i+2}), \quad (4)$$

SUBJECT INDEX

A

Adaptive mesh refinement 211
 Advantages of Chebyshev nodes 70
 Amplitude 86, 115, 146, 147
 and frequency 115, 146
 and growth rate 147
 functions 86
 Angular momentum 239
 Angular velocity 8, 28, 29, 42
 fluid-rotational 8
 pseudo average 42
 pseudo time-average 29
 spatial average 42
 spatial mean 28, 29
 time-average 29
 Antisymmetric shear vector 18
 Axisymmetric water jet 280

B

Base flow 63, 77, 81, 82, 85, 97, 100, 104,
 106, 107
 cylindrical 85
 quasi-rotational 104
 velocity profiles 104
 Basic reference frame (BRF) 3, 156
 Blasius 2, 76, 155, 179, 202
 boundary layer 2, 155
 solution 76, 179
 velocity 202
 Boundary 57, 60, 61, 64, 146, 179, 205, 207,
 211, 229
 immersed 211
 outflow 75, 179
 ribbed 229
 up-stream 207
 Boundary conditions 64, 75, 78, 80, 84, 90,
 94, 95, 191, 215, 216, 226, 245
 discretized 80
 incoming flow 216

 periodic 191
 thermal wall 245
 Boundary layer 60, 76, 116, 190, 191, 195,
 245, 265, 266, 278
 flat plate 76
 hypersonic 245
 laminar separation 116
 parameters 60
 supersonic 245
 transitional 116
 velocity profile 190
 zero-pressure gradient 266
 Boundary-layer flows 60, 71
 flow transition 71
 Boundary layer interaction 54, 293
 turbulence 293
 Boundary layer transition 5, 25, 48, 51, 62, 63
 late natural 5

C

Calculation Procedure for Liutex 9
 Cartesian tensor form 294
 Cauchy-Stokes decomposition 1, 4, 9, 11, 12,
 18
 Cavitation 233, 240
 bubbles 233
 Channel 56, 155, 211, 226, 227, 228, 229,
 233, 234, 244
 cooling 227
 flow 56, 155, 244
 impeller 234
 inlet 228
 internal cooling 211
 ribbed 226, 227, 228, 229
 vortex 233
 Characteristics 229, 243, 244, 245, 250, 259,
 266
 statistical 243, 244, 245, 250, 259
 streaky 229
 structural 266
 Chebyshev 63, 64, 69, 79
 discretization 64, 79

expansion 79
 spectral method 63, 69
 Chebyshev collocation 59, 68, 79, 99, 100, 111
 approach 68
 method 79, 99, 100, 111
 spectral method 59
 Chebyshev nodes 66, 67, 70, 79, 92, 95, 99, 104
 distribution 92
 Chebyshev polynomials 64, 65, 66, 67, 68, 69, 70, 79, 80, 90, 91, 94, 98, 99, 104, 111
 monic 70
 nth-degree 69
 quadratic Shifted 98, 99, 104, 111
 Coefficients 45, 94, 142, 182, 184, 189, 255, 270
 cross-correlation 255
 dynamic viscosity 270
 spanwise-averaged skin-friction 182, 184
 Coherent structures 21, 100, 116, 187, 281, 289
 fundamental 100
 movement of 187, 289
 multi-dominant 116
 small-scale 281
 Compact finite difference scheme 293, 297
 Comparison 13, 15, 81, 82
 of Liutex lines and vorticity lines 13, 15
 of spectrums 81, 82
 Complex conjugates 22, 144, 146, 165
 eigenpair 165
 Complex 9, 11, 29, 30, 39, 42, 156, 158, 161, 164, 165, 166, 167, 168, 211
 eigenvalues 9, 11, 29, 30, 39, 42, 156, 158, 161, 164, 165, 166, 167, 168
 geometries 211
 Components 8, 24, 35, 84, 118, 121, 125, 132, 143, 144, 159, 211, 227, 233, 234, 235, 236, 238
 aero-engine 211
 antisymmetric 118
 dominant frequency 236, 238
 fluid passing 233
 imaginary 144
 low-frequency 236
 off-diagonal 24, 160
 steady-state 84
 temperature gradient 227
 vorticity tensor 8

Compressible boundary layer stability
 equations 76
 Compressible flows 17, 72, 154, 157, 162, 163, 165, 166, 167, 168, 172, 243, 244, 257, 293, 294, 306
 three-dimensional 72
 weak 257
 Compressible turbulent channel flow 244, 245, 259
 Computational domain 54, 56, 71, 72, 117, 118, 178, 179, 207, 248, 302
 length of 72, 118, 178, 207
 upstream boundary of 72, 118
 Computation fluid dynamics (CFD) 115, 116, 122, 293, 294
 Constant 24, 41, 151, 165, 246, 247, 282, 283, 289
 angle 24
 gas 246
 pressure 246
 rotation matrix 41
 spatial rotation 41
 time evolution 151
 velocity 282
 water level 283
 Coordinate rotation 21, 22, 158
 Coordinates 23, 24, 25, 26, 27, 41, 59, 63, 64, 72, 74, 83, 85, 86, 90, 92, 111, 180, 239, 271, 276, 281, 289
 axial 281
 curvilinear 72, 74, 180
 cylindrical 59, 63, 64, 83, 85, 90, 111
 polar 92
 radial 86, 239
 vertical 271, 276

D

Decay oscillation 146, 147
 Decomposition 3, 11, 41, 115, 120, 121, 138, 156, 157
 dynamic mode 115, 138
 fluid velocity 11
 modal 115, 120
 proper orthogonal 115, 121
 real Schur 213, 214
 singular value 121
 triple 3, 156
 Definition of Rortex 213, 214

- Derivatives 68, 69, 79, 80, 87, 95, 96, 167, 297
 - one-sided 69
 - second-order 167
 - Derivative term
 - first 97, 295
 - second 97, 295
 - second-order 306
 - Development 227, 265, 278
 - temporal-spatial 265, 278
 - thermal boundary layers 227
 - Differential equations 68, 87, 90, 94, 164, 157, 301
 - first-order ordinary 164
 - fourth-order ordinary 87
 - ordinary 90, 94, 157, 301
 - partial 68
 - Dimensional 53, 121, 125, 126
 - reduction 121, 125, 126
 - criterion 53
 - Direction 8, 24, 27, 33, 60, 75, 86, 178, 188, 191, 201, 202, 207, 212, 213, 245, 257, 272
 - normal-to-wall 60, 75
 - opposite 8, 24, 202
 - orthogonal 33
 - path 272
 - rotational 27, 201
 - spatial 212
 - stream-wise 245, 257
 - tangential 86
 - temporal 213
 - wall-normal 178, 188, 191, 207
 - Discontinuous Galerkin method (DGM) 293
 - Discrete orthogonality relation 67
 - Discretization 90, 295, 298, 301
 - explicit 301
 - Discretization method 301, 302
 - Distribution 136, 227, 252
 - fluctuation 136
 - instantaneous temperature 227
 - of RMS velocity fluctuations 252
 - DMD 116, 139, 140, 144, 145
 - applied 116
 - algorithm 140, 144
 - analysis for late transition flow 144
 - eigenvalues 144, 145
 - in shallow water 116
 - method 144
 - technique 139
 - DMD modes 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151
 - Imaginary part of DMD mode 148
 - DNS 12, 97, 100, 106, 108, 117, 186, 191, 205
 - and Experiment Results 191
 - code 12, 117
 - domain 186, 205
 - flow field 108
 - observation and numerical results of
 - stability 100
 - observations 97, 100, 106
 - ring-like vortices cases 106
 - DNS case 63, 71, 178
 - setup and code validation 178
 - DNS data 3, 112, 116, 120, 121, 191, 194, 195, 196, 198, 205, 244, 247
 - for TPG and CPG 247
 - of compressible turbulent channel flow 244
 - DNS of turbulence 294, 299
 - flow 294
 - DNS result(s) 176, 178, 182, 183, 186, 190, 192, 194, 195, 196, 198, 204, 205, 206, 244
 - of boundary layer 244
 - of multilevel vortex structures 196
 - DNS visualization 76, 205
 - matching 205
 - method 76
 - Downstream 48, 51, 116, 187, 191, 139, 217, 232, 233, 238, 271, 272, 276, 285, 286
 - fluid 286
 - mixing layer 116
 - region 48
 - section 191
 - systems 139
 - tracers move 271
 - vortex pair move 276
 - Dynamic mode decomposition (DMD) 115, 116, 120, 121, 138, 140, 144, 151
- E**
- Eddy cascade 205
 - Eddy motions 281
 - big-scale 281
 - small-scale 281
 - Effects 24, 29, 210, 234, 236, 237, 272, 273, 278, 294
 - compressing 273

- computational 294
 - modulation 29
 - negative 234, 237
 - non-linear 24
 - of load variations 236
 - of vortex 234, 235
 - strong shear 272, 278
 - surface roughness 210
 - Eigenfunctions equation 69
 - Eigenvalues 22, 25, 30, 40, 43, 59, 63, 78, 98,
 - 99, 104, 105, 106, 123, 140, 141, 142, 143, 144, 146, 161
 - complex conjugate 25, 30, 43
 - complex conjugated 22
 - imaginary parts of 143, 146
 - perturbation equation 59, 63
 - smallest positive imaginary 104
 - Eigenvalue-based 161, 154, 156, 157, 161, 163, 167, 172
 - criteria 154, 156, 157, 163, 167, 172
 - vortex identification criteria 161
 - Eigenvectors 9, 12, 22, 25, 30, 40, 42, 140, 141, 142, 143, 144, 156, 161, 165
 - complex conjugate 144
 - corresponding 25
 - corresponding real unit 22
 - Ejection 204, 205
 - events, multilevel vortex 204
 - multilevel hairpin vortices 205
 - Energy 122, 124, 126, 128, 151, 198
 - cumulative 126
 - kinetic 122, 151
 - large eddies pass 198
 - relative 124, 128
 - Energy content 115, 116, 132, 144, 151
 - kinetic 116, 132, 144
 - Engulfing 280, 281, 284, 285, 286, 287, 291
 - entrainment mode 286, 291
 - total proportion 287
 - Environmental perturbations 60
 - Epsilon 48, 50, 54
 - determination 48, 50, 54
 - function in Omega method 48
 - Eulerian vortex identification methods 4
 - Euler method 187
 - Evolution 2, 3, 183, 186, 265, 266, 267, 271, 275, 276, 277, 278, 281
 - dynamic 266
 - fast-moving vortex structure 267
 - Expansion 64, 70, 72, 79, 157, 163, 168
 - truncated 64
 - truncated series 64
 - Exposure time 189, 187, 207, 265, 267, 271, 275, 281, 282, 283, 285, 289
 - single frame 189, 271, 275
- F**
- Field 75, 116, 167, 190, 229, 271, 275, 283, 285, 293, 294, 302
 - density sine fluctuation 302
 - temperature gradient 229
 - Finite difference method (FDM) 64, 293, 294
 - Finite difference schemes 293, 294, 295, 296, 297
 - fourth-order 296
 - high-order central 294, 295
 - sixth-order 296
 - Finite volume method (FVM) 293
 - Flexible-cycle additive-correction (FCAC) 213
 - Flowmeter 268, 283
 - floater 268
 - Flow patterns 158, 227
 - typical backward-facing step thermal 227
 - Flow Reynolds number 226
 - Fluctuations 5, 84, 136, 198, 221
 - negative streamwise 198
 - positive streamwise 198
 - Fluctuation variables 84
 - Fluctuation velocity 286
 - non-dimensional 286
 - Fluid 2, 22, 42, 47, 60, 155, 176, 177, 199, 202, 203, 205, 206, 266, 272, 273, 283, 285, 286
 - high-speed 198, 266
 - irrotational 155
 - low-speed 266
 - low streamwise momentum 22
 - non-turbulent 285, 286
 - rotational motion of 2, 155
 - viscous 60
 - Fluid element 3, 76, 156
 - motion 76
 - Fluid flows 18, 48, 59, 115, 116, 122, 136, 235, 236, 238, 239
 - directions 239
 - structure 115
 - Fluid particles 3, 26, 29, 30, 59, 62, 164, 165
 - neighboring 164

reference 164
 surrounding 29
 Fluid rotation 2, 5, 8, 11, 12, 17, 18, 22, 155, 157, 213
 local 2, 8, 17, 22, 155, 157
 axis 213
 Fluid stiffness 47, 77
 Fluid transition 60
 Fourier analysis 297
 Fourier cosine expansion 70
 Fourier method 64
 Fourier series 70
 Fourier transform, fast 280
 Fractional step method 213
 Frame 7, 9, 22, 23, 42, 74, 158, 159, 189, 190, 267, 271, 272, 275, 276, 280, 283
 consecutive 267, 272
 position 74
 single 280
 Free stream velocity 60, 207
 approximate 60
 Frequency 76, 86, 90, 115, 138, 142, 143, 144, 146, 151, 207, 234, 235, 236, 237
 angular 86, 90
 blade passing 234, 236
 characteristic 234, 236, 237
 impeller rotational 234, 236
 Function 27, 45, 47, 48, 68, 70, 94, 187, 233, 297, 298, 299
 case-independent 45
 epsilon 48
 negative flux 298
 periodic 70
 positive flux 297
 smooth 167

G

Galerkin approach 64
 Galilean 3, 41, 188, 267, 289
 and rotational transformations 3
 decomposition 188
 transformations 41
 velocity decomposition 267, 289
 Galilean invariant 1, 4, 5, 17, 41, 42, 157
 definition for fluid rotation 17
 Gaseous jet 280
 Gas models 243, 245, 247, 248, 249, 250, 255, 259, 260
 Gauss 59, 66, 67, 68, 79

collocation points 59
 -lobatto points 66, 67, 68
 points 66, 79
 Generalized curvilinear coordinates 72
 Generation of ring-like vortex 108
 Gradient tensor 23, 27, 40, 157, 159, 156, 166, 172
 corresponding velocity 159
 evaluating velocity 40
 local velocity 157
 new velocity 23, 27
 original velocity 166, 172
 Grid 215, 248
 spacing 248
 structure 215
 Grid system 48, 54
 body-fitted 54

H

Hairpin vortex 16, 21, 100, 177, 178, 199, 200, 202, 266, 276
 coarsen 276
 Hairpin vortex ejection(s) 176, 178, 199, 206
 event 199
 multilevel 176, 206
 Heat 210, 227, 296
 conduction intensity 227
 conductivity 296
 exchange 210
 Heat flux 243, 246, 248
 conductive 246
 Heat transfer 210, 211, 212, 221, 227, 228, 229
 enhancement of 210, 229
 Heat transfer effects 211
 Helmholtz velocity decomposition 46
 Hermite interpolation method 297
 High- and low-speed streaks 265
 High order 71, 293, 295, 296, 302
 finite difference scheme 293
 method 295, 296
 polynomial interpolation 71
 Runge-Kutta methods 302
 High shear 62, 106, 205, 206, 228
 concentrations 228
 layer formation 62
 stress 106
 High-speed 203, 206, 272, 276, 278
 flow 203, 206

streaks 272, 276, 278
 zone outboard 203
 Hybrid techniques 293
 Hybrid Weno schemes 299
 Hydrogen bubble technique 266
 Hydroturbine performances 234
 Hyperbolic function 97

I

Imaginary eigenvalues 98, 104
 stable 104
 Impeller rotation 240
 Implicit large-eddy simulation (ILES) 54, 167
 Inflow boundary 72, 118, 179, 191, 207
 conditions 179, 191
 Instability waves 61
 linear 61
 small-amplitude 61
 Inviscid Euler equation 302

L

Lagrangian 155, 188
 methods 155
 type measurement technique 188
 Lagrangian coordinate 280, 289
 system 280
 Lambda vortex rotational direction 196
 Laminar 59, 62, 76
 two-dimensional 76
 Laminar flow 5, 59, 60, 62, 112, 182, 243, 272
 Laser induced fluorescence 281
 Linear 77, 83, 85, 98, 111, 112, 124
 combination of POD mode 124
 perturbation system 83
 stability equation 77, 83, 85, 111, 112
 transformation 98,
 Linearized perturbation equations 85
 Liutex
 based tensor decomposition 168
 iso-surface 12, 13, 14, 15, 16, 17, 168, 171
 Liutex vector 10, 12, 13, 14, 18, 21, 22, 24,
 25, 27, 30, 34, 39, 40, 41, 42, 157, 158,
 160
 and antisymmetric shear vector 18
 and lines 12
 calculating 25, 27, 40, 41
 computing 40
 magnitude of 27, 30, 34, 42

rotational 10
 Liutex vector field 13, 14
 and vorticity vector field 13
 Long-exposure measurement 268
 Low Reynolds number 176, 177, 178, 191,
 196, 197, 198, 206
 boundary layer 197
 Turbulent Boundary Layer 177, 178
 turbulent boundary layer flow 178, 196,
 197, 198, 206
 turbulent flows 176, 177, 191

M

Mach number 12, 72, 117, 118, 178, 207, 244,
 245, 246, 247, 252, 259, 306
 fluctuating 252
 free stream 12
 friction 248
 influence of 244, 245
 Mechanism 62
 of energy transfer paths 62
 of first ring-like vortex formation 62
 of high shear layer formation 62
 of multiple vortex 62
 of positive spike formation 62
 of small length vortices generation 62
 of U-shaped vortex formation 62
 Message Passing Interface (MPI) 72, 178
 Methods 64, 75, 151, 155, 156, 157, 191, 210,
 267, 281, 301, 302
 diagnostic-plot 191
 eigenvalue-based 157
 experimental 191, 267, 281
 explicit 301
 finite-element 64
 hybrid 302
 immersed-boundary 210
 iterative 301
 pseudo-spectral 75, 181
 vorticity-based 155, 156
 Micro vortex generator (MVG) 17, 54, 55, 56,
 116, 154, 157, 167, 168, 171, 172
 Modal decomposition method 138
 Modal energy 144, 145, 146, 147
 Modes 111, 116, 121, 125, 126, 127, 131, 132,
 133, 134, 135, 136, 137, 138, 142, 144,
 145, 146, 147, 150, 151, 233, 239, 240,
 284, 286, 287, 291
 coherent alternating entrainment 291

dominant modal energy 151
 energetic 151
 engulfing 286, 287
 five typical operational 240
 largest-energy 146
 low discharge 233
 lower amplitude 150
 nibbling 284, 286
 nibbling entrainment 291
 operational 239
 reverse pump 240
 spatial 116, 142, 144
 stable 111
 temporal 116, 138, 142, 145
 turbine brake 240
 zero-flow-rate 240
 Morkovin's hypothesis 244, 245
 Moving Single-Frame and Long-Exposure
 (MSFLE) 176, 178, 187, 191, 194, 265,
 266, 267, 278, 281, 282, 283, 289, 291
 Multilevel vortex 176, 194, 196, 198, 203, 206
 ejections and sweeps 206
 structures 176, 194, 196, 198, 203

N

Navier-Stokes equations 60, 73, 74, 244, 293,
 294, 295, 306
 compressible 73, 294
 time-dependent three-dimensional 245
 Non-oscillatory 293, 294
 hybrid Weighted Essentially 293
 Numerical dissipation 294, 300
 artificial 294

O

Omega 46, 59, 112, 194, 196, 197, 198, 243
 contour 194
 iso-surface 196, 198
 method and particle trace lines 194
 of vortex 112
 Origin reference frame 213
 Orr-Sommerfeld equation 63, 69, 78, 79, 90
 Orthogonal columns 141
 Orthogonality relation 67
 Orthogonal mode ranking 116
 Oscillation, spurious numerical 75, 181

P

Particle image velocimetry (PIV) 266, 267,
 281
 Particle 187, 206, 267, 281, 282
 tracking Velocimetry 281
 trajectory 206, 282
 velocity 187, 206, 267
 Phenomenon 25, 59, 138, 146, 194, 197, 198,
 229, 235, 236, 237, 285, 295
 eddy cascade 197
 heat-transfer 229
 thermal 229
 vortex-induced 235
 Physical conservation law 293
 POD 121, 125, 126, 136, 138, 140, 144
 algorithm 121
 method 126, 140
 ranking by DMD analysis 144
 snapshots 144
 time coefficients 136, 138
 Polynomials 64, 66, 90, 91, 92
 orthogonal 90
 trigonometric 64
 Power plant 233, 235
 pumped hydro energy storage 233, 235
 typical large-scale energy storage 235
 Power station 237, 238
 large-scale 237
 Pressure 1, 4, 84, 87, 179, 163, 181, 212, 268,
 295, 302, 305, 306, 307, 309
 ambient 163
 constant inlet 268
 contours 306, 307
 static 212
 Pressure fluctuations 233, 232, 234, 235, 236,
 238, 239, 240
 associated 232
 induced 232, 240
 observed large 239
 prominent 233, 234
 simulated 238
 vortex-induced 235
 Problem 3, 24, 25, 55, 60, 64, 90, 98, 116,
 156, 187, 295, 300, 304, 305, 306, 308,
 309
 complex flow 309
 compressible fluid 295
 expensive optimization 3
 non-periodic 64

periodic 64
 shock-tube 305, 306
 swirling flow 116
 viscous 308, 309
 Process 59, 60, 61, 100, 206, 228, 229, 265,
 267, 285
 heat-transfer 228, 229
 multilevel sweep 206
 multilevel vortex ejection 206
 Proper orthogonal decomposition (POD) 115,
 116, 121, 125, 138, 139, 151
 Property 4, 59, 62, 64, 65, 67, 70, 78, 177, 267
 new Lagrangian 176
 tracking 267
 transport 4
 Prototype power stations 232

Q

Quadratic 85, 98, 100
 fluctuation terms 85
 transformation 98, 100
 Quasi-streamwise vortices 22, 100, 155, 177
 counter-rotating 100
 surrounding staggered 22

R

Reference frame, rotating 4
 Refractive index 268
 Regeneration cycle 21
 Resolution 178, 187, 188, 189, 268, 282, 283,
 289, 297, 299
 sensor 189
 sharp 299
 spectral 297
 Reversed flow forms 217
 Reversible pump turbine 46, 232, 233, 234,
 239, 240
 Reynold 5, 211, 215, 221, 222, 243, 286
 numbers 211, 215, 221, 222, 286
 analogy 243
 stress and Liutex 5
 Reynolds number 61, 62, 72, 81, 82, 83, 84,
 117, 178, 210, 211, 221, 229, 280, 284,
 287, 291
 friction 248
 Rib 210, 211, 225, 226
 surface 225
 tabulator 210, 211

 temperatures 226
 Ring-like vortex 13, 15, 59, 100, 108, 111,
 186
 experiment 186
 Ring-like vortices 106, 109, 176, 183
 chains 183
 problems 106
 RK method 301
 fourth-order 301
 third-order 301
 RMS 251, 252, 253, 260
 mach number fluctuation 253
 velocity fluctuations 251, 252, 260
 R-NR decomposition 1, 4, 11
 of velocity gradient tensor 4
 Rortex 213, 228, 229
 and Shear decomposition 228, 229
 method 213
 Rotating vortex flow 240
 Rotation 4, 6, 29, 63, 158, 197, 202, 276
 angular 4
 axis 6, 29, 158
 direction 197, 202, 276
 flow 63
 Rotational 10, 18, 103
 streamline 103
 tensor 10, 18
 transformations 3
 Rotational motion 25, 29, 30, 42, 155
 instantaneous 42
 real 155
 Rotational strength 12, 17, 22, 24, 157, 158,
 160
 local 12, 157
 vortical 22
 Rotational vortex 202, 272
 clockwise 202
 Rotation angle 27
 clockwise 27
 in-plane 27
 Runge-Kutta method 247, 301, 302
 Runge's phenomenon 70

S

SFLE method 187, 190, 267, 270
 SFSE method 187
 Shear 10, 59, 62, 63, 100, 112, 156, 157, 177,
 214, 228, 272, 273
 anti-symmetric 10

- distribution of 100, 228
- Shear Flow 59, 77
 - two-dimensional 59, 77
- Shear-heat transfer correlation 229
- Shearing 154, 156, 166, 167, 168, 171, 172
 - components 166, 168, 172
 - effects 167, 168
- Shear layer 2, 62, 155, 177
 - instability 62, 177
 - region 2, 155, 281
- Shear layers 60, 78, 106, 156, 176, 205, 206, 266, 272
 - strong 176, 205, 206, 272
- Shear stress 83, 101, 246
 - correlated 101
 - tensor 246
- Shifted Chebyshev 90, 91, 92, 93, 97, 98, 99
 - of linear transformation 98
 - polynomials 90, 91, 92, 93, 97, 99
- Shock oscillation 299
- Single-frame 282, 289
 - multiple-exposure 282
- Singular value decomposition (SVD) 121, 140
- Snapshots 282, 284, 285, 286, 291
 - consecutive 282, 284, 285, 291
 - nibbling 286
- Spalding 190, 270
 - formula 190
 - law 190, 270
 - theory 190, 270
- Spanwise 62, 71, 75, 100, 103, 117, 151, 177, 178, 181, 191, 198, 271, 276
 - directions 71, 75, 117, 178, 181, 191, 271, 276
 - plane 198
 - spacing 177
 - structures 151
 - tubes 100
 - velocity 103
 - vortex tube formation 62
 - vorticity rollup 62
- Spectral collocation method 79
- Spectral volume/difference Method (SVM) 293
- Spectrums 5, 81, 82, 106, 238
 - frequency-domain 238
 - turbulence energy 5
- SRA 244, 245, 253, 255, 260
 - modifying 260
 - and Morkovin's hypothesis 244, 245
- form 255
- relations 253
- Stability 64, 71, 78, 79, 143, 146, 151, 299
 - calculations 64
 - characteristics 79, 143, 146, 151
 - incompressible temporal 78
 - numerical 299
 - problem, linear 71
- Stability analysis 59, 63, 94
 - linear 94
- Staggered-rib arrangement 227
- Stokes equations for viscous fluid motion 60
- Streamline topology 41
- Streamwise 48, 54, 56, 71, 75, 117, 132, 151, 178, 181, 187, 188, 203, 204, 206, 207, 219, 266, 268, 271
 - migration 187
 - streaks 203, 204, 206
 - structures 132, 151
 - turbulence intensities 219
- Streamwise direction 72, 117, 120, 182, 184, 188, 191, 192, 195, 271, 275, 276, 278
 - domain decomposition 72
- Stroboscopic cinematography 280
- Strong Reynolds analogy (SRA) 244, 253, 254, 260
- Strong vortex neck rotation 176
- Structures 116, 125, 132, 142, 146, 191, 192, 210, 211, 214, 215, 226, 227, 260, 266, 280, 281, 284, 285, 286, 293
 - asymmetric 116
 - complex fluid 293
 - dominant energy 125
 - fierce entrainment 285
 - heat-transfer 227
 - horseshoe 191, 192
 - larger-scaled 146
 - rib tabulator 210
 - rib-tabulator 214
 - special pathline 284, 285, 286
 - streaky 227
 - streamwise-aligned 266
 - stream-wise vortices 260
- Supersonic 54, 154, 244, 260
 - flow 260
 - microramp vortex generator 154
 - ramp flow 54
 - turbulent boundary layer flows 244
- Sutherland's law 246
- SVD of matrix 121

T

Tensor 10, 11, 12, 18, 46, 118, 163
 antisymmetric 18
 anti-symmetric 12, 46
 non-rotational 10, 18
 strain rate 161, 163
 symmetric 11, 12, 46, 118
 Theory 61, 62, 63, 77, 79, 158, 162, 164, 203
 classical stability 62
 critical point 158, 164
 linear hydrodynamic stability 61
 linear stability 61, 63, 77, 79
 simple inducement 203
 simple vortex inducement 203
 Thermally perfect gas (TPG) 243, 244, 245, 246, 247, 248, 249, 253, 259, 260
 Tollmien-Schlichting waves 61
 Total variation diminishing (TVD) 75, 181, 302
 Trajectories 27, 28, 29, 164, 187, 192, 267, 281, 289
 elliptic 27, 28
 instantaneous 164
 Transition 60, 61, 116, 183
 process 61, 183
 stage 116
 theory 60
 Trigonal geometry 55
 Trigonometrical formulas 67, 92, 93
 Turbine 211, 212, 233, 235, 236, 237, 238, 239, 240
 blade 211, 212
 operations 237
 performances 240
 pump 233, 240
 Turbulence 1, 2, 4, 210, 216, 228, 265, 266, 294
 energy-containing 266
 fully-developed 216
 kinetic energy 265
 muscle of 1, 2, 4
 near-wall 21
 simulating 294
 thermal 210, 228
 Turbulence intensity 210, 219, 220, 222, 223, 224, 229
 maximum streamwise 222
 stream-wise 220, 229

Turbulent 21, 116, 182, 251, 281
 flow velocity profile 182
 intensity 251
 jet flows 281
 momentum transport 21
 pipe flow 116
 Turbulent flows 59, 61, 62, 112, 116, 118, 154, 155, 177, 182, 186, 190, 191, 194, 197, 243, 244, 266, 294
 multi-scale 294
 small-scale 294
 wall-bounded 177, 266
 Two-dimensional measuring method 272

V

Vector 5, 10, 11, 17, 18, 30, 32, 34, 76, 154, 157, 166, 172, 246
 body force 246
 non-rotational shear 10
 orthogonal 32
 selected real 11
 Velocity 1, 2, 4, 6, 21, 63, 97, 101, 158, 190, 202, 212, 213, 239, 243, 244, 248, 250, 251, 269, 270, 271, 272, 282, 283
 average 270
 bulk 248
 fluctuating 243, 244, 251
 friction 248, 270
 gradient field 21
 nozzle 282
 profile equation 190, 270
 statistical 260
 tangential 239
 transformed 250, 251
 vorticity equation 63
 Velocity components 6, 60, 84, 88, 89, 159, 179
 azimuthal 84
 longitudinal 60
 Velocity gradient tensor 1, 4, 6, 7, 9, 10, 11, 17, 18, 22, 40, 42, 43, 46, 156, 157, 158, 161, 165, 166
 decomposition based on liutex 11
 Velocity profile 63, 69, 94, 97, 104, 105, 106, 110, 182, 183, 186, 190, 191, 270
 basic-state 69
 measured 270
 quasi-rotation 63
 spanwise-averaged streamwise 182

- Vibrational energy 247, 249, 250, 260
 - excitation 249
- Vibrations of hydroturbines 235
- Viscosity 60, 189, 207, 212, 246, 247, 248, 271, 282, 296
 - dynamic 60
 - kinematic 189, 271, 282
 - molecular 212
- Viscous 22, 60, 270, 272
 - fluid motion 60
 - shear stresses 60
 - sublayer 22, 270, 272
- Vortex 13, 15, 63, 155, 197, 201, 202, 232, 234, 276, 286
 - cavitating 234
 - clockwise rotating 276
 - earphone 286
 - flow-induced 232
 - inter-blade 234
 - lambda 13, 15, 63
 - lower level 197
 - quasi-streamwise 155
 - second level 201, 202
 - tennis racket 286
- Vortex breakdown 3, 51, 60, 61, 63, 176, 197, 198, 205, 265
 - process 197
- Vortex dynamics 1, 2, 4, 11, 155, 237, 238
- Vortex formation 5, 62
 - tertiary 62
- Vortex generation 5, 197, 265
 - mechanisms 5
 - multilevel 197
 - small 5
- Vortex identification methods 45, 46, 48, 53, 100, 116, 118, 119, 155, 156, 157
- Vortex-induced vibration 235
- Vortex legs 194, 195, 196, 199, 200, 202, 203, 276
 - clockwise rotation 202
 - hairpin 194, 196, 199, 200, 203
 - multilevel hairpin 195
 - outboard hairpin 203
 - second level 202
 - tilted hairpin 276
- Vortex rings 48, 51, 54, 55, 112, 177, 201, 280, 281
 - gaseous turbulent 281
 - large-scale 55
- Vortex rope 232, 233, 238, 240
 - dynamics 238
 - low-frequency 240
- Vortex rotation 1, 4, 197
 - axis 4
 - strength 1, 4
- Vortex structure(s) 5, 13, 51, 55, 132, 136, 177, 194, 198, 205, 206, 257, 267, 269, 271, 275
 - coherent 194, 205
 - evolution process of 267, 269
 - generation 5, 271, 275
 - hairpin 177, 257
 - horseshoe 194
 - multilevel hairpin 206
 - non-physical 51, 55
 - of late boundary layer transition 13
 - shapes of 132
 - single hairpin 198
 - streamwise 136
- Vortex vector , 215
 - field 21
- Vortex visualization 16, 45, 48, 51, 55, 58, 120
 - methods 45
- Vortical structures 2, 3, 16, 154, 155, 156, 157, 167, 168, 280, 281, 282, 289
 - alternating 280
 - coherent 281, 282
- Vortices 2, 5, 21, 22, 46, 51, 59, 61, 71, 77, 100, 106, 119, 176, 194, 196, 197, 202, 205, 266, 272, 273, 276, 289, 291
 - co-rotating 176, 206
 - counterclockwise 196, 197, 202
 - counter-rotating 196, 276
 - generation of 61, 71, 77
 - horseshoe 177
 - quasi-longitudinal 22
 - rotating 289
 - tennis racket 291
 - three-dimensional 2
- Vorticity 1, 2, 3, 4, 5, 7, 9, 11, 13, 15, 17, 18, 29, 37, 40, 45, 47, 48, 54, 59, 62, 63, 76, 100, 155, 196
 - decomposition 3, 5, 7, 9, 11, 13, 15, 17
 - high concentration of 2, 155
 - mixture of 47, 48
 - non-rotational 59, 62
 - rotational 59, 62
 - spanwise 100
- Vorticity lines 2, 13, 15, 16, 155

for hairpin vortices 16
Vorticity tensor 10, 29, 156, 157, 161
norm 156
Vorticity vector 9, 10, 13, 14, 35, 36, 39, 40,
42, 161, 166
angle 155
field 13, 14
projected 35

W

Wall 21, 221, 222, 248, 250, 256, 270
scaling 250
shear stress 270
streaks 256
streamwise velocity 221, 222
turbulence 21
friction velocity 248
Wall-normal velocity 200, 201
Wall temperature 72, 178, 207, 243, 244, 245,
247, 249, 250, 251, 260
dimensional 247
Water 60, 239, 268, 269, 281, 282, 283, 291
density 239
jet flow 281
pump 268
transport 268
Water tank 268, 269
constant position 269
Water tunnel, low-speed circulating 188, 268
Weighted essentially non-oscillatory (WENO)
293, 300

Z

Zone 198, 217, 221, 222, 228, 272
buffer 222
concentrated 228
high-speed upper 198
separate 221
upwelling 272



Chaoqun Liu

Dr. Chaoqun Liu received both BS (1968) and MS (1981) degrees from Tsinghua University, Beijing, China and PhD (1989) degree from the University of Colorado at Denver, USA. He is currently a tenured and distinguished professor and the director of Center for Numerical Simulation and Modeling at the University of Texas at Arlington, Arlington, Texas, USA. He has worked on high order direct numerical simulation (DNS) and large eddy simulation (LES) for flow transition and turbulence for almost 30 years starting from 1989. As PI, he has been awarded by the NASA, US Air Force and US Navy with 50 federal research grants of over 5.7⁷10⁶ US dollars in the United States.. He has published 11 professional books, 120 journal papers and 145 conference papers. He is the founder and major contributor of the third generation of vortex identification methods including the Omega, Liutex/Rortex, Liutex-Omega, Modified Liutex-Omega, Liutex Core Line methods, RS vorticity decomposition and R-NR velocity gradient decomposition.



Yisheng Gao

Dr. Yisheng Gao received his Bachelor of Engineering degree in aircraft design and engineering (2007), Master of Engineering degree in fluid mechanics (2009) and Ph. D. degree in fluid mechanics (2016) from the Nanjing University of Aeronautics and Astronautics University, China. His research is concerned with computational fluid dynamics, including direct numerical simulation and discrete adjoint methods.