

Numerical Methods and Implementation in Geotechnical Engineering — Part 2



Y.M. Cheng, J.H. Wang
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Numerical Methods and Implementation in Geotechnical Engineering – Part 2

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PREFACE

For most of the geotechnical problems, particularly those related to real life problems, analytical solutions are usually not available. For both research and practical applications, numerical methods and computer programs are required for many cases. In the recent forty years, many numerical methods have evolved for various kinds of engineering problems. Engineers are now well adapted to the uses of different computer programs for the solution of engineering problems. There is however a major drawback in the current engineering practice in that most of the engineers are not familiar with the basics of the numerical methods, the methods of implementations and the limitations of the numerical methods/programs. In fact, to a certain extent, the methods of implementations and the limitations of the numerical methods are related. In many internal studies using different commercial numerical programs, the authors sometimes found noticeable or even completely different results with different programs or the same program with different default setting for a given problem, and this situation is not uncommon. For a problem with unknown solution, how an engineer assess the acceptability of the computer results is a difficult issue that needs serious attention. In several technical meetings in the Hong King Institution of Engineers, the authors have discussed with some engineers about the appreciation of the limitations of the daily-used engineering programs. If two computer programs can produce significantly different results, how an engineer determine the acceptability of the results actually require deeper knowledge about the basics of the numerical methods and implementations. Interestingly, the authors like to ask the students a question “Different answers can be obtained from different commercial programs. Which results should be accepted, and why should those results be accepted?”. In general, the authors challenge the students (undergraduate and graduate students) every year for this question, and virtually this question is never answered properly. The problems in the assessment of the numerical results will also be discussed in this book, which is seldom addressed in other books or research papers.

The authors have participated in different types of geotechnical research and consultancy works in different countries, and has written a book *Frontier in Civil Engineering, Vol. I, Stability Analysis of Geotechnical Structures*, which is well-favored by many students, engineers and researchers. Most of the books on numerical methods seldom address the actual procedures in numerical implementations, but many postgraduates actually need to develop computer programs to consider special constitutive models, loadings, numerical methods, boundary conditions and other effects. In view of the limitations of most of the books at present, the authors would like to write a new book on numerical methods and the implementations based on their previous works, and this new book should be useful for senior undergraduates, postgraduates, engineers as well as researchers.

In this book, finite element method, optimization method, plasticity based slip line method, limit analysis method, distinct element method, Smoothed-Particle Hydrodynamics Method, Spectral Element Method and Material Point Method will be introduced. The present book will not cover dynamic problems which is a big topic, and hopefully this will be covered later by the authors in another book. The authors will also try to explain the methods of implementation for some of these methods through sample computer programs. Sample programs are given and discussed to assist students in developing programs for their own uses. These programs are not meant to be efficient or up-to-date, but will help the students in learning about the implementation of some numerical methods. This book should not be taken as a classical textbook, as the authors do not intend it to be. There are many new contributions to numerical methods in geotechnical engineering over the last 30 years, and many topics can be covered by individual books for detailed discussion. There is also no way for the authors to

cover all numerical methods in details in this book. This book is a basic introduction to some more commonly used numerical methods in geotechnical engineering which have been used by the authors for teaching and research, with the discussion of some common commercial program problems, programming techniques and applications.

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CONFLICT OF INTEREST

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CHAPTER 1

Distinct Element Method and Other Numerical Methods

Abstract: In this chapter, the basic formulation of the distinct element method is introduced, which will be followed by more discussions on the blocky and particle formulation of the DEM. This will be followed by some applications of the DEM to several types of problems. As the alternatives to the DEM, DDA and manifold for two-dimensional and three-dimensional problems will be introduced with case studies. There are also many new numerical methods that have evolved in the last twenty years. For example, some recent continuity based methods as such as the SEM, SPH and MPM will also be introduced in this chapter, with applications to some slope failure problems in Hong Kong.

Keywords: DDA, Distinct element, Manifold, Meshless, MPM, Particle flow, SEM, SPH.

1.1. INTRODUCTION TO DISTINCT ELEMENT METHOD

While the finite element is the most popular method adopted by the engineers for practical works, there are some other numerical methods which are under active research by various researchers. These methods are generally not mature enough or difficult to be used for general engineering problems, nevertheless, they can be applied in many geotechnical problems governed by large displacement, discontinuity or even separation of materials. The authors do not aim at introducing all of these special numerical methods, but will discuss some of these methods with applications.

The most commonly used numerical methods for continuous systems are the finite difference method (FDM), the FEM and the boundary element method (BEM). The basic assumption adopted in these numerical methods is that the materials concerned are continuous throughout the physical processes. This assumption of continuity requires that at all points in a problem domain, the material cannot be torn open or broken into pieces. The neighbourhood of all material points will remain unchanged throughout the whole physical process. Some special algorithms have been developed to deal with material fractures in continuum

mechanics based methods, such as the special joint elements by Goodman (1976) and the displacement discontinuity technique in BEM by Crouch and Starfield (1983). However, these methods can only be applied with limitations (Jing *et al.* 1993):

- (1) large-scale slip and opening of fracture elements are prevented in order to maintain the macroscopic material continuity;
- (2) the amount of fracture elements must be kept to relatively small so that the global stiffness matrix can be maintained well-posed, without causing severe numerical instabilities; and
- (3) complete detachment and rotation of elements or groups of elements as a consequence of deformation are either not allowed or treated with special algorithms.

The authors have adopted the zero thickness interface element by Goodman (1976) in developing a soil-structure interaction program. One of the practical problems in using this joint element is the difficulty in mesh generation. Currently, most of the engineers will specify a thin layer of material to simulate the interface instead using the joint element directly in finite element analysis. Rock masses are always dissected by joints, faults, cracks or other discontinuities which control the failure and sliding of the masses. The complicated structure of rock masses and the discontinuities lead to a complicated non-linear mechanical response. Research work in the deformation in jointed rock mass is very important in modeling complicated important geotechnical problems. For such system, the use of the classical continuity based numerical methods will face great difficulty if the deformation is large. The use of the discontinuity based approach will be more appropriate under such condition.

Before a slope starts to collapse, the factor of safety serves as an important index in both the LEM and SRM to assess the stability of the slope. The movement and growth after failure have launched which are also important and in many cases cannot be simulated on the continuum model. This should be analyzed by the discontinuous mechanics. The commonly used discontinuous numerical methods include: Finite Element Method (FEM) with Joints, Discrete Element Method (DEM), Rigid-Block Spring Method (RBSM), Contact-Spring Model, Discontinuous Deformation Analysis (DDA) and Numerical Manifold Method (NMM). Joint element method with zero thickness was proposed by Goodman, Taylor and Brekke in 1968. It is very effective for problems with only few discontinuities and small deformation. The conditions of no penetration and no tension at the interface however cannot be strictly satisfied. Sometimes, it may lead to great errors between computation results and actual measured results.

There was still a major progress in modelling discontinuous problem at that time. The extension of the Goodman joint element to higher order element for improved accuracy has been achieved by many researchers. Desai (1984) also proposed joint element where a small thickness is allowed. Desai's element is easier for programming purpose, but the formulation of the stiffness matrix needs some special treatments as the thickness of the element is very small. A major limitation of the joint element method is that if there are many discontinuities presented in a rock mass, the use of joint element is practically very difficult to be adopted. The separation of elements also cannot be modeled by the use of joint element, as the method of analysis is still based on the continuum finite element method. Besides the blocky structure of rock, masonry and similar structures, discontinuous medium in form of assembly of grains is also commonly found. There are hence, two major discontinuity based numerical approach: blocky and granular. Actually, the granular approach is a special case of the blocky type problem, but due to the shape of the grains, contact detection can be a much easier task, and special techniques are available for the solution which is not possible for the blocky type problem.

In continuum description of soil material, the well-established macro constitutive models and the parameters can be measured experimentally. On the other hand, a discrete element approach will consider the material as distinct grains or particles that interact with each other. The micro-parameters are difficult to be measured directly. The commonly used distinct element method is an explicit method based on the finite difference principles which is originated in the early 1970s by a landmark work on the progressive movements of rock masses as 2D rigid block assemblages (Cundall, 1971). Later, the works by Cundall are developed to the early versions of the UDEC and 3DEC codes (Cundall, 1980; Cundall and Hart, 1985, 1992). The method has also been developed for simulating the mechanical behavior of granular materials (Cundall and Strack, 1979), with an early code BALL (Cundall, 1978) and Trubal (Strack and Cundall 1984) which later evolved into the codes of the PFC group for 2D and 3D problems of particle systems (Itasca, 1995). Lemos (1983) developed a coupled DEM/boundary element formulation. Later, Lorig (1984) developed the pre and post-processor for DEM analysis, and the code is later modified to HYDEBE (Hybrid discrete element boundary element) which is written in Fortran IV. Through continuous developments and extensive applications over the last three decades, a great body of knowledge and a rich field of literature about the distinct element method have been accumulated. The main trend in the development and application of the method in rock engineering is represented by the history and results of the code groups UDEC/3DEC.

Based on Trubal, Thornton and Randall (1988) further developed the contact

Optimization Analysis in Geotechnical Engineering Problems

Abstract: Many engineering problems can be formulated as an equivalent optimization problem. In fact, the popular finite element method is a form of optimization problem, where local interpolation function is used to represent the local effect within an element. In limit equilibrium and limit analysis, the use of global optimization search for the critical slip surfaces and the failure mechanism is required for many slope stability, lateral earth pressure and bearing capacity problems. For pile driving back analysis, the whole analysis is just actually an optimization process. In this chapter, some of the classical gradient type, as well as the modern heuristic optimization methods will be introduced.

Keywords: Control variables, Global optimization, Gradient, Local optimization, Objective function.

2.1. INTRODUCTION

Many engineering problems can be formulated as a form of optimization analysis. In fact, the popular finite element method can be viewed as a special form of optimization problem, where the energy of a system is minimized. There are also many types of problems for which, the optimal solutions have to be evaluated. In the past 30 years, there are tremendous amounts of works in the area of optimization for constrained/unconstrained convex, non-convex, non N-P types of problems, integer problems, multi-criteria problems and others. In this chapter, several types of geotechnical problems will be considered from an optimization point of view, and some of the methods that the authors have used, will be discussed in this chapter. It should be noted that there are over hundreds of method that have evolved for the past 30 years, and they can be grouped accordingly to different criteria. In general, the use of the classical calculus in optimization analysis has only limited uses in geotechnical application, and this chapter will mainly concentrate on the numerical optimization methods.

There are three major methods in geotechnical engineering – the slip line method,

limit analysis method and the limit equilibrium method. A brief introduction about these methods and the relationship of these methods with optimization analysis will be discussed.

At the instant of impending plastic flow, both equilibrium and yield conditions are satisfied. For soils, the Mohr-Coulomb criterion is widely used for the yield condition. Combining the Mohr-Coulomb criterion with the equations of equilibrium will give a set of hyperbolic differential equations of plastic equilibrium. With the known stress boundary conditions, this set of differential equations can be used to obtain the stresses at the ultimate condition. For example, the bearing capacity of footing or the lateral earth pressure behind a retaining wall can be determined by slip line analysis (Cheng 2003, 2005, 2007). To solve the hyperbolic differential equations, this set of equations is transformed to curvilinear coordinates whose directions at every point in this yielded region, coincide with the directions of failure or slip plane. The locus of the failure planes is known as the slip lines while the network of failure planes is called the slip-line field. Kötter (1903) was the first to derive these slip-line equations for the case of plane deformations. Prandtl (1920) was the first to obtain an analytical closed form solution for a footing on a weightless soil. The weight of soil will, however create further complication in the solution of the hyperbolic differential equations, and no analytical solution will be available if the weight of soil is considered. Sokolovskii (1965) adopted a numerical procedure based on a finite difference approximation of the slip-line equations. He solved a number of interesting problems on bearing capacity of footings or slopes, as well as the lateral pressure on the retaining walls, for which it is impossible to find closed form solutions. In a slip-line solution, the domain under consideration is assumed to be completely in the state of plastic equilibrium (Chen 1972), and partial yielding can only be considered with some approximate assumption (Cheng and Au 2005), and this is one of the limitation of the slip-line method.

The limit equilibrium method has traditionally been used to obtain approximate solutions for stability problems in soil mechanics. Currently, most of the engineers still rely on the use of the limit equilibrium method to provide approximate solution to bearing capacity, lateral earth pressure and slope stability problems. Although this method has greatly simplified the stresses at the ultimate limit state and assumptions are required to solve the problems, the concept of this method is simple to be understood by most of the engineers and is hence, a popular tool for normal analysis and design. It has also been demonstrated that this approach can give reasonably acceptable solution for most cases. Based on this approach, the stability problem is reduced to the determination of the most dangerous location for the failure surface, and the use of various optimization methods have been proposed by different authors for such application.

The limit analysis method considers the stress–strain relationship of a soil in an idealized manner. This idealization, termed normality (or the flow rule), establishes the limit theorems on which limit analysis is based. Within the framework of this assumption, the approach is rigorous and the techniques are competitive with those of limit equilibrium, in some instances being much simpler. The plastic limit theorems of Drucker *et al.* (1952) may be used to obtain the upper and lower bounds of the collapse load for stability problems. The conditions required to establish an upper- or lower-bound solution are essentially as follows:

Lower-bound theorem - The loads, determined from a stress distribution that satisfies: (a) the equilibrium equations; (b) stress boundary conditions; and (c) nowhere violates the yield criterion, are not greater than the actual collapse load. The distribution of stress satisfying item (a), (b) and (c) has been termed a statically admissible stress field for the problem under consideration. Hence, the lower-bound theorem is equivalent to the condition that if a statically admissible stress distribution can be found, uncontained plastic flow will not occur at a lower load. Although the lower bound theorem is difficult to be applied except for simple problem, the authors will demonstrate that the use of modern optimization method will help to extend the capability of the lower bound theorem.

Upper-bound theorem - By equating the external rate of work to the internal rate of dissipation according to a prescribed failure mode (or velocity field) that satisfies: (a) velocity boundary conditions; and (b) strain and velocity compatibility conditions, the loads as determined are not less than the actual collapse load. A velocity field satisfying the above conditions has been termed a kinematically admissible velocity field. If a kinematically admissible velocity field can be found, uncontained plastic flow must impend or have taken place previously. The upper-bound technique considers only velocity or failure modes and energy dissipations while the stress distribution need not be in equilibrium, and is only defined in the deforming regions of the mode. Since a prescribed failure mode is required to be defined for the load computation, the critical collapse load is required to be determined from an optimization analysis where different shapes and locations of the failure surface have to be considered.

Both the lower and upper bound methods are useful tools to the engineers, and for practical problems, both methods will become an optimization problem where the solution will be given by the maxima/minima of a functional. Since it is difficult to evaluate the maxima/minima of the functional, the use of modern optimization methods will become particularly attractive for complicated real problems, and the principles and applications of some optimization methods will be discussed in the following sections.

Reliability Analysis in Geotechnical Engineering

Abstract: This chapter will give an overview about the application of reliability analysis in geotechnical engineering problems, and slope stability problem will be chosen for the illustration.

Keyword: Finite element, First order reliability method, Monte Carlo simulation, Reliability index, Risk, Response surface method, Second order reliability method.

3.1. INTRODUCTION

In many geotechnical analysis and design works, deterministic methods with suitable factors of safety are used by many engineers with general satisfaction in different countries. In this approach, loads, the strengths, failure modes or collapse mechanisms are assumed to be known or can be approximated with good accuracy, and the difference between the design load and the characteristic strength is large enough to provide adequate factors of safety, which are usually determined based on long term observation and research. Under such condition, the safety level of a system is not explicitly known (Vrijling 1998). For many practical structural and geotechnical problems, this approach is good enough for most of the practical works, and has proved to be acceptable in general.

Traditionally, the geotechnical adequacy is commonly expressed by the concept of safety factor. A safety factor can be expressed as the ratio of capacity to demand. Since there may be different definitions to the meaning of capacity and demand, the safety factor will vary according to the definition, and a unique value may not exist in some cases. The safety factor concept, however, has shortcomings as a measure of the relative reliability of geotechnical structures. A primary deficiency in this factor is that the control parameters (material properties, strengths, loads, *etc.*) must be assigned single precise values, but there are actually various uncertainties associated with these parameters. The use of precisely defined single values in an analysis is required in the *deterministic* approach. The safety factor using this approach will then reflect the engineer's judgment, and the degree of conservatism incorporated into the parameter values.

In general, deterministic approach may be applicable to most of the structural engineering problems, provided that appropriate factor of safety can be defined. On the other hand, geotechnical engineering is different from some engineering disciplines in that the properties of geomaterials and the distribution of the materials are highly uncertain by nature, and the uncertainties are much higher as compared with those in structural engineering. Although the presence of geotechnical uncertainty has long been recognized, most of the geotechnical engineering design works (including slope engineering) are still based on the concept of overall factor of safety, for which the controlling parameters are assumed to be deterministic while the uncertainties are accounted for the use of empirical safety factors which are large enough to cover the uncertainty. The safety factors in geotechnical engineering are hence greater than those in the structural engineering discipline. For example, a factor of safety of 3.0 and 2.0 is applied to shallow foundation and deep foundation in Hong Kong, which are much greater than the safety factor in structural or bridge design. Although the use of partial safety factors concept has appeared in Euro code as well as other design codes which has partially considered the uncertainties of some material properties and applied loadings, such concept is however more appropriate to linear system (typically in structural engineering works) than nonlinear systems in geotechnical engineering works. Also, the spatial variability of the material properties and distribution cannot be reflected by the use of partial safety factors. Safety factors are determined based on experience and long term observation and applications, but do not absolutely guarantee the safety or satisfactory performance under all cases. They also provide no information on how the uncertainties of different parameters influence the safety of the system. Some engineers argue that the classical approach has been used with satisfaction for a long time, as long as an acceptable factor of safety can be defined. This argument is basically correct, except that it is also observed in Hong Kong that about 5% of the stabilized slopes will still eventually fail under the current analysis and design practice. For cases where the appropriate factor of safety cannot be determined due to high uncertainties, engineers tend to use very high factors of safety to cover all these problems (Three Gorge project in China and other cases).

Probabilistic design approach with reliability and risk analysis concepts deal with the uncertainties in the geomaterial parameters, distribution, loadings and even failure mechanism. Such analysis will inevitably be more complicated, and due to a lack of sufficient information available for many practical projects, reliability analysis is not commonly carried out in geotechnical works.

Probabilistic analysis can bring rationality to the consideration of uncertainty in the analysis and design, provides more information about the system behavior, the influence of different uncertain variables on the system performance, and the

interaction between different system components (Haldar and Mahadevan 2000a, 2000). Due to the high uncertainty associated with the ground conditions, material properties, ground water conditions, loadings, computational models and other factors, reliability analysis in geotechnical engineering is more important than that in many other disciplines. During the past three decades, there are increasing demand and research works for the probabilistic analysis of the stability of slopes. It is well accepted that the probabilistic methods can play a complementary role to the conventional deterministic factor-of-safety method, and will be useful for some important projects where the consequence of failure can be high.

The following four levels of approach in determination of the safety of a structure is suggested by TAW (1985):

- Level 0: Deterministic approach, for which the design is based on the average situations;
- Level I: Semi-probabilistic approach, for which a characteristic value is used in the design, like the load which is not exceeded in 95% of the cases, or the strength which is available for 95% of the construction material;
- Level II: Probabilistic approach with statistical distributions of all variables are taken into account. Level II comprises a number of approximate methods in which the distribution functions are transformed into standard normal, Gaussian or lognormal distributions.
- Level III: the highest level probabilistic approach and the probability distribution functions of the stochastic variables are fully taken into account.

For practical purposes it is adopted there are three sources of uncertainty (Morgenstern 1995):

- I. Parameter uncertainty
- II. Model uncertainty
- III. Human uncertainty

Parameter uncertainty is readily understood and has received considerable attention in the geotechnical literature. It is concerned with input variables such as the spatial variations of parameters like strength or compressibility and the lack of data for key parameters. Many examples exist in the literature in which the statistical distribution, say, of strength is specified and the traditional Factor of Safety is replaced by a probability of failure. The model, an equation for Factor of Safety based on limit equilibrium assumptions, is taken as certain. However, if,

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