RADIAL BASIS FUNCTION METHODS FOR LARGE-SCALE WAVE PROPAGATION

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PREFACE

With the rapid development of computer technology, the radial basis function (RBF) method has become an important computational tool in addition to the finite element and boundary element methods. In practical applications of the RBF, constructing efficient and stable algorithms for solving complex scientific and engineering problems is a key issue. Efficient solutions of fully dense matrices, ill-conditioned matrices, and stable simulation of wave propagation at low sampling frequencies are all critical issues. Research till date into RBF methods and the corresponding multilevel meshless methods has resulted in the publication of a considerable amount of new information, leading to improvements in design and fabrication practices. Articles have been published in a wide range of journals, attracting the attention of both researchers and practitioners with backgrounds in the mechanics of solids, applied physics, applied mathematics, mechanical engineering, and materials science. However, extensive and detailed treatment of the subject is not available. It now appears timely to collect available information and to present a unified treatment of these useful results. These valuable results should be made available to professional engineers, research scientists, workers and students in applied mechanics and material engineering, e.g., physicists, metallurgists, and materials scientists. The objective of this book is to fill such a gap so that readers can access sound knowledge of the RBF and its algorithm implementation in wave propagation problems. This book details the development of various techniques and ideas, beginning with a description of the basic concept of the radial function method from the mathematical viewpoint. The derivation and construction of RBFs are then presented for large-scale wave propagation problems, including singularity problems, high frequency problems and large-scale problems, and are shown to arise naturally in the response of engineering structures to external loads such as external mechanical force.

Our presentation of RBF for large-scale wave propagation is written for researchers, postgraduate students and professional engineers in the areas of solid mechanics, physical science and engineering, applied mathematics, mechanical engineering, and materials science. Only simple mathematical knowledge and the usual calculus are required, although conventional matrix presentation is used throughout the book.

The outstanding features of our book include detailed derivation of formulations used in the RBF, simplified explanations of complex problems, combined uses of

RBF and other numerical methods. Methods and analyses are described in a way that makes them accessible to research scientists, professional engineers, and postgraduate students. Furthermore, widely ranging numerical examples are added to the relevant chapters to demonstrate applications of the formulation. Most numerical results presented in this book are important in their own right and serve as test problems for validating new formulations.

This book, consisting of 9 chapters and 2 appendices, covers the fundamentals of the RBF and its methodologies, with extensive applications to various engineering problems. In Chapter 1, the historical background of the RBF is presented, with an overview of the RBF in large-scale and high-frequency wave propagation computing. Chapter 2 discusses the singular boundary method (SBM) for water wave problems. Chapter 3 discusses the applications of SBM to 3-D low-frequency and middle-frequency acoustic problems. Chapter 4 discusses the RBF based on the modified fundamental solutions for high-frequency acoustic problems. Chapter 5 introduces a modified multilevel fast multipole algorithm based on the potential model. Chapter 6 constructs a dual-level fast multipole boundary element method based on the Burton-Miller formulation. Chapter 7 describes a time-dependent SBM for solving scalar wave equations. Chapter 8 proposes a regularized method of moments for time-harmonic electromagnetic scattering. Chapter 9 reviews recent advances and emerging applications of the RBF for large-scale and high-frequency sound wave propagation.

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CONFLICT OF INTEREST

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CHAPTER 1

Introduction to Radial Basis Function Method

Abstract: This chapter presents an overview of the radial basis function (RBF) method and its application in wave propagation problems. First, the historical background development of the radial basis function method is reviewed. Then, applications of RBF to problems of large scale high-frequency wave propagation are described. Strategies are presented for reducing sampling frequency and computational and storage complexity while maintaining high stability, accuracy, and efficiency. This is followed by a list of methods using the RBF in simulating wave propagation problems, including the boundary element method, method of fundamental solutions, singular boundary method, boundary knot method, method of moments, and fast multipole method.

Keywords: Fundamental solution, Radial basis function, Wave propagation.

1.1. HISTORICAL BACKGROUND

With the rapid development of computer technology, numerical simulation has become the third scientific research tool besides theoretical analysis [1-10] and experimental research [11-16], in which [1-7] various analytical solutions of piezoelectric materials [8], detailed numerical solutions of biological materials, and [9,10] described finite element solutions of compsite materials are focused on. The radial basis function (RBF) discussed in this book is a basis function based on the Euclidean distance [17, 18]. There are two versions for the origin of the radial basis function. One version is known as the Kriging method. In 1951, Kriging treated the deposition of mineral deposits as a kind of isotropic Gauss function and proposed the well-known Kriging method [19] for analysis of mineral deposits. The other version considers the radial basis function presented in 1971. Hardy proposed the multi-quadratic (MQ) function [20] for surface fitting of aircraft contour.

A milestone in the development of RBF is the publishing of the article ‘Scattered data interpolation: tests of some methods [21] which was published in *Mathematics of Computation*. In that article, Franke compared different interpolation algorithms and showed that the MQ function and thin-plate spline function [22] had the best fitting effect. Since then, the RBF has been widely used in the field of data processing and interpolation. The other milestone is attributed to articles published in *Computers and Mathematics with Application* [23, 24]. Kansa combined the collocation method with the RBF to form a new method for computational fluid

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dynamics. That was the application of the RBF to the field of numerical solution of partial differential equations. Since then, many RBF have been proposed to deal with different partial differential equations encountered in scientific and engineering computing.

Although the RBF has the advantage of being meshless and easy to use, the resulting fully dense and highly ill-conditioned matrix becomes the biggest barrier to the application of the RBF to large scale scientific and engineering computing. To bypass this limitation, various techniques have been proposed. These techniques can be briefly divided into three categories:

1) Transfer the global support structure of the method to a local support structure;

2) Transfer the fully dense matrix to a sparse matrix;

3) Expand the effective storage digits of the computer.

Essentially, numerical simulation is a computer-based technology for solving partial differential equations (PDE) as depicted in Fig. **(1)**. Generally speaking, the first step to deal with a physical problem is to construct the corresponding PDE to describe the physical phenomenon, such as the wave equation and the Maxwell's equations. In step 1, some simplified conditions are inevitably added to construct the mathematical model. Therefore, the model error is generated. Secondly, the PDE is discretized into linear equations by the corresponding numerical method, such as the finite element method (FEM) and the boundary element method (BEM). In step 2, the mathematical model with infinite degrees of freedom (DOF) is discretized into linear equations with finite DOFs. Inevitably, therefore, a discretization error is generated. Finally, the linear equations are solved by an appropriate numerical solver, such as the Gauss solver and the generalized minimal residual algorithm (GMRES) [25]. Step 3 inevitably generates truncation error due to the limitation of the computer's effective storage digits. Model error, discretization error, and truncation error constitute the three major error sources in numerical simulation. How to efficiently reduce these errors and balance their influence on results directly determines the effectiveness and accuracy of numerical simulations.

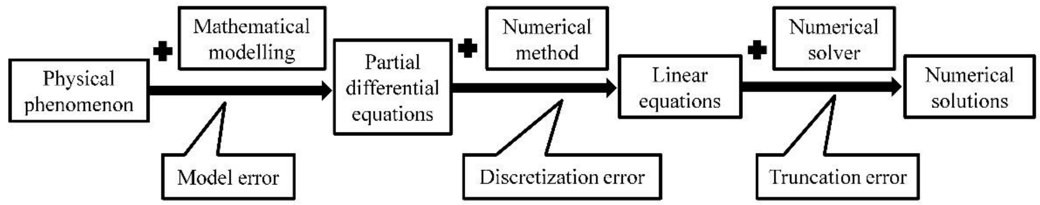

Fig. (1). Process of numerical simulation.

This book takes large scale high-frequency wave propagation as the physical background. A collection of the newly emerging numerical techniques for simulation of wave propagation are discussed. These works mainly focus on how more efficiently to deal with above steps 2 and 3 in numerical simulation, *i.e.*,

1) How to discretize PDEs into more appropriate linear equations;

2) How to solve linear equations more efficiently.

Several novel strategies and ideas for dealing with the three major computational bottlenecks encountered in simulation of wave propagation are discussed in this book, *i.e.*,

1) How to greatly reduce the computational and storage complexity caused by the fully dense matrix;

2) How to efficiently solve linear equations that consist of highly ill-conditioned and fully dense matrices;

3) How to stably simulate wave propagation at low sampling frequency.

The bottlenecks discussed above also limit the development of computational mechanics. Therefore, if a method can efficiently, accurately, and stably simulate large scale high-frequency wave propagation, the method will be powerful for simulating complex scientific and engineering problems. The wave problem is used as the physical background of the book, not only increasing its scientific research significance, but also providing examples to deal with practical engineering problems by efficient numerical methods. Because these three computational bottlenecks also exist widely in various scientific and engineering computing, this book may provide a reference for simulating other more complex scientific and engineering problems.

CHAPTER 2

Singular Boundary Method Analysis of Obliquely Incident Water Wave Passing through Submerged Breakwater

Abstract: Further to the introduction of radial basis function (RBF) methods given in Chapter 1, this chapter presents a brief description of the application of RBF to two-dimensional Helmholtz problems. The background of this chapter is that, in ocean engineering, a submerged breakwater have the advantages of allowing water circulation, permitting fish passage, and providing economic protection. Therefore, such breakwaters have been widely applied to reduce the energy of transmitted water waves. Under the assumption of linear wave theory, the simulation of a water wave passing through a submerged breakwater can be simplified for solving modified Helmholtz equations. In this chapter, the analysis of an oblique incident water wave passing through a submerged breakwater is taken as the engineering background for investigating the use of the singular boundary method for two-dimensional modified Helmholtz problems. The origin intensity factors for the 2-D Laplace equation, Helmholtz equation, and modified Helmholtz equation are listed.

Keywords: Radial basis functions, Singular boundary method, Water wave propagation, Fundamental solution.

2.1. MATHEMATICAL FORMULATION

2.1.1. Govern Equation

Consider a 2-D obliquely incident water wave passing through a submerged barrier, as shown in Fig. **(1)** [1-5]. A water wave with the incident wave angle θ propagates toward the submerged structure at a constant water depth h, where L is the half-length of the region II, D denotes the distance between two barriers, d is the height of the barrier, and b the width of the barrier.

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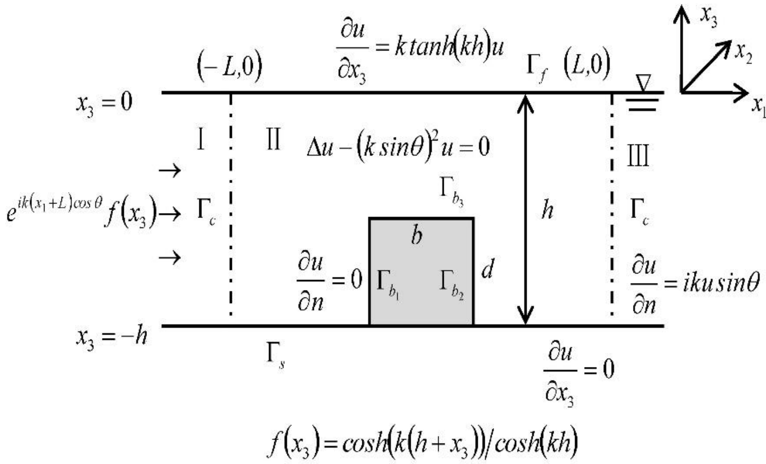

Fig. (1). Water wave passing through a submerged barrier [1].

Under the assumptions of linear water wave theory, the water waves considered here satisfy the following three basic assumptions:

1) The fluid is ideal incompressible. The gravity cannot be ignored.

2) The movement is non-rotational. There exists velocity potential.

3) The waves are linear waves. The depth of the water is much greater than the wavelength.

Assuming an inviscid, incompressible fluid and an irrotational flow, the wave field is represented by the velocity potential $\Phi(x_1, x_2, x_3, t)$, which satisfies the Laplace equation:

$$\nabla^2 \Phi(x_1, x_2, x_3, t) = 0\,. \tag{1}$$

Based on the simple harmonic assumption, the potential can be expressed as:

$$\Phi(x_1, x_2, x_3, t) = \phi(x_1, x_3) e^{(\lambda x_2 - \sigma t)i}\,, \tag{2}$$

where $\lambda = k\sin(\theta)$, k is the wavenumber and satisfies the dispersion relation:

$$\sigma^2 = gk \tanh(kh), \tag{3}$$

with g being the acceleration of gravity. Substituting Eq. (2) for Eq. (1), the governing equation is simplified to the modified Helmholtz equation:

$$\nabla^2 \phi(x_1, x_3) - \lambda^2 \phi(x_1, x_3) = 0 \tag{4}$$

2.1.2. Boundary Conditions

The boundary conditions of the domain of interest can be written as follows:

(1) Linearized free water surface boundary condition

$$\frac{\partial \phi(x_1, x_3)}{\partial x_3} - \frac{\sigma^2 \phi(x_1, x_3)}{g} = 0, (x_1, x_3) \in \Gamma_f \tag{5}$$

(2) Seabed boundary condition

$$\frac{\partial \phi(x_1, x_3)}{\partial x_3} = 0, (x_1, x_3) \in \Gamma_s. \tag{6}$$

(3) Submerged breakwater boundary conditions

(3a) Rigid boundary condition

$$\frac{\partial \phi(x_1, x_3)}{\partial x_1} = 0, (x_1, x_3) \in \Gamma_{b_1}, \tag{7}$$

$$\frac{\partial \phi(x_1, x_3)}{\partial x_1} = 0, (x_1, x_3) \in \Gamma_{b_2}, \tag{8}$$

$$\frac{\partial \phi(x_1, x_3)}{\partial x_3} = 0, (x_1, x_3) \in \Gamma_{b_3}. \tag{9}$$

CHAPTER 3

Singular Boundary Method for Three-Dimensional Low and Middle Frequency Acoustic Problems

Abstract: As an extension of the methods presented in Chapter 2, this chapter takes acoustic problems as the physical background to apply the singular boundary method to the three-dimensional Helmholtz equation. Based on the Burton-Miller formulations, the Burton-Miller singular boundary method is constructed to deal with the non-uniqueness difficulties at resonance frequency. Two seta of origin intensity factors based on the subtraction and adding-back technique are derived. In addition, for efficient evaluation of near-boundary and boundary solutions of the Helmholtz equation with wideband wavenumbers, this chapter proposes a regularized singular boundary method and derives a set of near intensity factors based on the subtraction and adding-back technique.

Keywords: Singular boundary method, Sound wave propagation, Origin intensity factor, Near intensity factor.

3.1. INTRODUCTION

The singular boundary method (SBM) is a strong form boundary collocation semi-analytical radial basis function (RBF) method, as described in [1]. Gu and Chen [2] applied the SBM to potential problems in 2013. Fu *et al*. [3] constructed the SBM based on the Burton-Miller formulation for acoustic problems. Unlike in the finite element method [4-6] and method of fundamental solution [7, 8], the core idea of the SBM is to use the origin intensity factor (OIF) to replace the diagonal terms of an interpolation matrix. The key issue is to derive the OIF of different partial differential equations [9-11]. By using the appropriate OIF, the SBM only needs 6 degrees of freedom (DOF) in each direction per wavelength to simulate sound wave propagation. Because the OIF has the effect of correcting the boundary discrete error [12], the SBM consumes much less CPU time and calculation than the linear boundary element method (BEM) to achieve higher accuracy and convergence rate. The SBM achieves a good balance between stability, accuracy, and complexity. Therefore, the SBM is very suitable to be combined with the fast algorithm for large-scale engineering problems.

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The key issue of the SBM is to derive the appropriate OIF for the studied problems. Gu and Chen [2] derived the OIF for 3-D potential problems based on the subtraction and adding-back technique (SAB) of the Laplace equation. Later, based on the similarity of the fundamental solution of the Laplace equation and Helmholtz equation at origin, Fu *et al*. [3] transformed OIFs of the Laplace equation to the OIFs of the Helmholtz equation by adding a constant. Many numerical experiments [13] indicated that the OIF derived by Fu *et al*. could accurately evaluate low and middle frequency acoustic problems. However, the accuracy of the OIF they derived behaved poorly in high frequency situations. Based on the work in [3], Li *et al*. [14] proposed a set of OIFs using the subtraction and adding-back (SAB) technique of the Helmholtz equation. On the other hand, the slope of the fundamental solutions of the Helmholtz equation changes dramatically when the field point moves closer to the boundary or is on the boundary. Therefore, the conventional SBM cannot produce a reasonable solution at regions near the boundary. To bypass this limitation, Gu *et al*. [15] proposed the concept of a 'near intensity factor' (NSF, here called 'near intensity factor') to replace the corresponding near singular terms in the interpolation matrix of the SBM. However, the NSF for Helmholtz equation is still an open issue to be investigated.

In this chapter, the SBM based on the Burton-Miller formulation is introduced in section 3.2. Two sets of OIF based on the SAB technique of the Laplace equation and Helmholtz equation are provided in section 3.3. A regularized SBM [16] is proposed to determine the NSF in section 3.4. Finally, two benchmark examples are used to test the SBM in simulating acoustic problems.

3.2. SINGULAR BOUNDARY METHOD BASED ON THE BURTON-MILLER FORMULATIONS

The 3-D Helmholtz equation is expressed as

$$\nabla^2\phi(x) + k^2\phi(x) = 0, \forall x \in \Omega, \tag{1}$$

$$\phi(x) = \bar{\phi}(x), \forall x \in S_1, \tag{2}$$

$$q(x) = \bar{q}(x), \forall x \in S_2, \tag{3}$$

where ∇^2 is the Laplacian operator, $\phi(x)$ is the physical variable, $q(x)$ the normal derivative of $\phi(x)$, k denotes the wavenumber, S is the boundary of the domain Ω.

The SBM uses the Burton-Miller formulations [3] to deal with well-known non-uniqueness difficulties. The interpolation formulation of the SBM based on the Burton-Miller formulations is

$$\phi(x_m) = \sum_{j=1}^{N} \beta_j \left[G(x_m, y_j) + \alpha \frac{\partial G(x_m, y_j)}{\partial n^e(y_j)} \right], x_m \in \Omega, \tag{4}$$

$$q(x_m) = \sum_{j=1}^{N} \beta_j \left[\frac{\partial G(x_m, y_j)}{\partial n^e(x_m)} + \alpha \frac{\partial^2 G(x_m, y_j)}{\partial n^e(y_j) \partial n^e(x_m)} \right], x_m \in \Omega, \tag{5}$$

where β_j is an unknown coefficient, x the collocation point, y the source point, $\alpha = i/(k+1)$ [17]. The superscript e represents the exterior domain and the corresponding fundamental solutions are

$$G(x, y) = \frac{e^{ikr}}{4\pi r}, \tag{6}$$

$$K(x, y) = \frac{\partial G(x, y)}{\partial n^e(x)} = \frac{e^{ikr}}{4\pi r^3}(ikr - 1)\left\langle (x, y) \cdot n^e(x) \right\rangle, \tag{7}$$

$$F(x, y) = \frac{\partial G(x, y)}{\partial n^e(y)} = -\frac{e^{ikr}}{4\pi r^3}(ikr - 1)\left\langle (x, y) \cdot n^e(y) \right\rangle, \tag{8}$$

$$H(x, y) = \frac{\partial^2 G(x, y)}{\partial n^e(y) \partial n^e(x)} = \frac{e^{ikr}}{4\pi r^3} \begin{bmatrix} (1 - ikr)\left\langle n^e(y) \cdot n^e(x) \right\rangle \\ +(k^2 - 3/r^2 + 3ki/r)\left\langle (x, y) \cdot n^e(y) \right\rangle \left\langle (x, y) \cdot n^e(x) \right\rangle \end{bmatrix}. \tag{9}$$

It is noted that the fundamental solutions encounter singularities and hypersingularities when $x_i = y_j$. The SBM uses the OIF to replace the

CHAPTER 4

RBF Based on the Modified Fundamental Solutions for High-Frequency Acoustic Problems

Abstract: The conception of modified fundamental solution of the 3-D Helmholtz equation is described in this chapter. Based on the modified fundamental solution, a modified singular boundary and a dual-level method of fundamental solutions are constructed. The merits of the proposed methods are that they inherit the high efficiency and accuracy of the boundary knot method and the method of fundamental solutions, whereas the high stability of the singular boundary method is not affected. For illustration, several acoustic radiation and scattering examples are investigated. Numerical experiments show that the present radial basis function (RBF) method based on the modified fundamental solutions only needs to set about 3 degrees of freedom in one wavelength per direction to produce highly accurate solutions for three-dimensional acoustic problems. At the end of this chapter, the influence of the fictitious boundary on results is analyzed in an appendix.

Keywords: Singular boundary method, Boundary knot method, Method of fundamental solutions, Sound wave propagation.

4.1. INTRODUCTION

It is well known that the core difficulty in a simulation of sound wave; simulation is the high sampling frequency required by the RBF [1]. As boundary collocation RBF, the boundary knot method (BKM) [2, 3] and the singular boundary method (SBM) [4, 5] share some common properties but have their own merits and demerits as well. The merits of the BKM are high accuracy and efficiency. However, marked ill-conditioning restricts its application to engineering problems. On the other hand, the SBM has high stability but still needs 6 degrees of freedom (DOF) in one wavelength per direction to generate accurate solutions for 3-D acoustic problems. In this chapter, a modified singular boundary method (MSBM) [6] is considered. The MSBM uses a modified fundamental solution of the 3-D Helmholtz equation as the basis function. The concept of the modified fundamental solution is based on the assumption that the numerical characteristics of the RBF are determined by the property of the basis function. Therefore, the basis functions of the SBM and the BKM are combined to generate a modified fundamental solution of the 3-D Helmholtz equation. Numerical investigations show that the MSBM can produce highly accurate solutions while maintaining high stability and low sampling

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frequency. However, the modified fundamental solution used in the MSBM abandons the imaginary part. Thus, the MSBM cannot directly simulate exterior wave propagation. To overcome this drawback, a dual-level method of fundamental solutions (DLMFS) [7, 8] is constructed. The DLMFS shares an idea similar to the MSBM, that is, to use the modified fundamental solution to replace the fundamental solution of the 3-D Helmholtz equation as the basis function. The DLMFS significantly improves the stability and inherits the high efficiency of the method of fundamental solution (MFS) [9-12], because the modified fundamental solution used in the DLMFS can automatically satisfy the radiation boundary condition at infinity. Therefore, the DLMFS can directly deal with exterior problems.

4.2. MODIFIED SINGULAR BOUNDARY METHOD

The governing equation of the propagation of the steady sound wave in the isotropic medium is reduced to the Helmholtz equation with a constant wavenumber.

$$\nabla^2\phi(x,y,z)+k^2\phi(x,y,z)=0,(x,y,z)\in\Omega^e, \quad \textbf{(1)}$$

$$\phi(x,y,z)=\overline{\phi}(x,y,z),(x,y,z)\in S, \quad \textbf{(2)}$$

$$q(x,y,z)=\overline{q}(x,y,z)=\frac{\partial\overline{\phi}(x,y,z)}{\partial n},(x,y,z)\in S, \quad \textbf{(3)}$$

In the SBM and the BKM, a linear combination of the basis functions corresponding to different source points is adopted to approximate the physical variable,

$$\phi(x_m)=\sum_{j=1}^{N}\beta_j K(x_m,y_j), x_m\in\Omega, \quad \textbf{(4)}$$

where N is the number of DOF, β_j the unknown coefficient, and K the basis function.

In the SBM, $K_{SBM}=\frac{\cos(kr)}{r}$, which is the real part of the fundamental solution of the 3-D Helmholtz equation. Considering that K_{SBM} has a singularity at the origin, the SBM uses the origin intensity factor (OIF) of the 3-D Helmholtz equation [13, 14] to evaluate singular terms when $r \to 0$. The interpolation formulation of the SBM is

$$\phi(x_i) = \sum_{j=1 \neq i}^{N} \beta_j K_{SBM}(x_i, y_j) + \beta_i K_{SBM}(x_i, y_i), x_i \in S. \tag{5}$$

In the BKM, $K_{BKM} = \frac{\sin(kr)}{r}$, which are the general solutions of the 3-D Helmholtz equation. Because the general solution has no singularity when $r \to 0$, the interpolation formulation of the BKM is

$$\phi(x_i) = \sum_{j=1}^{N} \beta_j K_{BKM}(x_i, y_j), x_i \in S. \tag{6}$$

Because the BKM uses the general solution without singularity at origin as the basis function, 2 DOF are required in each wavelength per direction to generate highly accurate solutions. However, severe ill-conditioning constrains its application to complex engineering problems. On the other hand, the SBM has a much lower number of conditions and avoids the time-consuming singular integration by using the OIF. Unfortunately, 6 DOF are still required in each wavelength per direction, creating heavy computing loads for 3D high-frequency acoustic problems.

It is reasonable to consider the merits of combining the desirable features of the BKM and SBM. Therefore, a modified fundamental solution of 3-D Helmholtz equation is constructed:

$$K_{MSBM} = \frac{\sin(kr + \varphi)}{r}. \tag{7}$$

Considering the trigonometric function,

$$\begin{cases} a\sin(kr) + b\cos(kr) = \sqrt{a^2 + b^2}\sin(kr + \varphi) \\ \tan\varphi = \dfrac{b}{a} \end{cases}, \tag{8}$$

CHAPTER 5

Modified Dual-Level Fast Multipole Algorithm For Three-Dimensional Potential Problems

Abstract: A modified dual-level fast multipole algorithm is constructed for analyzing three-dimensional (3D) potential problems. The core idea of the method is to use a dual-level structure for handling the excessive storage requirement and ill-conditioning caused by the fully populated interpolation matrix. The algorithm uses the fast multipole method to expedite matrix vector multiplication processes. The boundary element method (BEM) is used as the basic method in the algorithm. The 3D potential model is used as the physical background to illustrate this novel algorithm. The complexity analysis shows that the method has $O(N)$ operations and low memory requirements for a 3D potential model.

Keywords: Boundary element method, Fast multipole method, Modified dual-level algorithm, Three-dimensional potential model.

5.1. INTRODUCTION

Highly accurate simulation of large-scale engineering problems is an important task in the field of computational science. In the boundary collocation method [1], the number of degrees of freedom (DOF) increases very quickly due to the $O(N^2)$ computational and storage complexity. Therefore, the major challenge of this large-scale computation is to efficiently reduce the computational complexity and storage complexity. In past decades, many fast algorithms have been proposed to overcome this challenge, such as the fast multipole method (FMM) [2] and the fast Fourier transform (FFT) algorithm [3]. Liu combined the BEM [4] with the FMM to construct a fast multipole boundary element method (FMBEM) [5]. Gu and Qu combined the singular boundary method (SBM) [6, 7] with the FMM to propose a fast multipole SBM [8, 9]. Li combined the SBM with the FFT to develop a fast Fourier transform SBM [3, 10]. All these works are useful in the field of large-scale computing.

This chapter introduces a modified dual-level fast multipole algorithm (MDFMA) for potential 3D problems. The core idea of the MDFMA [11-13] is to transform the large-scale fully populated matrix into a sparse matrix by ignoring the residuals of the far-field contribution. The MDFMA introduces the coarse mesh to provide

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an initial solution. The final accurate solution is obtained by several recursive calculations and correction calculations. For the convenience of physical deduction, a 3D potential model is taken as an example to construct the MDFMA in this chapter.

5.2. BASIC FORMULATIONS OF THE BOUNDARY ELEMENT METHOD

Consider the following 3D Laplace equations:

$$\nabla^2\phi(x) = 0, \forall x \in \Omega. \quad \textbf{(1)}$$

$$\phi(x) = \bar{\phi}(x), \forall x \in S_1, \quad \textbf{(2)}$$

$$q(x) = \frac{\partial\phi}{\partial n}(x) = \bar{q}(x), \forall x \in S_2. \quad \textbf{(3)}$$

The corresponding boundary integral equation (BIE) is

$$\phi(x) = \int_S \left[G(x,y)q(y) - F(x,y)\phi(y)\right] dS(y), \forall x \in \Omega, \quad \textbf{(4)}$$

where

$$G(x,y) = \frac{1}{4\pi r} \quad \textbf{(5)}$$

$$F(x,y) = \frac{\partial G(x,y)}{\partial n(y)} = -\frac{1}{4\pi r^2}\frac{\partial r}{\partial n(y)} \quad \textbf{(6)}$$

are the fundamental solutions of the 3D Laplace equation.

When $x \to S$, the following integral equation is obtained:

$$C(x)\phi(x) = \int_S \left[G(x,y)q(y) - F(x,y)\phi(y)\right] dS(y), \forall x \in S, \quad \textbf{(7)}$$

where $C(x) = \frac{1}{2}$ when the boundary S is smooth at the source point x.

The following discretized form of the BIE is obtained by using the constant elements and collocation methods:

$$\sum_{j=1}^{N} f_{ij}\phi_j = \sum_{j=1}^{N} g_{ij} q_j \ , \tag{8}$$

where

$$g_{ij} = \int_{\Delta S_j} G(x_i, y) dS(y), \ f_{ij} = \frac{1}{2}\delta_{ij} + \int_{\Delta S_j} F(x_i, y) dS(y), \tag{9}$$

with Δs_j being an element, N the number of boundary elements, and x_i the ith source point.

Then Eq. (8) can be reformulated into the form

$$A\lambda = b \tag{10}$$

where A is the coefficient matrix, λ represents the unknown vector and b is the known right-hand side vector.

5.3. BASIC FORMULATIONS OF THE FAST MULTIPOLE BOUNDARY ELEMENT METHOD

To achieve the desired multipole algorithm, the fundamental solution G is expanded by [2, 5, 11]

$$G(x, y) = \frac{1}{4\pi|x-y|} \cong \frac{1}{4\pi}\sum_{n=0}^{p}\sum_{m=-n}^{n} \overline{S}_{n,m}(x, y_c) \times R_{n,m}(y, y_c), |x - y_c| > |y - y_c|, \tag{11}$$

where y_c represents the center of expansion, p is the number of truncated terms, and $\overline{(.)}$ denotes the complex conjugate. $S_{n,m}$ and $R_{n,m}$ are called solid harmonic functions [14], where

$$R_{n,m}(x) = \frac{1}{(n+m)!} P_n^m(\cos\theta) e^{im\varphi} r^n \ , \tag{12}$$

CHAPTER 6

Modified Dual-Level Fast Multipole Algorithm Based on the Burton-Miller Formulation for Large-Scale Sound Field Analysis

Abstract: This chapter extends the modified dual-level fast multipole algorithm (MDFMA) to the field of acoustics. The Burton-Miller formulation is applied to overcome non-uniqueness at internal resonance. The MDFMA based on the Burton-Miller formulation is here tested by a series of complex engineering cases. It is observed that the MDFMA performs 44% faster than COMSOL in the analysis of acoustic scattering characteristics of an A-320 aircraft, and 56% faster than the fast multipole boundary element method in the analysis of underwater acoustic scattering characteristics of the Kilo-class submarine.

Keywords: Modified dual-level algorithm, Boundary element method, Fast multipole method, Three-dimensional sound field.

6.1. INTRODUCTION

Based on the MDFMA [1-4] constructed in Chapter 5, this chapter applies the MDFMA to large-scale sound field problems. The Burton-Miller formulation [5] is combined with the MDFMA to overcome the non-uniqueness issue when evaluating exterior acoustic problems [6]. In this chapter, a variety of complex engineering cases are investigated using the MDFMA, such as the underwater acoustic scattering characteristics of a Kilo-class submarine and the acoustic scattering characteristics of an A-320 aircraft. In particular, comparisons are drawn between the MDFMA and the mature commercial software COMSOL to show the application potential of the MDFMA for large-scale sound field analysis.

6.2. FORMULATIONS OF THE BOUNDARY ELEMENT METHOD

The propagation of a sound wave in an isotropic medium is governed by the Helmholtz equation

$$\nabla^2\phi(x)+k^2\phi(x)=0, \forall x\in\Omega, \tag{1}$$

$$\phi(x)=\overline{\phi}(x), \forall x\in S_1, \tag{2}$$

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$$q(x) = \bar{q}(x), \forall x \in S_2, \tag{3}$$

The acoustic pressure $\phi(x)$ is expressed by

$$\phi(x) = \int_S \left[G(x,y)q(y) - \frac{\partial G(x,y)}{\partial n(y)}\phi(y) \right] dS(y) + \phi^I(x), \forall x \in \Omega, \tag{4}$$

where $\phi^I(x)$ represents the incident wave, x and y are the source point and field point, respectively. The fundamental solutions of the 3-D Helmholtz equation are

$$\begin{cases} G(x,y) = \dfrac{e^{ikr}}{4\pi r} \\ F(x,y) = \dfrac{\partial G(x,y)}{\partial n(y)} = \dfrac{e^{ikr}}{4\pi r^2}(ikr-1)\langle (x,y)\cdot n(y) \rangle \end{cases}, \tag{5}$$

where $i = \sqrt{-1}$, $r = |x-y|$ and $n(y)$ is the outward normal at point y.

When $x \rightarrow S$, the conventional boundary integral equation (CBIE) and hypersingular boundary integral equation (HBIE) are written as

$$C(x)\phi(x) = \int_S \left[G(x,y)q(y) - \frac{\partial G(x,y)}{\partial n(y)}\phi(y) \right] dS(y) + \phi^I(x), \forall x \in S, \tag{6}$$

$$C(x)q(x) = \int_S \left[\frac{\partial G(x,y)}{\partial n(x)} q(y) - \frac{\partial^2 G(x,y)}{\partial n(y)\partial n(x)}\phi(y) \right] dS(y) + q^I(x), \forall x \in \mathrm{S}, \tag{7}$$

where $C(x) = \dfrac{1}{2}$ when the boundary S is smooth, and

$$\begin{cases} K(x,y) = \dfrac{\partial G(x,y)}{\partial n(x)} = -\dfrac{e^{ikr}}{4\pi r^2}(ikr-1)\langle (x,y)\cdot n(x) \rangle \\ H(x,y) = \dfrac{\partial^2 G(x,y)}{\partial n(y)\partial n(x)} = \dfrac{e^{ikr}}{4\pi r^3}\begin{bmatrix} (1-ikr)\langle n(y)\cdot n(x) \rangle \\ +(k^2r^2 - 3 + 3kri)\langle (x,y)\cdot n(y) \rangle \langle (x,y)\cdot n(x) \rangle \end{bmatrix} \end{cases}, \tag{8}$$

The MDFMA combines Eqs. (6) and (7) to overcome the non-uniqueness at internal resonance, *i.e.*, the Burton-Miller formulation,

$$\begin{aligned}&\left[\int_S \frac{\partial G(x,y)}{\partial n(y)}\phi(y)dS(y)+C(x)\phi(x)-\phi^I(x)\right]+\alpha\int_S \frac{\partial^2 G(x,y)}{\partial n(y)\partial n(x)}\phi(y)dS(y)\\&=\int_S G(x,y)q(y)dS(y)+\alpha\left[\int_S \frac{\partial G(x,y)}{\partial n(x)}q(y)dS(y)-C(x)q(x)+q^I(x)\right],\forall x\in S\end{aligned}, \quad \textbf{(9)}$$

where α=i/k [7] is the coupling constant.

The discretized form of CHBIE is given by Eq. (10).

$$\sum_{j=1}^{N} f_{ij}\phi_j=\sum_{j=1}^{N} g_{ij}q_j+\hat{b}_i\ , \quad \textbf{(10)}$$

where $\hat{b}_i$ is the value of the incident wave at the *ith* node, and

$$f_{ij}\phi_j=\int_{\Delta S_j}\frac{\partial G(x,y)}{\partial n(y)}\phi_j dS(y)+\frac{1}{2}\delta_{ij}\phi_j+\alpha\int_{\Delta S_j}\frac{\partial^2 G(x,y)}{\partial n(y)\partial n(x)}\phi_j dS(y)\ , \quad \textbf{(11)}$$

$$g_{ij}q_j=\int_{\Delta S_j}G(x,y)q_j dS(y)+\alpha\left[\int_{\Delta S_j}\frac{\partial G(x,y)}{\partial n(x)}q_j dS(y)-\frac{1}{2}\delta_{ij}q_j\right]. \quad \textbf{(12)}$$

Eq. (10) is reformulated as

$$A\lambda=b \quad \textbf{(13)}$$

by moving the unknown and known terms to left-hand side and right-hand side, respectively.

CHAPTER 7

Time-Dependent Singular Boundary Method for Scalar Wave Equation

Abstract: This chapter presents a time-dependent singular boundary method for solving two- and three-dimensional scalar wave equations. In contrast with previous chapters focused on frequency domain computation, this chapter focuses on the solution of the scalar wave equation instead of the Helmholtz equation. The time-dependent fundamental solution is used here as the basis function. Considering the difference between two- and three-dimensional wave equations, the time-dependent fundamental solution is integrated along with time for solving two-dimensional scalar wave equations. A time-successive evaluation approach without complex mathematical transforms is applied for three-dimensional wave equations.

Keywords: Scalar wave equation, Time-dependent fundamental solution, Time-dependent singular boundary method, Origin intensity factor.

7.1. INTRODUCTION

In this chapter, we consider the simulation of wave propagation in terms of time domain computation [1-5]. In contrast with previous chapters focused on frequency domain computation, this chapter studies the solution of scalar wave equations [6, 7] instead of the Helmholtz equation [8-11]. A time-dependent singular boundary method (SBM) [12, 13] is constructed based on the time-dependent fundamental solution [14-16]. In the simulation of wave equations, the three-dimensional (3-D) wave is totally different from the two-dimensional (2-D) wave. In the 2-D wave equation, the wave has aftereffect. It can be seen that the fundamental solution of 2-D wave equation has product factor $H(ct\text{-}r)$. It means that the wave will have continuous impact on the observation point after the wave passing. Therefore, the time-dependent SBM can easily use finite DOF to describe waves. However, the 3-D wave does not have an after-effect. The product factor of the fundamental solution of 3-D scaler wave equation is $\delta(ct-r)$ instead of $H(ct-r)$. It means that the wave only creates an effect at the observation point when the wave passing the point. After the wave passing the observation point, the wave will not have any aftereffect on the point. Therefore, it is very difficult to describe the 3-D waves by finite DOF. The details about the aftereffect of waves can be seen in [S. Z. Xu, The boundary element method in geophysics. Beijing: Science Press, 1995] and [C. A.

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Brebbia, Progress in Boundary Element Methods: Volume 2. New York: Springer, 1983].

The 2-D wave has an after-effect phenomenon, which means that the wave has a continuous effect on the observation point after the wave surface passes through the observation point. Therefore, the time-dependent fundamental solution is integrated along with time for solving 2-D scalar wave equations [17, 18]. The 3-D wave has no after-effect phenomena, which means that the wave has an effect on the observation point only when the wave surface passes through the observation point. Therefore it is difficult to discretize 3-D wave equations. To overcome this problem, a time-successive evaluation approach without complex mathematical transforms is used to solve 3-D wave equations.

7.2. TIME-DEPENDENT SINGULAR BOUNDARY METHOD FOR 2-D WAVE EQUATIONS

The scalar wave equation is written as

$$\phi = \begin{cases} \Delta\phi - \dfrac{1}{c^2}\dfrac{\partial^2\phi}{\partial t^2} = 0, X \in \Omega, t > 0 \\ \phi\big|_{\Gamma} = \bar{\phi}, X \in \Gamma, t > 0 \\ \phi\big|_{t=0} = \phi_0, \dfrac{\partial\phi}{\partial t}\Big|_{t=0} = \upsilon_0, X \in \Omega, t = 0 \end{cases} \quad \textbf{(1)}$$

where Ω is the computational domain with the boundary Γ, c is the wave speed, t denotes time. The fundamental solution of the 2-D scalar wave equation is

$$G(t,r) = \frac{c}{2\pi\sqrt{c^2t^2 - r^2}} H(ct - r), \quad \textbf{(2)}$$

where $H(x)$ is the Heaviside function

$$H(x) = \begin{cases} 0, x < 0 \\ 0.5, x = 0 \\ 1, x > 0 \end{cases}. \quad \textbf{(3)}$$

Eq. (1) can be interpreted as the initial boundary value problem. Based on the superposition principle, Eq. (1) can be split into a boundary value problem and an initial value problem. After Eqs. (4) and (5) are solved by the time-dependent SBM respectively, the solution of Eq. (1) can be obtained simply by the addition of these two solutions: $\phi = \phi_1 + \phi_2$.

$$\phi_1 = \begin{cases} \Delta\phi_1 - \dfrac{1}{c^2}\dfrac{\partial^2\phi_1}{\partial t^2} = 0, X \in \Omega, t > 0 \\ \phi_1\big|_{t=0} = \phi_0, \dfrac{\partial\phi_1}{\partial t}\Big|_{t=0} = \upsilon_0, X \in \Omega, t = 0 \\ \phi_1\big|_{t=0} = 0, \dfrac{\partial\phi_1}{\partial t}\Big|_{t=0} = 0, X \notin \Omega, t = 0 \end{cases}, \tag{4}$$

$$\phi_2 = \begin{cases} \Delta\phi_2 - \dfrac{1}{c^2}\dfrac{\partial^2\phi_2}{\partial t^2} = 0, X \in \Omega, t > 0 \\ \phi_2\big|_{\Gamma} = \overline{\phi}_2 = \overline{\phi} - \overline{\phi}_1, X \in \Gamma, t > 0 \\ \phi_2\big|_{t=0} = 0, \dfrac{\partial\phi_2}{\partial t}\Big|_{t=0} = 0, X \in \Omega, t = 0 \end{cases}. \tag{5}$$

The initial value problem Eq. (4) can be solved directly using the 2-D Poisson's formulation

$$\phi_1 = \frac{1}{c^2}\iint_{C_{ct}^M} \phi_0 \frac{\partial G}{\partial n} ds + \frac{1}{c^2}\iint_{C_{ct}^M} \upsilon_0 G ds, \tag{6}$$

where C_{ct}^M represents the circle domain with radius ct and center M . C_{ct}^M is the range of influence of computational point M .

Then consider the boundary value problem Eq. (5). The time-dependent SBM uses the fundamental solution integrated with time as the basis function.

$$G_0 = \int_0^t G d\tau = \frac{1}{2\pi}\left[H(c(t-\tau)-r)\cosh^{-1}(\mathrm{c}(t-\tau)/r)\right]\Big|_{\tau=0}^{\tau=t}. \tag{7}$$

CHAPTER 8

Regularized Method of Moments for Time-Harmonic Electromagnetic Scattering

Abstract: This chapter extends the application of the theory of the radial basis function method to computational electromagnetics. Several key mechanical issues involved in the process of high-precision calculation of electromagnetic scattering are discussed. A regularized method of moments based on the modified fundamental solution of the three-dimensional Helmholtz equation is constructed in this chapter. The origin intensity factor is used to evaluate the singular term of interpolation matrix. Non-uniqueness at internal resonance is avoided by using the modified fundamental solution as the basis function. The regularized method of moments reduces the consumed CPU time by half compared to the traditional method of moments, while stability and accuracy are not affected. Experiments indicate that the regularized method of moments can accurately evaluate the radar cross section of perfect conducting scatter over all frequency ranges.

Keywords: Electric field integral equation; Radar cross section; Method of moments; Modified fundamental solution.

8.1. INTRODUCTION

High-precision calculation of electromagnetic scattering of three-dimensional complex targets has important application in industrial fields. The chapter is based on the theory of the radial basis function method (RBF) [1, 2]. The main purpose of this work is to extend the application of the theory of the RBF to computational electromagnetics [3, 4]. Several key mechanical issues involved in the process of high-precision calculation of electromagnetic scattering are discussed, including how to eliminate non-uniqueness at internal resonance without increasing the number of calculations and storage requirements, and how to obtain an accurate evaluation of singular terms in matrix. A regularized method of moments (RMOM) [5] based on the modified fundamental solution of the three-dimensional (3-D) Helmholtz equation is constructed for evaluating the radar cross section (RCS) [6] of a perfect conducting scatter. The traditional method of moments (MOM) [7, 8] suffers the deficiencies of singularity at origin and non-uniqueness at internal resonance [9, 10]. The traditional strategy combines the electric field integral equation (EFIE) [11,12] with the magnetic field integral equation (MFIE) [13] to remove non-uniqueness at a given resonance frequency. The RMOM uses the

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origin intensity factor (OIF) [14, 15] to replace the singular term in the interpolation matrix. The modified fundamental solution of the 3-D Helmholtz equation is used as the basis function to remove the non-uniqueness at internal resonance. The amount of calculation and storage space is not affected by the modified fundamental solutions. Compared to the combined field integral equation (CFIE) [16, 17], the consumed CPU time of the RMOM is reduced almost by half. As shown in the following examples, the RMOM can accurately calculate the RCS of complex targets at all frequency ranges.

8.2. THE REGULARIZED METHOD OF MOMENTS FOR THREE-DIMENSIONAL ELECTROMAGNETIC SCATTERING

The electric field integral equation (EFIE) of a perfect conducting scatter is written as

$$\bar{E}^S(\bar{r}) = i\omega\bar{A}(\bar{r}) - \nabla\Phi(\bar{r}), \tag{1}$$

where ∇ is the Laplace operator and $\bar{E}^S$ is the scattered electric field. ω denotes the angular frequency. A bar over a variable indicates that the variable is a vector. The magnetic vector potential $\bar{A}$ is expressed as

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi}\int_S \bar{J}(\bar{r}')G(\bar{r}')dS', \tag{2}$$

and the scalar potential Φ is written as

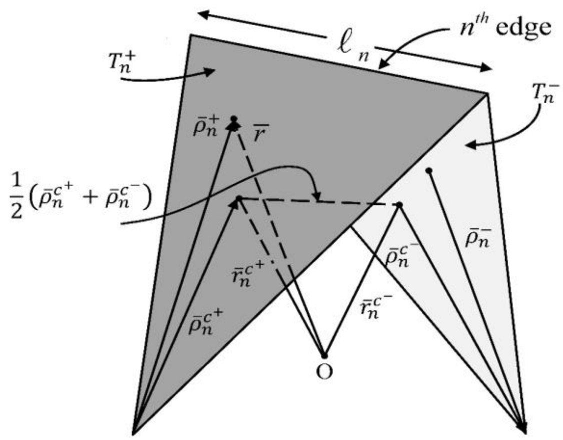
$$\Phi(\bar{r}) = \frac{1}{4\pi\varepsilon}\int_S \sigma(\bar{r}')G(\bar{r}')dS', \tag{3}$$

where $\bar{r}$ denotes a collocation point and $\bar{r}'$ represents a source point. $k = \omega\sqrt{\mu\varepsilon} = 2\pi/\lambda$ is the wavenumber, λ is the wavelength. $\mu = 4\pi\times10^{-7}$ and $\varepsilon = 8.854187817\times10^{-12}$ are the permeability and the permittivity respectively of the surrounding free space.

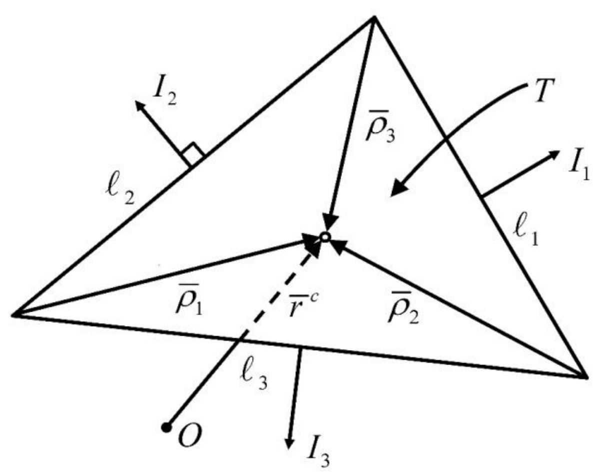
$\bar{J}$ is the induced surface currents:

$$\bar{J} = \sum_{n=1}^{Nl} I_n \bar{f}_n(\bar{r}), \tag{4}$$

where I_n is the unknown coefficient. $\bar{f}_n$ is the RWG vector basis function associated with the nth edge, as depicted in Fig. (**1**).



(a)



(b)

Fig. (1). RWG vector basis function.

CHAPTER 9

Recent Advances and Emerging Applications of the Radial Basis Function Method for Simulation of Large-Scale and High-Frequency Sound Wave Propagation

Abstract: With the rapid development of computer technology, numerical simulation has become the third scientific research tool besides theoretical analysis and experimental research. As the core of numerical simulation, the construction of efficient, accurate and stable numerical methods to simulate complex scientific and engineering problems have become a key issue in the field of computational mechanics. This chapter outlines the application of the radial basis function method for the simulation of large-scale and high-frequency sound wave propagation. All the techniques and methods discussed in the book are reviewed in this chapter. This collection can provide a reference for the simulation of other more complex wave propagation.

Keywords: Radial basis function method, Singular boundary method, Computational acoustics, Origin intensity factor, Helmholtz equation.

9.1. OVERVIEW OF THE SBM IN LOW AND MIDDLE FREQUENCY SOUND FIELD CALCULATIONS

The singular boundary method (SBM) is a strong form boundary collocation semi-analytical radial basis function method. Presented in 2009 [1], the SBM has become a popular computational tool in engineering. Gu and Chen [2] applied the SBM to potential problems. Later, Fu *et al.* [3] constructed the SBM based on the Burton-Miller formulation for acoustic problems. The core idea of the SBM is to use the origin intensity factor (OIF) to replace the diagonal terms of an interpolation matrix. The key issue is the determination of the OIF for different PDEs [2-6]. With the use of appropriate OIF, the SBM needs only 6 DOFs in each direction per wavelength to simulate the propagation of a sound wave.

The SBM has higher stability and wider applicability as compared to the MFS [7, 8]. In comparison with the BEM [9-11], the SBM is free of integration and mesh. As a result, it is much easier to program. Because the OIF has the effect of correcting boundary discrete errors, the SBM can consume much less CPU time

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and amount of calculation than the linear BEM, achieving higher accuracy and a superior convergence rate. The SBM achieves a good balance between stability, accuracy, and complexity. Therefore, the SBM is very suitable for combining with fast algorithms in solving large-scale engineering problems. Another advantage of the SBM is that it is very easy to program, as mentioned above. Scientific research is a process of constant trial and error. Usually, when new physical problems are explored, they cannot be directly simulated by commercial software such as COMSOL Multiphysics®. Much valuable time can be saved by using the SBM as the basic algorithm to explore new physical problems.

In the frequency domain, the governing equation of an acoustic problem is a 3-D Helmholtz equation. The SBM based on the Burton-Miller formulation proposed by Fu *et al.* [3] is the standard interpolation format of the SBM for 3-D acoustic problems. The key issue of the SBM is accurate evaluation of the OIF. The OIF assumes that there exists a set of OIF that can perfectly replace the singular terms in the matrix. The OIF should be easy to obtain and be free of integration. It can make the SBM reach optimal accuracy and convergence rate without destroying stability. The first OIF technique was the inverse interpolation technique (IIT) proposed by Chen and Wang [1]. The IIT needs to solve linear equations twice by introducing a set of sample points to obtain the approximate value of the OIF. The IIT technique enables the SBM to obtain a very high accuracy and convergence rate. However, the IIT technique lacks theoretical analysis. It is difficult to obtain a set of appropriate sample points for a complex boundary due to the random deployment of sample points. Therefore, the IIT technique is highly unstable. Later, Gu and Chen [2] derived the OIF of Neumann boundary conditions for 3-D potential problems based on subtraction and adding-back technique (SAB). With this set of OIFs, the SBM can achieve accuracy similar to that obtained with the IIT while avoiding the inverse interpolation. The SAB technique accurately evaluates the OIF of Neumann boundary conditions. However, evaluation of the OIF of Dirichlet boundary conditions still requires an inverse interpolation. Based on the work of Gu, Fu *et al.* [3] converted the OIF of the Laplace equation to the OIF of the Helmholtz equation by adding a constant. The mathematical basis of the conversion is the similarity of the fundamental solution of the Laplace equation and the Helmholtz equation at the origin. Many numerical experiments [12] indicate that the OIF thus determined by Fu *et al.* can accurately evaluate low-frequency and intermediate-frequency acoustic problems.

Inspired by the works of Fu and Gu, a set of regularized OIF [13] based on the SAB technique was derived in Chapter 3. With the construction of general solutions that satisfy a certain boundary condition and then substituting them into the boundary integral equation or hyper boundary integral equation, unnecessary singular terms can be eliminated. The set of regularized OIFs inherits the advantages of the OIF formulas and avoids their disadvantages, as listed in Table 1. It is the best form of expression of the OIF for acoustic problems.

Table 1. Advantage and disadvantage of three sets of mathematical formulas.

Items / OIF	**Accuracy**	**Stability**	**Applicability and Generality**	**Reference**
IIT OIF	High	Low	Low	[1]
SAB OIF	High	Medium	Medium	[3]
Regularized OIF	High	High	High	[13]

From the source of derivation, the OIF can be summarized in the following three forms: the mathematical formula [13], the experimental formula [5, 6] and the physical formula [14, 15]. In parallel with the aforementioned mathematical formula derived based on the SAB technique, Wei *et al.* [5] and Li *et al.* [6] provided a set of experimental formulas based on a large number of experiments. Although these experimental formulas lack theoretical analysis, they are easier to obtain and use. Numerical experiments show that the experimental formulas enable the SBM to achieve accuracy and convergence rate similar to mathematical formulas. As a mathematical supplement to the SBM theory, In Ref [14], Li derived the error bound of the SBM and explained the reason for the high accuracy and convergence rate of the SBM; namely, the OIF has a corrective effect on boundary discrete errors. The OIF is composed of two parts. The first term represents the effect of the source point on itself. The second term is considered as the correction factor for a boundary discrete error. That is the essential difference between the OIF and singular integration. Based on this understanding, Li *et al.* [15] further derived a set of physical formulas of the OIF according to physical derivation. All the MATLAB code of those OIF formulas can be downloaded in the singularity toolbox at https://doi.org/10.13140/RG.2.2.13247.00162. One interesting

APPENDIX A

Code of the Origin Intensity Factor Based on the Matlab 2016b

```
function [ff]=G_xiyi(x,y,z,nx,ny,nz,S,kappa)

%This function is to evaluate the non-singular formulation of G(xi,yi)

%This program is written by Junpu Li, Email:junpu.li@foxmail.com

%(x,y,z):coordinate of boundary nodes

%(nx,ny,nz):outer normal vector at (x,y,z)

%S:range of influence of boundary nodes

%kappa:wavenumber

len=length(x);

for ii=1:len

temp_x=x(ii)–x;

temp_y=y(ii)–y;

temp_z=z(ii)–z;

R_X=nx.*temp_x;

R_Y=ny.*temp_y;

R_Z=nz.*temp_z;

P_sjxi=(R_X+R_Y+R_Z);

clear R_X R_Y R_Z

R_R=sqrt(temp_x.^2+temp_y.^2+temp_z.^2);

C_sjxi_nsj=P_sjxi./R_R;

C_sjxi_nsj(ii)=1;
```

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```
clear temp_x temp_y temp_z
G0=sin(kappa.*(x–x(ii)))*nx(ii)+sin(kappa.*(y–y(ii)))*ny(ii)+sin(kappa.*(z–z(ii)))*nz(ii);
Q=–(–exp(R_R.*kappa.*1i)./R_R.^2+(kappa.*exp(R_R.*kappa.*1i).*1i)./R_R).*C_sjxi_nsj;
P2=G0.*Q.*S;
P2(ii)=0;
G=exp(1i.*kappa.*R_R)./R_R;
Q0=kappa.*(cos(kappa.*(x–x(ii))).*nx.*nx(ii)+cos(kappa.*(y–y(ii)))…
.*ny.*ny(ii)+cos(kappa.*(z–z(ii))).*nz.*nz(ii));
P1=G.*Q0.*S;
P1(ii)=0;
P_P=(P2–P1);
ff(ii)=sum(P_P)./S(ii)./kappa./4./pi;
end
end
function [ff]=F_xiyi(x,y,z,nx,ny,nz,S,kappa)
%This function is to evaluate the non-singular formulation of F(xi,yi)
%This program is written by Junpu Li, Email:junpu.li@foxmail.com
%(x,y,z):coordinate of boundary nodes
%(nx,ny,nz):outer normal vector at (x,y,z)
%S:range of influence of boundary nodes
%kappa:wavenumber
len=length(x);
for ii=1:len
```

```
temp_x=x(ii)–x;

temp_y=y(ii)–y;

temp_z=z(ii)–z;

R_X=nx.*temp_x;

R_Y=ny.*temp_y;

R_Z=nz.*temp_z;

P_sjxi=(R_X+R_Y+R_Z);

clear R_X R_Y R_Z

R_R=sqrt(temp_x.^2+temp_y.^2+temp_z.^2);

clear temp_x temp_y temp_z

C_sjxi_nsj=P_sjxi./R_R;

C_sjxi_nsj(ii)=1;

G0=sin(kappa.*R_R)./R_R;

Q=–(–exp(R_R.*kappa.*1i)./R_R.^2+(kappa.*exp(R_R.*kappa.*1i).*1i)./R_R).*C_sjxi_nsj;

P2=G0.*Q.*S;

P2(ii)=0;

C_xisj_nsj=–C_sjxi_nsj;

G=exp(1i.*kappa.*R_R)./R_R;

Q0=((kappa.*cos(R_R.*kappa))./R_R–sin(R_R.*kappa)./R_R.^2).*C_xisj_nsj;

P1=G.*Q0.*S;

P1(ii)=0;

P_P=(P1–P2);

ff(ii)=sum(P_P)./kappa./S(ii)./4./pi;
```

NOMENCLATURE

A	interpolation matrix	b	known right-hand vector
c	wave speed	C	sparse matrix
g	acceleration of gravity	k	wavenumber
$\mathbf{n}$	unit outward normal on the physical boundary	R	reflection coefficient
S	boundary of domain Ω	T	transmission coefficient
x	collocation point	y	source point
Ω	Computational domain	β	unknown vector
ω	angular frequency	σ	Surface charge density
δ	Dirac delta distribution	Nl	number of triangular patches
$\phi(x)$	physical variable	$q(x)$	normal derivative of $\phi(x)$
$H(x)$	Heaviside function	Tol	preset convergence criterion
A_j	area of the *j*th element	Γ_c	pseudo-boundary
G_2	absorbing parameters of the rear side of the barrier.	G_1	absorbing parameters of the front side of the barrier.
y_c	expansion center	$()_2$	the fine level mesh subscript
$P_n(x)$	Legendre polynomials of degree *n*	P_n^m	associated Legendre function
$\bar{E}^I$	incident electric field	$\bar{E}^S$	scattered electric field
∇^2	Laplacian operator	$\bar{J}$	induced surface currents
χ^k	the *kth* accurate potential residual vector	α^k	the *kth* residual potential solution
I^+	positive projection operator	I^-	negative projection operator
V^k	the *kth* approximate potential residual vector	γ^k	the *kth* accurate potential solution
$\bar{f}_n$	RWG vector basis function	λ^k	the *kth* approximate potential solution
$Rerr^k$	the *kth* local average relative error	$\begin{pmatrix} * & * & * \\ * & * & * \end{pmatrix}$	Wigner 3*j* symbol
$M_{n,m}$ and $\widetilde{M}_{n,m}$		multipole moments	
$\Phi(x_1, x_2, x_3, t)$		velocity potential	
$\mu = 4\pi\times10^{-7}$		Permeability of the surrounding free space	
$\varepsilon = 8.854187817\times10^{-12}$		Permittivity of the surrounding free space	

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Jun-Pu Li

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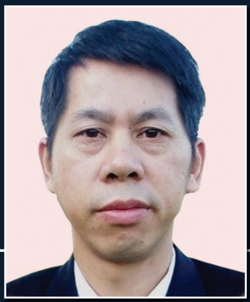

Qing-Hua Qin

Qing-Hua Qin is a Professor at Shenzhen MSU-BIT University. He has worked in several leading Australian and Chinese institutions and was awarded a Queen Elizabeth II fellow and a professorial fellowship at the University of Sydney and Australian National University. His research interests include meshless methods, hybrid finite element methods, piezoelectric materials, metamaterials, and composite materials. He has published more than 300 journal papers and 8 English monographs in the field of applied mechanics.