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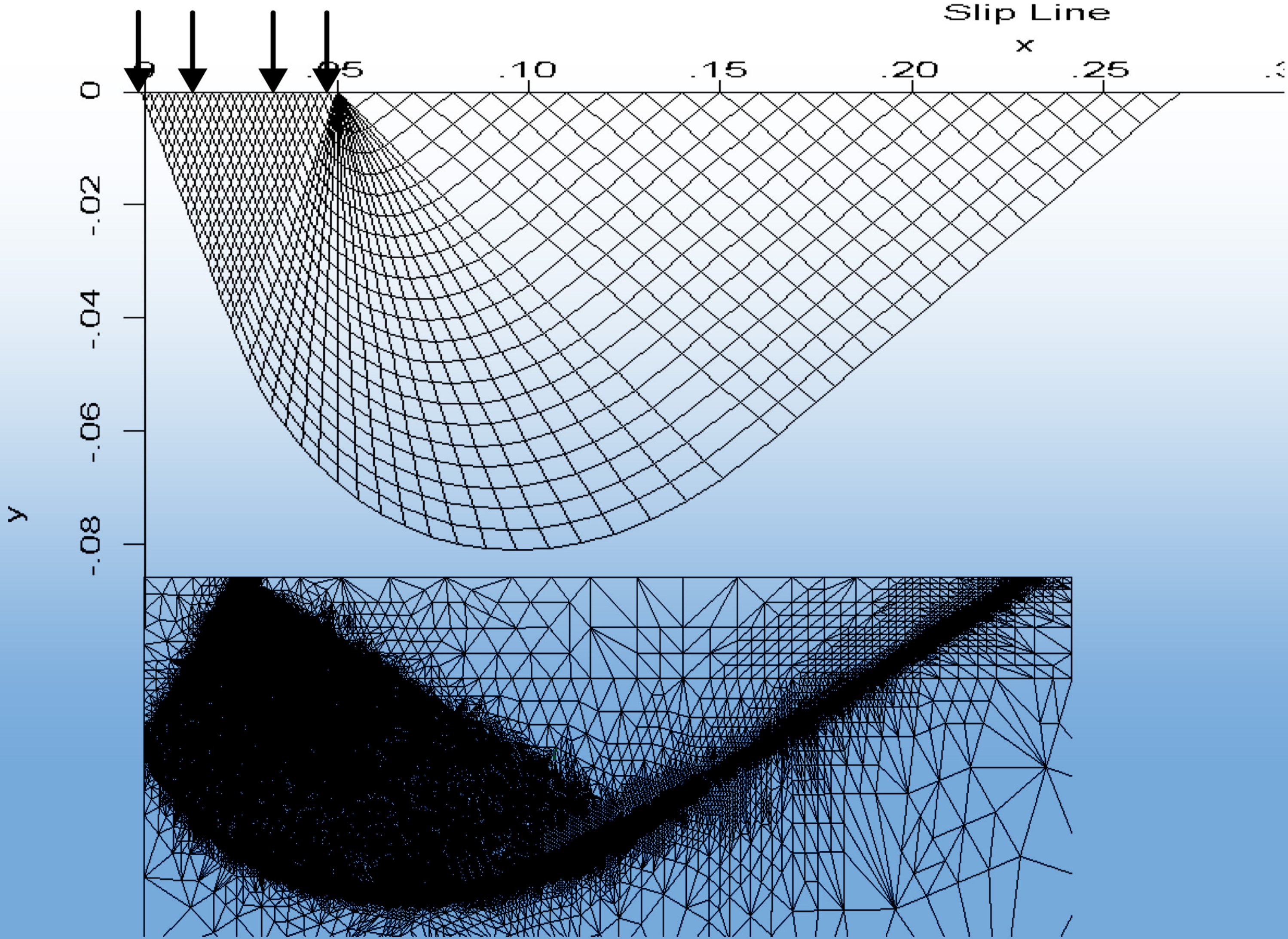
Stability of Geotechnical Structures Theoretical and Numerical Analysis



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Frontiers in Civil Engineering

Volume 1

Stability of Geotechnical Structures: Theoretical and Numerical Analysis

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PREFACE

Due to the difficulty to define the initial stress and boundary conditions, the loading paths as well as the constitutive model of geomaterials, stability analysis has always been a very important discipline in geotechnical engineering. Towards this, engineers will assess the ultimate conditions where the strength of the system is fully mobilized. The facture of failure or the collapse load will then be assessed based on the ultimate analysis which is considered to be unaffected by the initial conditions of the system.

This book will introduce the fundamental concepts and applications of plasticity theory, limit equilibrium, and limit analysis in geotechnical engineering. These concepts will be illustrated using analytical examples whenever possible in order to enhance understanding at a fundamental level and can also be used to make preliminary estimates of geotechnical stability. In parallel, suitable numerical methods and advanced computational tools will be introduced for the engineers to solve theoretical and geotechnical problems of practical interests which require greater detailed consideration.

This book deals with the challenging subject matter in a systematic fashion, from a theoretical standpoint to practice in the real world. For this reason, the book is divided into 3 parts. In part 1, the fundamental concepts in plasticity, limit equilibrium, limit analysis and instability for geomaterials are presented as a first step in introducing readers as the theoretical basis. Analytical and semi-analytical solutions are then discussed in Part 2, with liberal use of illustrative examples, as a further step to shed insights and reinforce the underlying principles embodied in the theory. Finally in Part 3, examples utilising advanced computational tools like the finite element and discrete element methods are covered for the purpose of elucidating the complexity of dealing with stability problems of the real world using numerical approaches.

A particular feature of this book is that it stresses the rigorous formulation as much as the computational techniques to tackle stability problems. It is noted that the solution of these problems is far from trivial. The search of failure load and the corresponding failure mechanism involve the constrained optimisation of discontinuous objective functions containing multiple optimum points. In short, this book is an attempt to present within a single volume the fundamentals as well as the practical developments of stability analysis in geotechnical engineering in an easily accessible manner. Most of the materials are either based on the research works from the authors or the teaching materials to the postgraduate students.

This book is aimed at researchers and engineers working in the field of geotechnical engineering having to make design decisions concerning the stability and the risk of failure of geotechnical structures. These include natural and man-made slopes, dams, shallow or deep foundations, soil retaining structures, embankments, road or railway tunnels, large scale underground structures including for underground storage of nuclear wastes or CO_2 sequestration.

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CONFLICT OF INTEREST

There is not any conflict of interest for the content of this book with any project, universities or parties.

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CHAPTER 1

Introduction

Abstract: Stability analysis has always been a critical issue in geotechnical engineering. Many of the design works by engineers are actually based on the ultimate condition which can be assessed without the need of the initial condition and sophisticated constitutive models. A brief introduction about the stability analysis will be given in this chapter.

Keywords: Close-form solution, Finite element, Limit analysis, Limit equilibrium, Stability analysis.

1.1. INTRODUCTION

Due to the difficulty to define the initial stress and boundary conditions, the loading paths as well as the constitutive model of geomaterials, stability analysis has always been a very important discipline in geotechnical engineering. Towards this, engineers will assess the ultimate conditions where the strength of the system is fully mobilized without the detailed information about initial condition. In fact, it is extremely difficult and expensive to determine the initial condition for a general problem, even if the ground condition is simple, and no continuous results will be obtained even if explorations are carried out. The initial condition can be affected by the soil formation process, development and loading of the site, ground water and other surrounding effects and possibly many other factors. The failure or the collapse load will be assessed based on the ultimate analysis which is considered to be unaffected by the initial conditions of the system. Although this approach appears to be highly simplified without considering the initial condition as well as the constitutive model of soil, it appears to be indispensable for most of the practical problems. Furthermore, the engineers

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are now well familiar with this approach, and many design and analysis have been carried out successfully with this approach.

This book will introduce the fundamental concepts and applications of stability analysis with particular reference to plasticity theory, limit equilibrium, limit analysis, finite element and discrete element methods in geotechnical engineering. Suitable numerical methods and advanced computational tools will be introduced for the engineers to solve theoretical and geotechnical problems of practical interests which require greater detailed consideration.

There are various advancement on stability analysis over last 40 years, and many results are developed and used for a variety of problems. It is impossible to cover all these results in the present work. This book is mainly based on the research and teaching materials by the authors, but sufficient background about the works by other researchers are also given for the readers.

1.2. BACKGROUND

Due to the growth of population and economic activities, terraces are created for buildings and infrastructures like quays, canals, railways and roads. Man-made cut and fill slopes have to be formed to facilitate such developments. In the past, the stability of many slopes, foundations and retaining structures are assessed by simple rules of thumb, due to the lack of adequate fundamental knowledge and computing power. There are various attempts to improve the rules of thumb approach in 20 century. One of the earliest attempts was by a French engineer Alexander Collin (Collin, 1846) which is not really better than the rules of thumb. In 1916, a series of quay failures had occurred in Sweden, and the Swedes had developed one of the earliest methods to assess slope stability using the method of slices and limit equilibrium method. The method is now called the Swedish

Method (or the Ordinary Method) of Slices (Fellenius, 1927) which is still used in limited sense up to present. A number of subsequent refinements to the method were made later: Taylor's stability chart (Taylor, 1937) based on moment equilibrium; Bishop's Simplified Method of Slices (Bishop, 1955) which is also based on the moment equilibrium; Janbu's method which is extended to the non-circular slip (Janbu, 1973); Morgenstern & Price (1965) which try to ensure forcing moments and forces to be simultaneously achieved; Spencer's parallel inter-slice forces (1967) as a special case of the Morgenstern & Price's method; and Sarma's method which is based on a horizontal earthquake approach (1973). These various methods are now basically unified under the Modern Generalized Method of Slices (GMS) (*e.g.* Low *et al.*, 1998).

In the classical Limit Equilibrium approach, the user has to define a slip surface before the stability analysis. There are different techniques to ensure a critical slip surface can indeed be identified. The finite element method (Griffiths & Lane, 1999) or the equivalent finite difference method (Cundall & Strack, 1979) are some of the modern computational methods which are used to evaluate the stability problems directly using the strength reduction algorithm (Dawson *et al.*, 1999). Zhang (1999) and others have proposed the rigid finite element method which is still limited to research purpose for the stability analysis up to now. The advantage of these methods is that there is no need to assume any inter-slice forces or slip surface which have to be prescribed in the classical limit equilibrium or limit analysis, but there are also various limitations to these methods.

Currently, most of the engineers are using computer methods which are commonly limit equilibrium or finite element method to solve different kinds of stability problems. However, every numerical method has its own assumptions and limitations. It is therefore necessary for the engineers and researchers to be fully aware of them so that the methods can be used within its limitations in real design situation. There is however one fundamental

CHAPTER 2

Upper and Lower Bound Approaches

Abstract: In this chapter, the basic theory about the lower and upper approaches will be introduced. These approaches will then be applied to various types of problems, and the applicability of the lower and upper bound approaches will be illustrated through many types of problems, for which analytical/rigorous solutions are available.

Keywords: Failure mechanism, Lower bound, Upper bound, Yield.

2.1. INTRODUCTION

To determine the collapse load of a structure composed of an elastic-plastic material and subject to a set of external forces (volume forces, surface tractions), there are essentially two different approaches: (1) an incremental load path analysis taking into account the complete stress-strain behaviour or (2) a direct limit analysis only taking into account the failure condition.

The first approach consists of performing an incremental elastic-plastic analysis up to failure to determine the collapse load. The material behaviour in terms of the stress-strain relationship up to failure is accounted for. Numerical computations are usually performed using computer programs based on finite elements or finite differences. Here failure is considered to take place when a representative displacement tends to increase without bound. Examples of this approach using finite element method can be found in Nagtegaal *et al.,* (1974), Chen (1975), Sload and Randolph (1982), Alehossein *et al.,* (1992), Yu *et al.,* (1993) and Potts and Zdravkovic (1999). The main inconveniences of this approach are that: it requires accounting for the complete stress-strain relations hence the amount of computations

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involved is usually large; it needs the material properties for each material component which are difficult and expensive to obtain; failure load is determined when the computer program diverges and stops to operate normally.

Alternatively, a second approach uses the bound theorems which only takes into account the strength criterion of the material, but not the stress-strain behaviour. The material strength is supposed invariant during straining in order that the method works; locally failure and yield are synonymous and is described by a yield function. This approach leads directly to the determination of the collapse load, requiring much less amount of computation hence cost-effective, which constitutes its main advantage. Note that in civil engineering problems, structural collapse is normally the main concern. Therefore, the non-consideration of displacement and strain fields is in many cases not an important issue, especially at the preliminary design stage where a choice needs to be made among different alternatives.

Chen (1975) and Chen and Liu (1990) used this approach to geotechnical problems. A more theoretical treatment of this subject can be found in the monograph of Salençon (1983). For simple geometries and homogeneous material domains, analytical solutions are available. A very classical and illustrative problem concerns the tunnel face stability for which many researchers have made contributions, for example: Davis *et al.*, (1980), Chambon and Corté (1994), Leca and Dormieux (1990), Subrin and Wong (2002), Wong and Subrin (2006), only to cite a few. For complex geometries and heterogeneous material domains, numerical procedures are necessary. A brief account on numerical bound analysis based on finite elements and linear programming can be found in Yu (2006), which summarized the contributions of previous authors like of Lysmer (1970), Bottero *et al.*, (1980), Sloan (1988, 1989), Sloan and Kleeman (1995). These works are based on a linearization of the strength criterion. More recently, non-linear programming was introduced: D.Z. Li and Y.M. Cheng (2013).

In this chapter, emphasis will be put on a clear presentation of fundamental tools and concepts. We therefore limit ourselves to simple geometries where analytical solutions are accessible. For numerical approach, the reader can refer to the work of Yu (2006).

2.2 MATERIAL STRENGTH AND ITS MATHEMATICAL DESCRIPTION

As aforementioned, the bound-approach to the determination of collapse loads is based on the consideration of material strengths, discarding the stress-strain relation. It is therefore appropriate to begin with the mathematical description of the admissibility condition of stress fields, at the same time fixing notations for future references.

At any material point inside the physical domain, in order that the stress be supportable by the material, it must remain inside a domain of "admissible stresses" G as shown in Fig. (**2.1**) which is a subdomain of the six-dimensional Euclidean space $\mathbb{R}^6$ for 3D problems. Restricting the analysis to isotropic materials, the spatial orientation of the stress tensor does not count, only 3 independent scalar invariants representing the stress intensities intervene, for example the 3 principal stresses. The domain G can then be represented as a subdomain of the three-dimensional Euclidean space $\mathbb{R}^3$. Thermodynamic stability conditions require this domain to be convex, which is commonly described using a convex function $f(\boldsymbol{\sigma})$ such that:

$$G = \{\boldsymbol{\sigma} \in \mathbb{R}^3 : f(\boldsymbol{\sigma}) \leq 0\} \qquad \textbf{(2.1)}$$

If the physical domain is not homogeneous, then G would be a function of space coordinates: $G = G(\boldsymbol{x})$, and so is the case of the yield function: $f = f(\boldsymbol{\sigma}, \boldsymbol{x})$. To simplify the presentation, we will omit this dependence on x and continue to write $f = f(\boldsymbol{\sigma})$, unless this dependence is explicitly required.

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CHAPTER 3

Slip Line, Limit Equilibrium and Limit Analysis Methods

Abstract: In this chapter, three major stability analysis methods are introduced. Each method will be discussed with many recent findings discussed. Based on the slip line method and extremum limit equilibrium method, the classical slope stability, lateral earth pressure and bearing capacity problems are unified under one formulation. A Fortran code is also provided for the lateral earth pressure analysis based on the limit analysis approach, and this code will be useful to many readers.

Keywords: Limit analysis, Limit equilibrium, Extremum principle, Slip line, Unification.

3.1. INTRODUCTION

For stability analysis, there are various methods available to the engineers, and the choice of the method will depends on the complexity of the geometry and the convenience in the solution. In this chapter, the slip line method, limit equilibrium method and limit analysis will be introduced for the lateral earth pressure, ultimate bearing capacity and slope stability problems. It is interesting to note that these three topics are usually considered separately in most of the books or research studies, and different methods of analyses have been proposed for individual problem even though they are governed by the same requirements for the ultimate conditions. Since the governing equations and boundary conditions for these problems are actually the same, Cheng and Li (2015) view that each problem can be viewed as the inverse of the other problems which will also be demonstrated in the present chapter. After the introduction of the three

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basic stability analysis methods, a section on the unification of the three most important stability problems will be discussed.

The three stability methods together with the corresponding numerical solution techniques will be discussed with the use of different computer programs developed by the author. The limit equilibrium methods as discussed in this chapter are available in the program SLOPE 2000 developed by Cheng, which can be downloaded at the site http://www.cse.polyu.edu.hk/~ceymcheng/. SLOPE2000 is also one of the analysis modules in the large scale geotechnical analysis and design package GEOSUITE 1.0/2.0. For the slip line and limit analysis programs which are used in this chapter, they are more suitable for single material problem and are not yet mature enough for general conditions, hence these programs are not available for general download but can be obtained from Cheng at ceymchen@polyu.edu.hk.

3.2. SLIP-LINE METHOD

At the ultimate condition, both equilibrium and yield conditions must be satisfied. Combining the Mohr-Coulomb yield criterion (which is generally adequate for soil) and the equilibrium equations, a set of hyperbolic partial differential equations of plastic equilibrium can be developed. In order to solve the governing partial differential equation, it is more convenient to transform the governing equations to curvilinear coordinates along the directions of the failure planes for mathematical convenience. Once the equations are solved, the failure modes with the corresponding systems of stresses will be automatically determined. The slip directions or slip lines constitute a network which is called slip-line field. The governing equations can be solved with adequate boundary conditions to investigate the stresses at the ultimate condition, and the solution of the problem is commonly taken as the rigorous solution, as the solutions are either similar to those from other methods or are better. Since the governing equations are written along the slip lines, the slip line fields corresponding to the solutions are commonly considered as the failure mechanism of the governing problem. For example, the bearing capacity of footing and the lateral earth pressure behind a retaining wall are commonly analyzed by the slip line analysis, but not for the slope stability problem.

Kötter (1903) was the first to derive the slip-line equations for two-dimensional ultimate problems, while Prandtl (1920) was the first to obtain an analytical solution for footing by assuming the weight of soil to be negligible. His results were then applied by Reissner (1924) and Novotortsev (1938) to different problems on the bearing capacity of footing on weightless soil. The inclusion of soil weight in the solution of the governing partial differential equation is analytically impossible, and Sokolovskii (1965) has proposed a finite difference approximation of the slip-line equations for which the accuracy can be further improved by an iteration scheme (Cheng 2003b), and such iteration to update the coordinates of the grid points on the slip line field has been demonstrated to be important for passive pressure evaluation. Sokolovskii (1965) has solved many types of problems on the bearing capacity of footings, slopes as well as the lateral earth pressure on retaining walls. De Jong (1957) has developed a graphical procedure for the solutions which appeared to be seldom used nowadays. There are other approximate solutions for the governing differential equations which include the applications of perturbation methods (Spencer, 1962) and series expansion methods (Dembick *et al.*, 1964), but these methods are not popular and versatile enough for more complicated problems and are seldom considered now. More recent results and numerical techniques are given by Cheng and Au (2005) for bearing capacity problem and Cheng (2003b) and Cheng *et al.*, (2007b) for lateral earth pressure problem.

CHAPTER 4

Numerical Methods – Finite Element and Distinct Element Methods

Abstract: In this chapter, the basic theory about the two most important numerical methods in stability analysis are introduced. After that, these two methods are applied to different cases, and some laboratory tests are also used for comparison. In general, it is found that both methods are useful, and each method has its own merits and limitations.

Keywords: Distinct element, Failure mechanism, Finite element, Numerical method.

Most of the problems in geotechnical engineering are difficult to be solved by nature. In particular, the irregular geometric domain, nohomogeneous ground conditions, the presence of water, external and internal loadings and structural elements and the complex mechanical response of soil and soil-structure interaction have created extreme difficulty in the analysis of real engineering problems. Towards this, the use of numerical method will be more appropriate. Currently, the use of finite element, finite difference, boundary element and distinct element methods are the most popular methods which are adopted by many engineering geotechnical programs. Boundary element method is most suitable for linear problem with elastic constitutive behavior, and it is not commonly adopted in geotechnical problems. The capabilities of finite difference and finite element methods are comparable, but there are only few general finite difference programs in the world due to various technical difficulties. Finite element method is the most popular numerical tool to solve many types of differential equations in various disciplines. Currently, there are hundreds of finite

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element programs available to the engineers for various applications, but there are also many limitations of FEM to geotechnical problems which include: (1) difficult to be applied for very large displacement problems, (2) generation and loss of contacts are difficult to be modelled, (3) fracture is difficult to be handled (but not impossible), (4) flow of the materials. Towards these limitations, the finite difference based distinct element method may be more suitable, though there are also many fundamental limitations in DEM which are still difficult to be resolved. There are also coupled methods based on DEM and FEM, and DEM is applied in special highly stressed/fractured region while FEM is applied to the general medium. In general, FEM and DEM are the two most important numerical methods in geotechnical engineering, and these methods will be introduced in this chapter. After the basic introduction to these two numerical methods, applications to some stability problems will be discussed.

4.1. PLANE STRAIN AND PLANE STRESS FOR CONTINUOUS PROBLEMS

For simplicity, two-dimensional problem will be discussed here. The extension of two-dimensional analysis to three-dimensional analysis is possible and is covered in many books which will not be repeated here. A problem is two-dimensional if the field quantities such as stress and displacement depend on only the two coordinates (x, y). In this sense, there are strictly no two-dimensional problems because every structure or loading system is three-dimensional in the real world. Many engineering problems can however be simplified approximately to a plane problem. Such approximation will greatly reduce the computational effort and can yield results satisfying practical requirements at the same time. Generally speaking, there are two kinds of elastic plane problems (Barber 2010): plane strain and plane stress, which will be introduced in the following subsections.

4.1.1. Plane Strain

To illustrate the problem, Fig. (**4.1**) shows a section of a structure with its cross-section independent of its length, except for the body force. On the surface of the structure, there may be surface force or constraint. It should be noted that both the internal and external forces are parallel to the cross-section and the values of them are invariable with the length normal to the section.

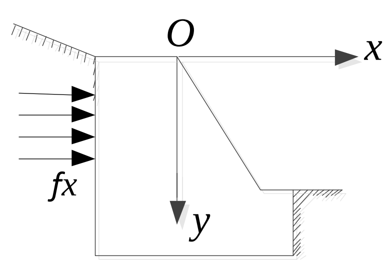

Fig. (4.1). Cross-section of an infinite long structure with external force.

Assume the length of the structure to be infinite, and consider any one of the cross-sections as the *xy*-plane, to which *z*-axis is perpendicular. All the field quantities (*e.g.*, stress, strain, and displacement, *etc.*) depend only on the two coordinates (*x*, *y*). Additionally, any cross-section is a symmetrical surface so that any point in this surface will only have *x*-displacement and *y*-displacement (*i.e.*, *z*-displacement is 0). For this condition, it follows that

$$\gamma_{zx} = \gamma_{zy} = \varepsilon_z = 0 \quad \textbf{(4.1)}$$

In view of the stress-strain relations, we can deduce that

$$\tau_{zx} = \tau_{xz} = 0, \tau_{zy} = \tau_{yz} = 0, \gamma_{xz} = \gamma_{yz} = 0 \quad \textbf{(4.2)}$$

Therefore, all the shear strain components equal to 0 except those related to *xy*-plane (*i.e.*, $\varepsilon_x, \varepsilon_y$ and γ_{xy}). This two-dimensional state is known as the plane strain condition.

CHAPTER 5

Numerical Techniques for Solution of Stability Problems

Abstract: Analytical solutions to most of the geotechnical problems are not available unless the geometry under consideration is highly simplified. For practical purposes, numerical method is indispensable. In this chapter, the numerical methods for some of the stability methods as discussed in previous chapters are elaborated.

Keywords: Discretization, Limit analysis, Limit equilibrium, Optimisation, Work done.

In the past few decades, great strides have been made in the computational methods of stability analysis. This chapter briefly discusses three main numerical-based methods for performing geotechnical stability analysis, namely:

(a) limit equilibrium,
(b) the displacement finite element method and
(c) lower- and upper-bound finite element limit analysis

5.1. LIMIT EQUILIBRIUM

Generally speaking, limit equilibrium methods assume that failure occurs across a predefined slip surface. It is a commonly used method for stability analysis in geotechnical context due to its simplicity, but the method may also be inadequate to analyse complex failure mechanisms not well defined by a failure surface (for instance progressive creep, soil liquefaction and brittle fracture). Sloan (2013) summarizes some shortcomings of the limit equilibrium technique as follows:

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(a) The resulting stresses do not satisfy equilibrium at every point in the domain.
(b) There is no simple means of checking the accuracy of the solution.
(c) It is hard to incorporate anisotropy and inhomogeneity.
(d) It is difficult to generalise the procedure from two to three dimensions.

One of the most common applications of the limit equilibrium method (LEM) in geotechnical engineering is for slope stability analysis. Several limit equilibrium methods have been developed for slope stability analysis and many are covered in standard geotechnical textbooks. These include the widely used ones: Bishop and Simplified Bishop method of slices (1955), Ordinary or Fellenius method of slices (1936), Spencer, Janbu (1973), and Morgenstern and Price (1965) methods. Broadly speaking, all limit equilibrium methods presuppose a failure surface and define the factor of safety, *F*, as the factor by which the shear strength of the soil would have to be factored down to bring the slope into a state of limiting equilibrium, that is to the limit of stability so that it is on the verge of failure. Thus, the factor of safety is given as,

$$F = \frac{\text{shear strength of soil}}{\text{shear stress at limiting equilibrium}} = \frac{\tau_{\text{ult}}}{\tau} \qquad \textbf{(5.1)}$$

that is,

$$\tau = \frac{\tau_{\text{ult}}}{F} = \frac{c + \sigma_n \tan\phi}{F} \qquad \textbf{(5.2)}$$

where c is the cohesion, ϕ is the internal friction angle and σ_n the normal stress. It is customary to apply a common "*F*" for both c and ϕ although it is not necessary the case. As the number of equations in limit equilibrium is less than the number of unknowns in slope analysis, the problem is in fact statically indeterminate. Various assumptions are then made in the various methods to render the problem determinate. Fellenius and Simplified Bishop methods make assumptions regarding the horizontal or vertical force equilibrium, while Bishop, Spencer, Janbu, and Morgenstern and Price methods satisfy all conditions of equilibrium.

As Duncan (1996) pointed out, the advent of computers has dramatically changed the computational approach for analysing slope stability. One of the most consequential changes is in respect of numerical optimisation techniques for determining the critical slip surface, as discussed below.

5.1.1. Critical Slip Surface

A fundamental part of slope stability analysis is to determine the slip surface that has the lowest, therefore the most critical, slip surface be it a circular or a non-circular surface. The trial slip surface is generally a pre-defined shape consisting of straight line segments or smooth curve or both. Search methods employing variational calculus (Baker and Gaber, 1978), dynamic programming (Baker, 1980), alternating variable methods (Celestino and Duncan, 1981), Monte Carlo technique (Greco, 1996) and genetic algorithm (Goh, 1999) are then used to identify the critical surface. The problem can be posed as illustrated in the following example (*e.g.* Sun *et al.*, 2008; Malkawi, 2001).

Fig. (**5.1**) shows a trial slip surface represented by n nodal points $(x_1, y_1), (x_2, y_2), \cdots\ (x_n, y_n)$ and n-1 segments. For sake of minimizing the number of variables, the nodal points are constrained to the same spacing along the abscissa so that,

$$x_i - x_{i-1} = \Delta x = \frac{1}{(n-1)}(x_n - x_1) \qquad i = 2, \cdots, n-1 \qquad \mathbf{(5.3)}$$

and the coordinates of the nodal point *1* and *n* on the slip surface can also be defined using the surface profile $y = h_0(x)$ (see Fig. **5.1**), namely:

$$y_1 = h_0(x_1); \qquad y_n = h_0(x_n) \qquad \mathbf{(5.4)}$$

CHAPTER 6

Applications of Plasticity Theory and Limit Analysis to the Bearing Capacity of Shallow Foundations

Abstract: In this chapter, the plasticity theory (slip line analysis) and limit analysis will be applied to the bearing capacity problems for illustration. Through these applications, the readers will be able to strengthen their understanding on the various types of stability analysis methods.

Keywords: Axisymmetry, Bearing capacity, Footings/foundations, Geotechnical centrifuge modelling, Method of characteristics, Numerical modelling, Physical modelling, Plane strain, Plasticity, Sands, Shear strength, Silts, Stress analysis.

6.1. OUTLINE

The method of characteristics which has been introduced in chapter 3 is used to establish consistent factors for the vertical bearing capacity of circular and strip footings on soil which satisfies the linear (c, Ø) Mohr-Coulomb strength criterion. This method of solution avoids the assumption of arbitrary slip surfaces, and failure zones are automatically generated within which equilibrium and plastic yield are simultaneously satisfied for the given boundary stresses. Although similar solutions have previously been published for circular footings, their application has been hindered by errors and confusions over terminology which are now resolved and is explained by the method of solution in this chapter. For bearing capacity problems, it is well-known that Terzaghi's superposition of bearing capacity terms containing N_q, N_γ, and N_c is both

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safe and sufficiently accurate for circular footings as well as for strip footings. The values to be adopted are tabulated as functions of Ø for ease of application. It is also found that the differences between the factors applicable to circular and strip footings far exceed the empirical shape factors commonly in use. Some new shape factors are suggested that can better represent the relationship between the limiting equilibrium of circular and strip foundations. Some current shape factors attempt to relate the axisymmetric (triaxial) and plane strain soil parameters. This approach cannot succeed, as the relationship between strength parameters depends also strongly on the relative density. The new bearing factors which are proposed facilitate a more rational approach in which the soil parameters appropriate to the geometry can first be determined and then used to find the appropriate bearing capacity factors.

The solutions so far were based either on a straight (c, Ø) envelope or simply on a constant angle of shearing, Ø. For granular soils, sec Ø usually varies linearly with the logarithm of mean effective stress. A new method of calculation permits Ø to vary throughout the stress field as an arbitrary (or empirical) function of stress. This method is verified for both plane-strain and axisymmetric conditions by forcing a variation in sec Ø equivalent to generating a constant-cohesion envelope, for which solutions already exist. The variable- Ø analysis is used to demonstrate the highly significant effect of stress variation around and beneath a footing. Finally, it is shown that an equivalent constant value $Ø_m$ can be derived empirically, using the new solutions to identify an equivalent mean effective stress p_m. However, only the variable- Ø solution can simultaneously capture the bearing capacity and the geometry of the bearing mechanism.

This approach is validated here for the case of model circular footings on dense beds of silica sand and silica silt. The models were tested at 1 **g** with surcharge to explore the N_q behaviour, and in a centrifuge to determine selfweight effects for N_γ. It is shown that triaxial Ø values expressed as a function of the logarithm of p can be used to predict model bearing capacities within a deviation in Ø of 2°.

It should be noted that part of this chapter has previously been published in Canadian Geotechnical Journal, 1993, 30(6): Geotechnique, 2011, 61(8): 627-638 and Geotechnique, 2011, 61(8): 634-650.

6.2. INTRODUCTION

The derivation of bearing capacity for foundations for frictional soil requires the relation between the strength parameters and the effective stresses, and then on the use of bearing capacity factors. This chapter will establish the approach of modelling the strength envelope by a simple constant- Ø relation; which is later expanded into a more general (c, Ø) envelope. The objective is to derive the corresponding estimates of the bearing capacity factors in both plane and axisymmetric load cases under a more realistic formulation.

The method of characteristics proposed by Sokolovskii (1960) is used here. This method assumes that limiting stresses have been reached at every point inside the solution domain, and the plastic equilibrium is determined with the applied load. The requirement of a trial slip surface as adopted in the limit equilibrium analysis by Terzaghi (1943) and Meyerhof (1951) can then be avoided. There are various doubts regarding the method of characteristics which include:

(i) The difficulty in the assigning boundary conditions, particularly where the mobilisation of the tangential friction should be such as to oppose the relative motion and when the kinematics of plastic soil strain is itself uncertain.

(ii) The difficulty in accepting the assertion that certain zones (*e.g.* in

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